Teacher's Guide Level 4

Metric Edition

SRA MATHEMATICS LEARNING SYSTEM TEXT

Teacher's Guide

Preface

How was the SRA Mathematics Learning System *developed?* Writing was almost the last step. We started by listening.

We visited schools of all kinds, from the inner city to remote rural areas. We sat in classrooms. Teachers, children, and parents told us about what they liked and didn't like about math programs.

We asked an independent research organization to interview supervisors and administrators. We talked to SRA Staff Associates about the needs they saw.

We reviewed all major basal math series. We studied standardized tests to see what children might be expected to know at various ages. Consultants evaluated existing SRA programs.

We also analyzed recommendations and reports from curriculum study groups, state and city adoption committees, and researchers in a variety of fields.

Then our authors, editors, and consultants worked together to prepare the rationale and learning objectives for the SRA Mathematics Learning System. Writing of the program did not begin until the entire scope and sequence had been defined by the learning objectives.

The manuscript was continuously reviewed, discussed, and revised by our development team. But it's arrogant for adults to sit in an office and predict what will work in the classroom. We needed answers to three questions:

- Will pupils attain the objectives?
- Will pupils develop positive attitudes toward the program?
- Will teachers find the program easy to use?

Our next step was to undertake two years of prepublication tryouts. We carefully selected classrooms across the United States and Canada to represent the broad range of pupil abilities, family backgrounds, and teaching styles. We visited, surveyed, listened, and tested. We rewrote and revised and retested before going to press with the program you see today.

We're confident that you'll find the SRA Mathematics Learning System effective, enjoyable, and easy to use. One of our tryout pupils wrote:

I liked the program and nothing was hard or dificalt. I think the book should come out to the world.

What makes the SRA Mathematics Learning System *a "system"?* The word *system* has many definitions. We call the SRA Mathematics Learning System a system because of the following five characteristics:

1. The entire program is based upon well-defined learning objectives.
2. Although the program can be enriched in many ways, the texts are complete in themselves. The teacher is not required to use any other materials.
3. There is a comprehensive evaluation program in each text.
4. Learning alternatives are provided for teachers and pupils who wish to use them.
5. The program provides information about the learners that will guide the teacher in altering or expanding a learning sequence.

What kind of objectives are there? Objectives are given for each chapter. Key to the program, however, are the year-end mastery objectives. These are the goals toward which instruction is directed.

Distinguished educator Ralph Tyler helped with the difficult task of defining learning objectives. He told us: "Remember the purpose of objectives. They are to guide, not dictate. Think of them as goals to be reached as a result of the teaching-learning process.

"Keep the number of objectives for any level under thirty, if possible. The teacher should be able to remember them all. A teacher who has to search through hundreds of objectives to figure out what to do cannot be free to teach anything more than bits and pieces.

"Avoid fashionable formulas for writing objectives. Fashions change. Avoid jargon. Keep the language simple. Objectives have to say something or their value is lost."

I liked the whole book because it told me things that I didn't even know about and it was very interesting

What are the texts like? The SRA Mathematics Learning System focuses on the real world and develops many concepts from real-world situations.

Compared with other programs, this program spends a longer time on an idea and its related skills before introducing another topic. This gives skill competency a better chance to develop.

I like the S.R.A. math book because it's not just plane old 5 x 3 it you can gives you more time to think.

There are many invitations for pupils to *think* as well as to do. Not all questions are meant to be answered. Some questions have many answers; some have none.

I liked this section. They had fun math sentences And they maid you think.

Math is getting a lot eazer and its funner to; math realy is fun

The language is informal. Purists may object to this departure from standard textbook English, but our tryout pupils responded enthusiastically to the style. Instead of stressing technical vocabulary and symbols, the program emphasizes skills and nonverbal understanding.

No pupil should have to moan, "Aw! It's the same old stuff," when he flips through his math book. The pages of the SRA MATHEMATICS LEARNING SYSTEM are varied. They're lively. They look like fun.

How are the texts organized? There are three major types of chapters.

Exploratory In an exploratory chapter pupils play with a big idea, think about it, and share their own ideas. The necessary vocabulary is introduced, along with some of the notation and operations related to the major idea.

Instructional In this type of chapter pupils begin the serious business of acquiring skills. Ideas are carefully sequenced into learning steps, and each learning step is accompanied by practice.

Review Here pupils must demonstrate understanding and skills. There is a review of the learning sequence, along with opportunities to explore applications.

No one text contains all three types of chapters for a single concept strand. For example, there are exploratory and instructional chapters on addition of whole numbers in level 1. The instructional chapters continue through level 4, and the review chapters start at level 5.

How are pupils evaluated? The evaluation program is built into each text. It allows a learner to check his own progress and determine his own strengths and weaknesses.

The first pages of an instructional chapter contain an informal survey to find out what the learner knows about the chapter to come. These pages indicate what the chapter is about, and they help to define the learning goal of the chapter.

The tests within an instructional chapter are called Progress Checks. A Progress Check identifies the knowledge that is a prerequisite for further work.

After learning from your math book I found this test very easy I found it easy because I learned from your book this is what made it easy.

A Progress Check can serve two functions:

1. If a pupil has gone through the preceding instructional pages, a Progress Check tells whether or not he has acquired the appropriate knowledge and skills. If he hasn't, he should try other kinds of instruction.
2. If a pupil seems to have prior knowledge of the chapter, as indicated by the survey at the start of the chapter, he may go directly to the Progress Check. It will determine if the preceding instructional pages can be skipped. If he's not successful on the test, he simply goes back to pages he skipped.

At the end of every chapter a Checkout lets the pupil and teacher know whether the learning objectives of the chapter have been reached.

What learning alternatives are there? There's a limit to the number of pages in a textbook. Beyond a certain length the book becomes too expensive and hard to use. So only a few pages can be devoted to extras for pupils who need more help or who would benefit from additional activities. To make sure these important extras are available, we've supplied a wealth of reinforcement and extension activities in the Teacher's Guides.

The SRA MATHEMATICS LEARNING SYSTEM *was built on some simple convictions.* Mathematics is relevant and vital. It's useful and interesting. Everyone should relax and enjoy it.

I like math a lot more than I did last year.

Some of the greatest learning opportunities are found in everyday things and are discovered when people talk together.

I liked the chapter becouse I think we learned the most and becouse we had a lot of discussions.

Mathematics doesn't have to be formal and abstract to be good mathematics.

I think this math is very good because I know what I'm doing

Everyone – students and teachers – should succeed in mathematics.

I liked it very much I never learned this kind of measures but I think it is good that you wrote it in the book because soon America will learn ~~the~~ a new kind of measure and you already are teaching it to us.

A textbook can't transmit the joy of learning as well as an enthusiastic teacher, but it can help.

Notes & Things

We have asked a lot of questions. We have listened to the answers. One set of questions had to do with the organization of a teacher's guide.

Teachers told us that they wanted the guide pages numbered the same as the pupil pages. That was not an easy request to handle, since there are teacher resource pages at the end of each chapter and some pages of introduction to the next chapter. But the request made sense, so we put the alphabet to work.

The pupil page number repeats at the end of a chapter like the tune of a broken record; but attached to the number is a letter, which changes to accommodate those extra guide pages. When we finally get to the first pupil page of the next chapter, the guide page number will once again match the pupil page number. This whole thing sounds terrible. But flip through the book. Keep an open mind. It won't be so bad once you get used to it.

As we listened to teachers we found that there were many possible classroom organizations. They varied from large, heterogenous grade-level groups to small, homogeneous groups composed of pupils of different ages. It was impossible to tailor a guide to fit all kinds of organizations. But it was possible to let you know the special features of each page so that you can quickly decide how you want to use that page.

Each pupil page appears, slightly reduced in size, on the guide page. And the answers are right in place. You will find comments beside each pupil page. They will be in categories like these:

lesson The pages indicated here are related but need not all be presented on the same day. They provide the continuity of experience necessary to get an idea established.

goal The words needed to turn this goal into a behavioral or performance objective have not been printed, although they could have been. The goal simply helps you pinpoint the learning task for the page.

memo These words will tell you if a discussion is needed to get the learners started in the right direction. Or they may warn you about a potential problem or provide an explanation of why something is done the way it is. Or they may be simply a suggestion to make your work easier.

things All materials that you'll need for the page activities are listed here.

warm-up You can guess where this label came from. If the mathematics idea is a new one or maybe a hard one, a suggestion is made that will help get the learners ready to learn.

page 1 The comments about the page itself may be introduced with traffic-light colors and will mean about the same things.

Caution—everyone needs most of this information, but the use of the page will vary according to the ability of individual pupils.

Stop—think about which pupils should do this activity. Your independent learners will have no trouble, but your pupils at the other end of the performance scale may benefit much more by doing other, more appropriate activities.

As you flip through the pages you will see a few sentences in *italic type*. These sentences are intended to give you some ideas about the words you might use to talk about the page. (How many times have we all thought, "I understand, but I don't know what words to use to explain it"?) In talking, probably nobody would use the exact words that are given. They are not meant to be a script. They are offered as a guide, nothing more.

You will find the new concept-development words in SMALL CAPITAL LETTERS, and some other functional words that deserve emphasis will be in **this kind** of type.

Now flip through the book again. Notice the copy below the pupil pages. There are 3 types of activities. Each type has its special symbol—one of the international traffic signals.

The old familiar red *stop* sign will signal an activity for the youngster who simply needs more work before he goes on to a new learning goal.

The yellow *divided highway* sign signals an activity for those youngsters who can accept a challenge beyond the learning expectations of the page.

The new, blue *rest area ahead* sign signals an activity you can use with any group. This type of activity departs from the standard math work and gives everyone a chance to have a break and hopefully some fun.

Here you will find the ideas that will extend the lessons and suggestions that offer specific help for those children who need it. Each activity was carefully selected so that you could have something special to personalize each child's learning.

You'll find a Resource Section at the end of each chapter. This provides alternate forms of all progress checks and the checkout and still more activities. Not every Resource Section is the same, but you will find references to additional learning aids that may be in your school and are just right to use. At the end of some chapters you will find a description of these learning aids. At the end of the guide you'll find a bibliography that lists children's books and film medium references. These features were planned to help you with your job of teaching and the children's job of learning.

A glossary of mathematical terms used in the text appears at the end of each pupil book. It is also reproduced at the end of each of the Teacher's Guides. Why not take time to look at it now, so you can get an overview of how the SRA Mathematics Learning System uses math words?

Canadian SI Edition of the SRA Mathematics Learning System

The Canadian SI pupil texts of the SRA Mathematics Learning System have been revised to be fully in accord with the specifications of the Canadian Metric Commission, the Canadian Government Specifications Board, and with teaching requirements of Canadian schools. In revising the Teacher's Guides, we have been governed by considerations of economy. In order to keep costs to schools at a reasonable level, the revision of the Teacher's Guides has been limited to what is necessary for teachers.

Where significant changes have been made on pages of the pupil's book, the Teacher's Guide has been revised accordingly. Where only minor changes have been made in the pupil's book, the Teacher's Guide has not been changed. You will find occasional references in the Teacher's Guide which are not applicable to schools in Canada. While the pupil's texts are fully metric, in the Teacher's Guides there are refences to non-metric units of measure or to working in both systems. In the pupil's book, all numbers of five or more digits are separated by spaces between millions, thousands, and so on, instead of the traditional commas; this change has not been made on pages of the Teacher's Guide that needed no other revision. Similarly, in the pupil's book there is always a zero before the decimal point where there is no other number; again, this change has not been made on otherwise unchanged pages of the Teacher's Guide. In the text, questions have been revised to avoid reference to non-metric units, but when the answers have remained numerically the same, the Teacher's Guide has not always been altered. Pages that have been revised to assist the teacher and avoid ambiguity will be easily recognized; these pages are printed in black and red.

Special Features of

Mathematics Learning System — Canadian SI Edition

— A *complete* instructional system which includes clear objectives, built-in evaluation, and alternate instructional and assessment procedures

— *Extensive field testing* before final publication to ensure a program that works

— A *real-world approach* that involves students in practical, everyday mathematics

— *Three types of chapters* to develop each content strand:
  (i) *Exploratory* chapters in which major concepts are introduced
  (ii) *Instructional* chapters in which specific learning and skill building takes place
  (iii) *Review* chapters in which the student's mastery of skills is asssessed and challenged

— *Complete SI* (International System of Measures) metrication, Levels 1 through 6

— Well-defined *mastery objectives* for each level and specific objectives at the beginning of each chapter

— *Flexible learning sequence* by which a teacher can meet the needs of individual students by selecting sequences of chapters that differ from the order in the text

— *Evaluation within the text* by means of
  (i) *Survey questions* leading into each chapter
  (ii) *Progress checks* - periodic trouble shooting within each chapter
  (iii) *Checkouts* - measurement of student achievement at the end of each chapter
  (iv) *Alternate checkouts* for each chapter in the Teacher's Guide

— *Practice Sheets* for each level that contain additional materials for skill building and evaluation

— *Teacher's Guides* giving comments and suggestions for every page of the text. Resources at the end of each chapter provide
  (i) alternate evaluation for Progress Checks
  (ii) activities
  (iii) additional learning aids, including specific card references to many related SRA educational materials, as well as to those of other suppliers
  (iv) in some chapters, games and and exercises, with permission to reproduce the pages for class use

— *Level placement tests* by which pupil readiness for a given level can be determined, or by which year-end mastery can be assessed

— *Chapter placement tests* by which enrichment or remedial needs can be determined

the curriculum

In order to maintain a balance of concept development and drill, each SRA Mathematics Learning System chapter is built around one big idea.

The three kinds of chapters discussed in the preface – exploratory, instructional, and review – serve to organize major mathematical ideas into continuous strands. The big-idea chapters in strand organization give you more control over the learning sequence. In cooperative planning with other teachers, you can safely change the order of chapters across several levels in order to emphasize a particular content strand at a given level.

The following chart shows the chapter organization for levels 1 through 5. Think of this chart as a road map. You will use only the part you need to get you where you want to go. The strands of content are listed down the side. The levels are listed across the top. The numeral in each rectangular shape tells the chapter number and the location of the rectangle itself signals what part of the strand's content is featured in each chapter.

If you want to emphasize the whole numbers for example, the chart will show you the sequence of the strand. If you wish to use a measurement chapter in some order other than that in the book, it's O.K. The chart indicates that certain chapters can be used much earlier than that designated by the printed sequence. Most important, the chart lets you see the flow of content from one level to another.

whole numbers	concept operations + − × ÷
fractions	concept operations + − × ÷ decimal notation operations + − × ÷
measurement	length mass capacity time money
geometry	3-d, 2-d shapes 1-dimensional measurement
statistics	collecting/recording interpreting information
probability	
rational numbers	concept

In planning your year's work with the SRA Mathematics Learning System, be sure to consult your local or provincial curriculum guidelines.


level 2
level 3
level 4
level 5
level 6

OBJECTIVES

This program was built upon learning objectives. Each objective clearly states what pupil behavior is to be observed, the conditions under which the pupil is to perform, and the criteria for acceptable performance.

These objectives by definition are limited to observable behavior; therefore they are limited to the cognitive domain. This is a severe limitation, for the field-test information reveals that the affective domain also has been penetrated. A greater pupil involvement in things relating to mathematics has been shown in the complete range of abilities. Attitudes have been changed. But writing objectives for the affective domain is a new endeavor, and frankly we have not yet mastered the art. The following, therefore, are learning objectives in the cognitive domain. Each objective states the performance expected at the completion of this level.

whole-number concepts

1. Given any one of the numerals 0, 1, 2, ..., 999 999, the learner can read it aloud or write it.
2. Given any number with 6 or fewer digits, the learner can tell the value of each digit.
3. Given a problem in estimation, the learner can round the numbers appropriately.

whole-number operations

1. Given any two 4-digit numbers, the learner can estimate and find their sum.
2. Given any four 3-digit numbers, the learner can estimate and find their sum.
3. Given any three 4-digit numbers, the learner can estimate and find their sum.
4. Given any 3-digit number subtracted from any 4-digit number, the learner can estimate and find the difference.
5. Given any two 2-digit numbers, the learner can estimate and find their product.
6. Given any 3-digit number and any 1-digit number, the learner can estimate and find the quotient and the remainder (if any).
7. Given any two 2-digit numbers, the learner can show that the order in which they are multiplied does not affect their product.
8. Given any three 1-digit numbers, the learner can show that the way they are grouped for multiplication does not affect their product.

sentences

1. Given a math sentence with one operation and one placeholder, the learner can find the solution.
2. Given a simple one-step word problem with any one of the four arithmetic operations, the learner can solve it.

fractional-number concepts

1. Given a fraction model (a number line, a region, or a set), the learner can identify the associated fraction; given a fraction, the learner can construct a fraction model.
2. Given two fractions and their models, the learner can compare the fractions and record the comparison by using the appropriate relation symbol.
3. Given a fraction with the same numerator and denominator, the learner can identify the fraction as a name for one.

fractional-number operations

Given two fractions with like denominators, the learner can find their sum or their difference.

geometry concepts

Given models of geometric solids, the learner can identify spheres, cones, cylinders, cubes, and rectangular prisms.

measurement

1. Given objects with different lengths, the learner can select an appropriate metric unit of measure.
2. Given an object whose mass is to be found with a metric unit of mass, the learner can select an appropriate unit of measure.
3. Given the word *centimetre, metre,* or *kilometre,* the learner can give an example of an object whose length would be appropriately measured with the given unit of measure.
4. Given a measurement expressed in centimetres or metres, the learner can name an equivalent measurement. (For example: 2 m = ______ cm)

measurement of events

1. Given several trials of a probability experiment to observe, the learner can prepare a tally chart to show the frequency distribution of the outcomes.

1 NUMERATION AND ROUNDING

before this chapter the learner has—

1. Read and written numerals from 0 through 9999
2. Told the value of each digit in a 4-digit number
3. Compared two 3-digit numbers using "greater than," "less than," or "equal to"

in chapter 1 the learner is—

1. Mastering telling the value of each digit of any 6-digit number
2. Mastering reading aloud or writing any numeral from 0 through 999,999
3. Rounding numbers to the nearest ten, hundred, thousand, or ten-thousand
4. Ordering a set of 3- or 4-digit numbers
5. Comparing any two numbers having up to 5 digits each
6. Determining when an estimate is sufficient and when an exact answer is necessary
7. Identifying a measurement range, stating "at least . . . but not more than" and recognizing what is "between"

in later chapters the learner will—

Round numbers appropriately to use in a problem of estimation

Notes & Things

The emphasis of this chapter is on building an understanding of our number system and on exploring some of the applications of estimation and approximate measures.

Estimation, which can be used daily both in and out of school, is perhaps one of the most important basic ideas to be taught in the elementary school. Because it is a big idea with so many uses, this chapter simply starts exploring the idea. A prerequisite for good estimation is knowledge of the rounding of numbers. And, of course, a learner can't round numbers without knowledge of place value—and so goes the chapter's organization.

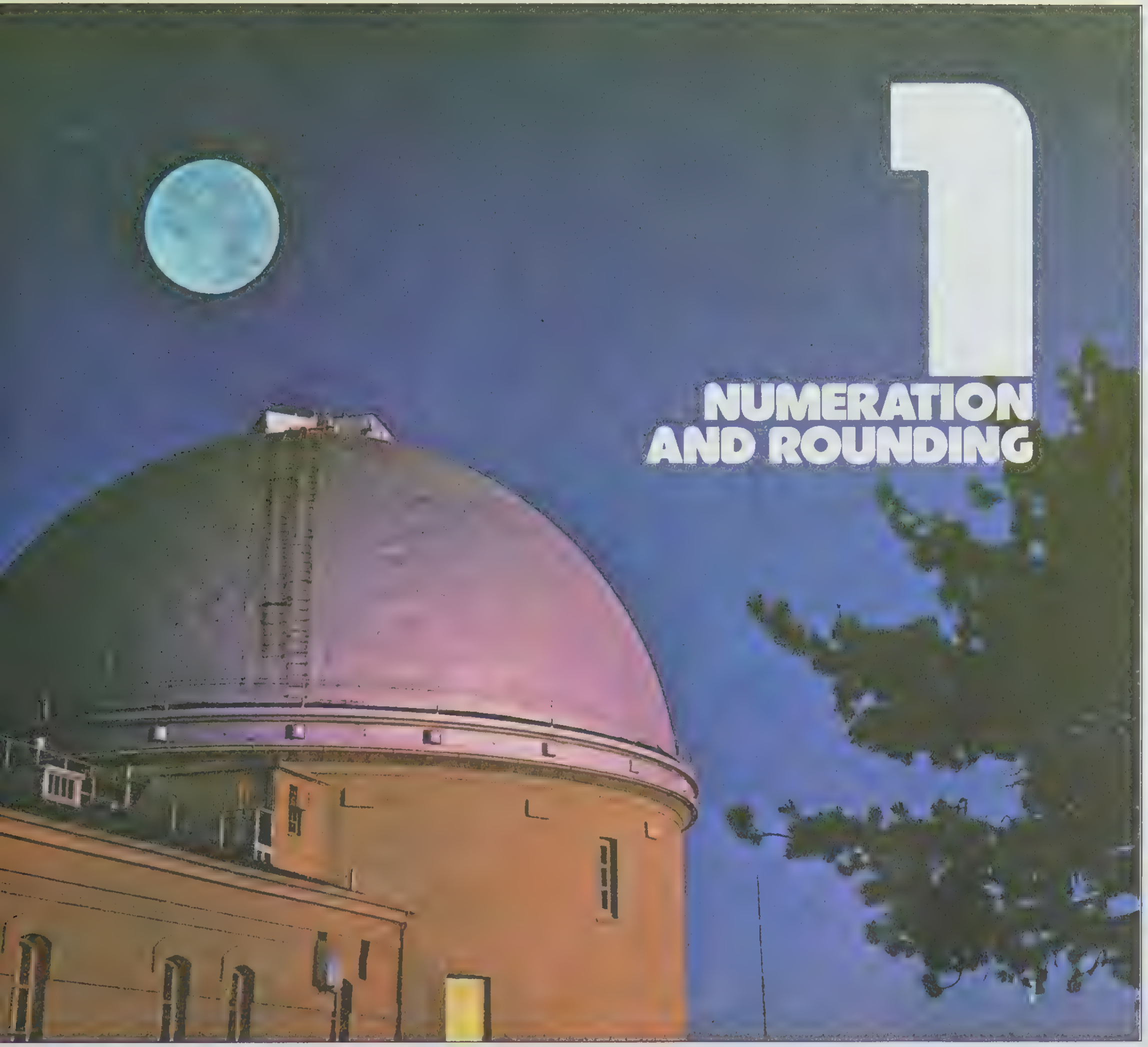
Awareness of numbers in everyday situations must be the beginning point, so that each pupil can personally accept the study of numbers as valid and relevant. We see numbers most frequently associated with measurement in the real world. The pupil pages present a variety of situations that serve to initiate individual research on related ideas. The individual's interests can be easily tapped throughout the chapter, but the pupil pages provide an opportunity to work in a large group most of the time. This will let you get acquainted with the mathematical thinking of the individuals in your group prior to the time skills are emphasized. Youngsters who have skill-development problems will have a chance to contribute their good ideas to discussions. Pupils who have well-developed arithmetic skills will be challenged to demonstrate the depth of their knowledge of elementary mathematics. The mix of abilities and ideas will start the study of this level's mathematics in a way that is intellectually exciting for everybody.

things

2-year calendar covering the current school year

For the extra activities you will want to have these things available:

- almanac, road atlas
- small plastic dog
- $\frac{1}{2}''$ graph paper
- 10 same-size boxes
- 6 wood cubes



goal Think about and explore ideas through a picture clue

page 1 Each chapter opens with a full-page photograph. Such a page contains much more than the identification of the chapter number and chapter title. This page, for example, can serve you in many ways.

It will help you get an idea of how sophisticated members of your class are about outer space, how much they know about astronomy, and how much information they have about the moon and why we considered it so important to land on its surface. You will gain insight into the special interests of some children and be able to identify the one you can depend upon as your "expert in residence."

This page may be the source for independent research ideas too. Self-study projects related to the math lessons, but apart from the book itself, are vital for your most able learners. These learners have the capacity to do so much more. Investigating applications of mathematics is one of the most worthwhile extensions of any math lesson.

It is obvious that large numbers will enter into the conversation and the discussion of them serves as the direct tie to the chapter itself.

goal **Survey**—knowledge of place value and numeration; appropriateness of a rounded number

memo Discuss. These are probably new ideas. Use the goal on the pupil page to kick off discussion.

page 2 Why aren't the numbers that measure the same distance alike? Which are probably most accurate? Is an accurate measure always necessary?

Agree on the meaning of **digits** before pupils handle any of the problems independently. This page will help identify those children who may already have a good knowledge of our numeration system. These pupils may be able to move faster than others.

How much do you know about numbers?

Don't expect correct answers to every problem on this page.

GOAL —find out!

How far is it from the earth to the moon?
We found these answers:

Newspaper article	240,000 miles
Science book	238,000 miles
NASA report	232,478 miles

Which is correct? All Which is most useful?
Depends on need for accuracy.

How far is it from New York to San Francisco?
We found these answers:

Airline map	2600 miles
Road map	3000 miles
Atlas	2650 miles
Motor club guidebook	2974 miles

Which is correct? All Which is most useful?
Discuss. It depends on need. Also air mileage is less than surface mileage.

1. How many different symbols do we need to write any whole number? 10

Each single symbol is called a digit.

2. There are 11,350,000 people in the world's largest city.
   - **a** How many digits are there in the numeral 11,350,000? 8
   - **b** How many *different* digits are there in this numeral? 4

3. How many digits are there in the numeral 10? How many different digits? 2
   - **a** How many digits are there in the numeral 100? 3 How many different digits? 2
   - **b** How many digits are there in the numeral 1000? 4 How many different digits? 2

4. How many *different* digits do you need to write your telephone number? Ask if they are coun[ting] in area code.
   - **a** How many digits do you think there are in your telephone book? (thousands, milli[ons])
   - **b** How many different digits are there in your telephone book? 10

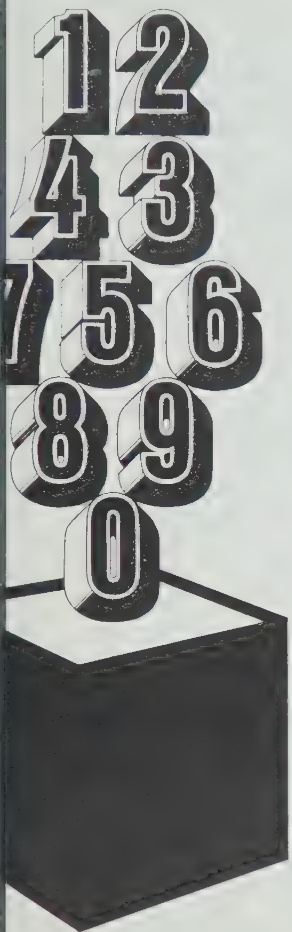
5. What makes one number bigger or smaller than another number? The place value of the digits

2

things references: almanac, encyclopedias, dictionary, road atlas

Let the junior researchers use available references to provide more examples of different measurements for the same quantities, distances, heights, weights, or time estimates. Consider population figures, distances between cities, altitudes, weight of a car, and so on.

References listed in the order of their probable accuracy: almanac—detail for current study; encyclopedia—overview study; dictionary—quick reference.



Because of our place-value system, only ten different digits are needed to name all the numbers we use.

1. Use each of the digits 1, 2, 3. Name as many 3-digit numbers as you can. 123, 132, 213, 231, 312, 321
   a Which number is the smallest? 123
   b Which is the largest? 321

2. Use each of the digits 1, 2, 3, 4.
   a Name the largest 4-digit number you can. 4321
   b Then name the smallest number you can with these four digits. 1234

3. Pick any four different digits. Use each one of them only once to— Answers will vary.
   a name the largest number you can.
   b name the smallest number you can.

4. Pick only one digit to write the largest 4-digit number you can. 9999

5. Pick only one digit to write the smallest 4-digit number. 1111

6. Use only the digits in each set below. Write the largest and the smallest 4-digit number you can.

		largest	smallest
a	0, 1, 2, 3	3210	1023
b	2, 4, 6, 8	8642	2468
c	1, 3, 5, 7	7531	1357
d	1, 2, 8, 9	9821	1289

7. Use the words "is greater than" or their symbol ">" or the words "is less than" or "<" to replace each ?.
   a 5436 ? 6543 <
   b 7654 ? 7564 >
   c 2461 ? 2416 >
   d 9876 ? 9871 >

3

See activity 1, page 20a.

lesson Pages 3, 4

goal Examining the relationship of the placement of a specific digit and its value in our system of numeration

warm-up Consider how football teams use numbers to call different plays. An 83 may be a pass play, whereas a 38 may be a running play. Here numbers are used to label a type of play. Does play 38 necessarily mean that there are 37 other plays that come before?

Now consider money. *What do you know about 83 cents and 38 cents? . . . If you have 38 cents, does it mean that you have 37 cents and 1 more? What about 83 cents?*

page 3 Let the abilities of your pupils and your classroom organization determine how the page is used—independent activity, small-group project, or discussion.

goal Determining the value of each digit in a number

page 4 Proceed as with page 3. Do not let reading interfere with success in math. Give all the help necessary. Consider a peer buddy—one who just happens to excel in reading.

River	Length
Danube	2842 km
Volga	3364 km
Amazon	6400 km
Congo	4350 km

Stop and think. Look at the table at the left.

A Does each 4 in the table have the same value? No.

B Which 4 has a value of thousands? 4350 (Congo River)

C Which 4 has a value of hundreds? 6400 (Amazon River)

D Which has a value of tens? 2842 (Danube River)

E Which has a value of ones? 3364 (Volga River)

Use the table at the right for the next questions.

	thousands	hundreds	tens	ones
a	3	7	1	0
b	7	0	3	1
c	1	3	0	7
d	1	0	7	3

1. What is the value of the 7 in each of the numbers?
a. 700, b. 7000, c. 7, d. 70

2. What is the value of the 0 in each of the numbers?
a. 0, b. 0, c. 0, d.0

3. What is the value of the 3 in each of the numbers?
a. 3000, b. 30, c. 300, d. 3

Copy and complete.

4. 8940→? thousands, ? hundreds, ? tens, ? ones
8 9 4 0

5. 5007→? thousands, ? hundreds, ? tens, ? ones
5 0 0 7

6. 3206→? thousands, ? hundreds, ? tens, ? ones
3 2 0 6

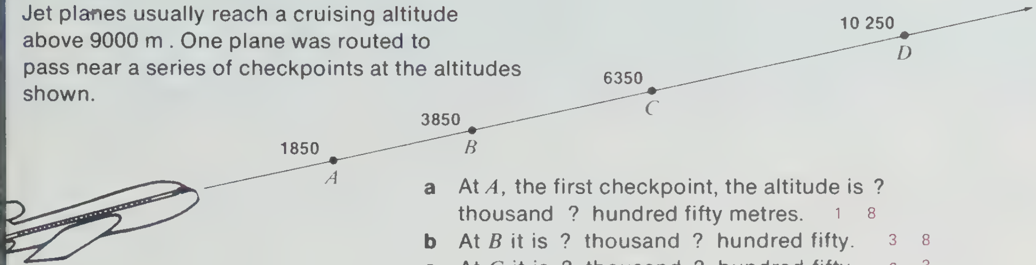
Each digit in the numeral for the length of the Danube has a different value.

2842
2000
800
40
2
2842

7. Tell the value of each digit in the lengths of the other rivers in the table at the top of the page.

Volga—3000, 300, 60, 4
Amazon—6000, 400, 0, 0
Congo—4000, 300, 50, 0

Jet planes usually reach a cruising altitude above 9000 m . One plane was routed to pass near a series of checkpoints at the altitudes shown.



a At A, the first checkpoint, the altitude is ? thousand ? hundred fifty metres. 1 8

b At B it is ? thousand ? hundred fifty. 3 8

c At C it is ? thousand ? hundred fifty. 6 3

d At D it is ? thousand ? hundred fifty. 10 2

thousands			ones			
hundred	ten	one	hundreds	tens	ones	
					1	one
				1	0	ten
			1	0	0	one hundred
		1	0	0	0	one thousand
	1	0	0	0	0	ten thousand
1	0	0	0	0	0	one hundred thousand

2. Look at the table on the left. Be ready to read each number aloud.

3. Is there a number greater than those given in the table? Name one. Answers will vary. Example: one million

4. Large numbers with more than four digits are usually written with a space to the right of the thousands place. So the last number in the chart would be written 100 000.

The 4-digit numbers may be written without spaces, as 1000. Why do you think this is done? Discuss.
Compliment those who decide a space isn't necessary for easy reading.

5. Use only the digits in each set below. Write the largest 5-digit number you can and the smallest.

a 0, 1, 2, 3, 4 **b** 4, 7, 9, 5, 1 **c** 7, 0, 1, 9, 8
43 210 10 234 (or 01 234) 97 541 14 579 98 710 10 789 (or 01 789)

6. Write the largest 6-digit number there is and the smallest.
largest: 999 999 smallest: 100 000

5

lesson Pages 5, 6

goal Practice in reading and writing 5- and 6-digit numbers

memo Discuss – think how awful it would be to write out the answers to the first question.

page 5 That little place-value chart on page 4 is growing rapidly. Comparing the ones, tens, and hundreds repeating in the thousands period should interest the pupils. Have some fun by having each person write the answers to the last two problems on a scrap of paper. Collect the responses and then redistribute. Have pupils decide if the answers they now have are correct or not and why. This correcting device saves embarrassment for those who don't know the answers and at the same time challenges those who know the answers to do a little thinking.

STOP

things numeral cards 0 through 9; cards with place-value positions, ones through hundred-thousands

Shuffle the numeral cards. Place them facedown in a stack. Repeat for the place-value cards. The pupil draws one card from each stack, then places the digit card (numeral card) in the position directed by the place-value card.

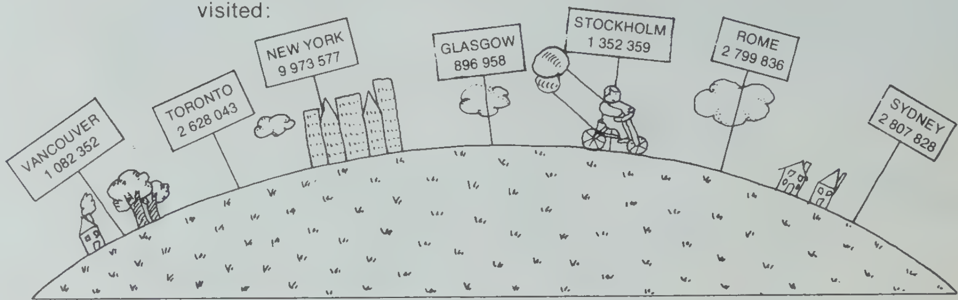
goal Practice in reading and writing 5- and 6-digit numbers

memo Again discuss the usefulness of numbers for writing large amounts.

page 6 Listen and make sure that the space is **not** read as **and.**

The special research problem marked with an * may be saved for another time. However it may be revealing to get some estimates on how many people the group thinks there are in the school, community, and country.

Kathy's father is an airline pilot. The family went around the world on holiday. Kathy found these population figures for cities they visited:



Look at the population for Stockholm, Sweden.
This is a big number. The first digit 1 has the value of 1 million.

1. Name one time when you have heard the number one million.
Answers will vary. Examples: bank deposits of $1 000 000; the one-millionth car on a toll road

2. Read the population numbers.
Example only: Vancouver: one million, eighty-two thousand, three hundred fifty-two

Note: "And" should not be read in a number.

3. Read each of these numbers.

a 365 227 **b** 450 220 **c** 700 530 **d** 698 000 **e** 40 019 **f** 520 416

Example only: a three hundred sixty-five thousand, two hundred twenty-seven b four hundred fifty thousand, two hundred twenty

4. Write the numeral for each number.

a fifty-six thousand twenty-four 56 024
b sixty-three thousand four hundred six 63 406
c one hundred thirty-six thousand 136 000
d eight hundred thousand six 800 006

***5.** When you have time, find out–

a how many people go to your school.
b how many people live in your community, in your country.

things 2 sets of numeral cards 0 through 9; space card (blank)

Pair pupils. One arranges a 3-, 4-, 5-, or 6-digit numeral, placing the space in position for 5- and 6-digit numerals. He then challenges the other to read the numeral formed. Pupils alternate roles.

PROGRESS CHECK

Write a riddle. Skill: Writing numbers

1. Write a number with four digits – the biggest on the line. Use only a bunch of zeros and the digit 9. 9000
2. Write a number with five digits – the smallest you can do. Use only a bunch of ones and the digit 2. 11,112
3. Write a number with six digits – the only one to write. Use the digit 3 because no others are in sight. 333,333

Skill: Comparing numbers

We have only 10 different digits. The location of each digit tells how to read the number. It also tells how many. Pretend these numbers tell how many pennies you have. Which would you rather have?

4. 19 or 91
5. 911 or 119
6. 191 or 119
7. 1191 or 1911
8. 9111 or 1911
9. 19,111 or 11,911

Skill: Ordering numbers

Write the numbers in each set in order from least to greatest.

10. 955, 559, 595, 95, 59
59, 95, 559, 595, 955
11. 4774, 4477, 744, 7744, 7474
744, 4477, 4774, 7474, 7744
12. 13,056, 30,561, 51,651, 56,301, 65,310
13,056, 30,561, 51,651, 56,301, 65,310

Skill: Telling the value of a digit

Match the place value listed in column 2 to the value of the digit 5 in each number in column 1.

Column 1		Column 2
13. 111,115	a	a ones
14. 511,111	f	b tens
15. 115,111	d	c hundreds
16. 111,151	b	d thousands
17. 151,111	e	e ten-thousands
18. 111,511	c	f hundred-thousands

7

lesson Page 7

goal **Progress Check** – reading, writing, and ordering numbers that have 1 to 6 digits

things 6 wood cubes

page 7 Make sure that pupils understand the directions for each exercise. Help with reading if necessary. This should be independent work. There are a lot of problems for some pupils. Be sure to allow enough time for everyone to complete the page

Provide additional help for pupils who score 0 through 15. Eliminate the possibility of a reading problem by questioning verbally. Look for a lack of understanding of place value. A place-value chart as shown on page 5 should prove helpful.

Here's a fun way to provide additional practice. Number each face of six cubes with a 1-digit numeral. Each pupil in turn rolls the cubes and arranges the largest number possible. The one who arranges the largest number wins the round. Vary the activity by arranging the smallest number possible.

STOP Anyone having trouble misreading numerals that look the same but are not? This spirit master may help.

Ring the numerals in each row that are the same.

a	43	34	43	54
b	39	93	63	39
c	583	583	538	583
d	502	520	502	205
e	988	889	898	988
f	4486	4468	4486	4864
g	32815	32851	32581	32815

lesson Pages 8, 9

goal Investigation of range—**at least . . . but not more than**

memo Discuss this new idea. The ability to round is necessary in estimation. This program places a heavy emphasis on estimation as an aid in computation and as a problem-solving tool. This exploratory approach to rounding is a very important preliminary activity that will make estimation easier to understand.

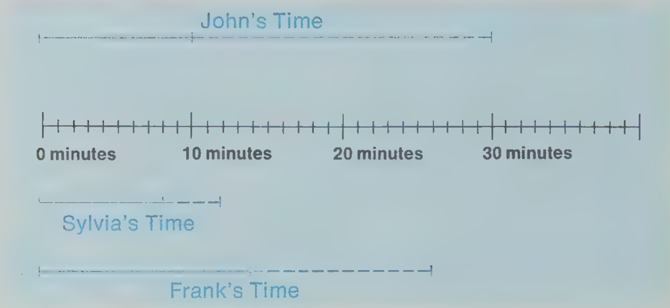
page 8 What things do you have to consider if you want to be at a certain place at 9:30? When should you start to get ready? How will you get there? How long will it take?

The planning of time as presented on the page requires some good thinking, especially on the last problem. There are many answers. Listen to all of them. Let the group decide the most probable reason.

When your mother fixes dinner, she tries to cook *at least* enough to feed everyone but *not more than* they can eat.

When you go on a trip, you try to take *at least* enough clothes for the different things you'll be doing but *not more than* the clothes you can easily carry.

1. John usually walks to school. This takes him *not more than* 30 minutes (unless he fools around on the way). In bad weather he rides the bus. This is faster. But it takes at least 10 minutes.

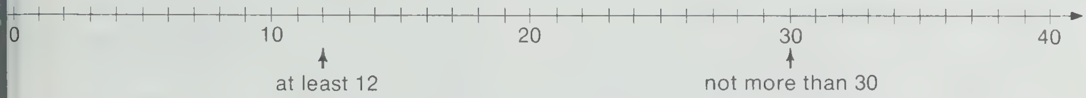


John takes at least 10 minutes but not more than 30 minutes to get to school.

a Frank lives farther away. He always rides the bus. Look at the chart. What is the least time that it takes him to get to school? the most time? 14 minutes 26 minutes

b Sylvia takes at least ? minutes but not more than ? minutes. Why do you think there is so little difference between least time and most time for her? 8 12 Accept reasonable answers. Example: She probably lives the closest to school.

1. Mary needs a ribbon for her ponytail. It should be *at least* 12 cm long but *not more than* 30 cm long.

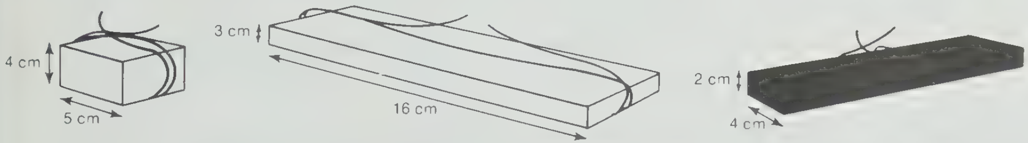


Which of the following ribbons can she use?

red: 18 cm	pink: 40 cm	brown: 8 cm
blue: 12 cm	green: 30 cm	white: 14 cm

2. These pictures show boxes of three sizes used by a department store. Pieces of string are to be cut and used to tie the boxes. The string is wrapped around the box twice.

Each piece should be long enough to wrap twice around and tie. It should not be so long that string is wasted. What length of string might you choose for each box?



	at least	not more than		at least	not more than		at least	not more than
a	45	50	d	19	38	g	35	40
b	28	40	e	35	75	h	20	30
c	58	75	f	90	110	i	50	60

9

goal Deciding the most appropriate range for a problem situation

things boxes
ball of string
scissors
metrestick

page 9 If page 8 went smoothly, you can use this for independent work. You may want the girls to explain why the ribbon has a minimum length and how one would decide the maximum length.

Problem 2 signals this activity. Each pupil takes, without actually wrapping or measuring, what he thinks will be enough string to do the task described on the page. After the string is tied, the pupil measures the piece(s) of wasted string and records that measurement on the chalkboard.

Examine the measurements recorded. Question whether the wasted string could be important in any way. Consider a large store that wraps an average of 1000 packages a day, 6 days a week.

lesson Pages 10, 11

goal Introduction to and development of the idea of ROUNDING 2-digit numbers to tens

memo Discussion is needed for this new idea.

page 10 Make sure that pupils understand the number-line markings. You may need to show more number-line examples. Key questions: The number is between which two multiples of 10? Which multiple of 10 is it closer to? **Rounding up** and **rounding down** make perfectly good sense to adults. They may not to children. Check by asking what these words could mean. This background is absolutely necessary for the development of successful rounding skills.

If you're painting a wall, you need at least enough paint for the job. It doesn't matter if you have more than enough. (But you don't want too much left over.)

1. When is it important to have at least enough but not so important to have more than enough

   a fence for a yard b light to read by

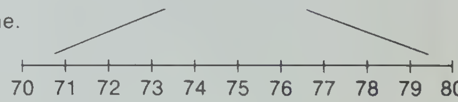
   c load to carry d money to pay the cashi

2. You are to be home between 5 and 6 o'clock. What is the last minute you could get home and not be in trouble? 5:59

3. When someone says it's 6:35 P.M., is it closer to 6:00 P.M. or 7:00 P.M.?
   Is 1:30 closer to 1:00 or 2:00? Neither
   Is 6:00 closer to noon or midnight? Neither

4. Here is a number line marked to show tens.

Here's a magnified view of part of the number line.



72 is between 70 and 80. Is it closer to 70 or 80?
Since 72 is closer to 70 than to 80, we say that 72 *rounded to the nearest ten* is 70.

   a What is 74 *rounded to the nearest ten*? 70 b What is 79 rounded to the nearest ten? 80

   c What is 76 rounded to the nearest ten? 80 d What is 71 rounded to the nearest ten? 70

5. The number line will help you answer the next questions.

   a 13 is between 10 and 20. Is it closer to 10 or 20? What is 13 rounded to the nearest ten? 10

   b 36 is between 30 and 40. Can it be rounded to 30 or 40?

   c Is 42 rounded to 40 or 50?

   d Is 18 rounded to 10 or 20?

   e Is 55 rounded to 50 or 60?

   f Is 96 rounded to 90 or 100?

10

The number line below shows hundreds.

Here's a magnified view of part of the number line.

Other numbers have been marked on the number line.
580 is between 500 and 600. Is it closer to 500 or 600?

Since 580 is closer to 600 than to 500, we say that 580 *rounded to the nearest hundred* is 600.

1. Round each of the following to the nearest hundred (500 or 600).
   a 575 600 **b** 525 500 **c** 561 600 **d** 590 600 **e** 540 500

2. Think of the part of the number line between 300 and 400.
   Round each of the following to the nearest hundred.
   a 306 300 **b** 386 400 **c** 346 300 **d** 320 300 **e** 370 400

3. A number line will help you answer the next questions, too.

a Is 120 rounded to 100 or 200? **b** Is 530 rounded to 500 or 600?
c Is 460 rounded to 400 or 500? **d** Is 955 rounded to 900 or 1000?
e Is 289 rounded to 200 or 300? **f** Is 306 rounded to 300 or 400?
g Is 749 rounded to 700 or 800? **h** Is 751 rounded to 700 or 800?

11

goal Extension of the idea of rounding to 3-digit numbers rounded to hundreds

memo Be prepared to continue discussion.

page 11 The number line now shows multiples of 100. The thrust is to place a number between two multiples of 100 first and then to decide which multiple the number is closer to. This sequence is important. The pupils are not ready for shortcuts. You may want to do several additional examples on the board to make sure that the idea of rounding is clear.

Have them make number lines to prove any incorrect answers for problems 1 through 3.

lesson Pages 12, 13

goal Refining the idea of rounding to that of halfway numbers; extension of rounding to thousands

memo Check again on the pupils' understanding of rounding **up** and **down.** It may need further clarification.

page 12 This is a crucial page. It is time to contrast multiples of 10 with multiples of 100. The **halfway number** is the key to understanding, but it is meaningless without the number line. If the halfway number is understood for tens AND hundreds, the pupils will have the basic tool for rounding **any** number.

Provide help with reading if necessary. Encourage the use of a number-line sketch. This extra step will ensure success.

1. Think tens. What number is halfway between 40 and 50? If you rounded this number to the nearest ten, you would round up to 50. 45

40 ? 50

a What other numbers would you round up to 50? Why?
46, 47, 48, 49 They are more than halfway between 40 a

b Would you round 41, 42, 43, or 44 up to 50 or down to 40? Why? They are less than h between 40 and 50.

2. Think tens some more. What number is halfway between 90 and 100? If you round this number to the nearest ten, you would round up to 100. 95

90 ? 100

a What other numbers would you round up to 100? 96, 97, 98, 99

b What numbers would you round down to
91, 92, 93, 94

3. Now think hundreds. What number is halfway between 200 and 300? If you rounded this number to the nearest hundred, you would round up to 300. 250

200 ? 300

a Which of these numbers round up to 30
279↑ 223↓ 249↓ 251↑ 260↑ 225↓ 280↑ 250

b Go back.
Which of the numbers round down to 20

4. You use the same kind of thinking when you round thousands. Start slowly. Sneak up on the idea.

a What number is halfway between 80 and 90? 85

80 ? 90

b What number is halfway between 800 and 900? 850

800 ? 900

c What number is halfway between 8000 and 9000? 8500

8000 ? 9000

d Which of these numbers would round up to 9000? 8552 8899 8900 8468 8154 8600

e Which of these numbers would you round down to 8000? 8256 8012 8471 8725 8392 84

12

5. Some people need to deal with thousands every day. Newspaper reporters are one group who "think" thousands all the time. It's part of their job.

a Do you think reporters need to know the exact number of people at a football game they report? No.

b Do they need to know the exact number of people at a car race? No.

c Do they need to know the exact number of people at a concert? No.

d Reporters estimate crowds. They determine about how many thousands of people attend sports events. Why? Accept reasonable answers. Example: To know which games are most popular

PROGRESS CHECK

Use a number line if you need help. Skill: Finding the number halfway between two numbers

What number is—

1. halfway between 0 and 10? 5
2. halfway between 0 and 100? 50
3. halfway between 0 and 1000? 500

What number is—

4. halfway between 50 and 60? 55
5. halfway between 500 and 600? 550
6. halfway between 5000 and 6000? 5500

Round each number to the nearest ten. Skill: Rounding to the nearest ten

7. 45 50 8. 61 60 9. 79 80 10. 55 60 11. 97 100

Round each number to the nearest hundred. Skill: Rounding to the nearest hundred

12. 150 200 13. 309 300 14. 777 800 15. 850 900 16. 915 900

Round each number to the nearest thousand. Skill: Rounding to the nearest thousand

17. 2500 3000 18. 4500 5000 19. 8051 8000 20. 9220 9000 21. 7576 8000

13

goal **Progress Check**—rounding to tens, hundreds, and thousands

page 13 Use the Progress Check to help pinpoint the specific type of rounding situation the learner has not yet mastered.

1. Is naming the halfway mark causing trouble?
2. Can the youngster name numbers that are greater than the halfway number? less than the halfway number?

The number line is the best test for the majority of pupils at this point, but with the more confident learners you can explore the place value of the digit that helps decide the rounding.

1. What is the key digit when rounding to tens? (Ones)
2. What is the key digit when rounding to hundreds? (Tens)
3. What is the key digit when rounding to thousands? (Hundreds)

See activity 2, page 20a.

See activity 3, page 20b.

lesson Pages 14, 15

goal Exploration of the concept "between"; extension of rounding to ten-thousands

memo Pupils who have found the work difficult may have problems with this page.

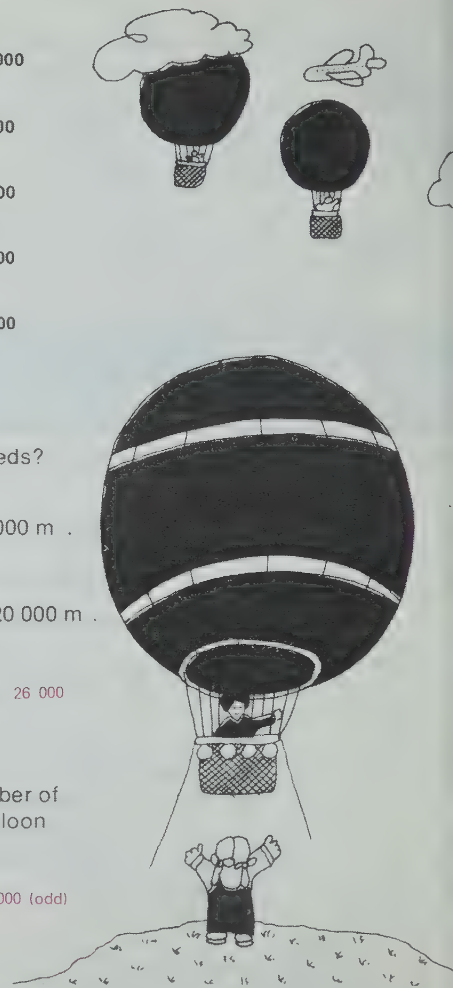
warm-up Check whether the group can identify **even** and **odd** numbers. Write the numbers 1, 2, 3, . . . , 28 on the board or on the overhead projector. Ask someone to ring the first 2 even numbers. Have another person make a ✓ under the first odd number. Continue by having someone ring the next 4 even numbers, make a ✓ under the next 3 odd numbers, and so on. Don't erase. Let this be a model for those pupils who have had no earlier experience with odd or even numbers. The ones digit signals whether a number having 2 or more digits is even or odd.

ones digit: 1, 3, 5, 7, 9 — number is odd
2, 4, 6, 8, 0 — number is even

page 14 The very capable pupils may be able to work independently on this page, but it is a relevant and interesting discussion page for all.

Be careful of problem 6. The digit 5 has been the key for halfway numbers on previous pages.

Before you start, make sure you know that 2, 4, 6, 8, and 10 are called even numbers. 1, 3, 5, 7, and 9 are odd numbers.



WHAT'S BETWEEN?

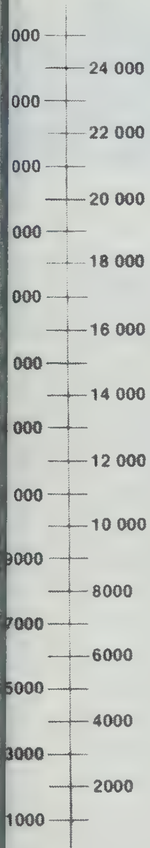
The Red Racers and the Purple Pirates are planning a race. To avoid confusion, the Red Racers' balloons will travel at even hundreds of metres above the ground (600, 800, 1000, and so on). The Purple Pirates' balloons will travel at odd hundreds of metres above ground (700, 900, 1100, and so on).

1. Why do you think 800 and 1000 are called even hundreds? Why are 700 and 900 called odd hundreds?
   8 and 10 are even numbers. 7 and 9 are odd numbers.
2. Name the even hundreds between ground level and 1000 m.
   200, 400, 600, 800 m
3. Name the odd hundreds between 500 and 1500 m.
   700, 900, 1100, 1300 m
4. Name the even thousands between ground level and 20 000 m.
   2000, 4000, 6000, 8000, 10 100, 12 000, 14 000, 16 000, 18 000 m
5. Name the odd thousands between 20 000 and 34 000.
   21 000, 23 000, 25 000, 27 000, 29 000, 31 000, 33 000
6. What number is halfway between 25 000 and 27 000? 26 000
   a halfway between 20 000 and 24 000? 22 000
   b halfway between 21 000 and 23 000? 22 000
7. A balloon might not be exactly at an even or odd number of hundred metres above the ground. For example, a balloon might be at 710 m.
   Is 710 closer to 700 (odd) or 800 (even)? 700 (odd)
   Is 25 100 closer to 25 000 (odd) or 26 000 (even)? 25 000 (odd)

14

STOP See activity 4, page 20b.

Extend activity 4 in the Resource Section by having the pupil join 2 even numbers. What kind of number is the answer? Join 2 odd numbers. What kind of number is the answer? Join an even and an odd. The answer is? (Odd) Experiment with other numbers. Is this always true?



1. On the number line above, *A* marks the point halfway between 21 000 and 22 000. What number is marked by point *A*? 21 500

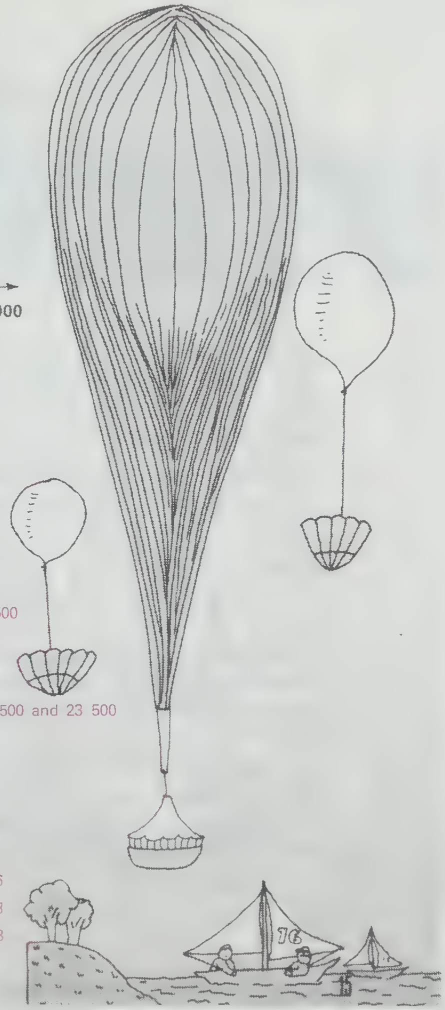
   a What number is marked by point *B*? That number is halfway between 22 000 and 23 000. 22 500

   b A weather balloon reaches an altitude of about 22 000 m. Its altitude should be between what two numbers? 21 500 and 22 500

   c Another unmanned balloon reaches an altitude of about 23 000 m. Its altitude should be between what two numbers? 22 500 and 23 500

2. Instruments reported weather balloons at the following altitudes. About how many thousands of metres high was each balloon? (Round to the nearest thousand.)

a 23 200	23	**b** 23 900	24	**c** 25 800	26
d 29 200	29	**e** 23 800	24	**f** 32 700	33
g 29 600	30	**h** 25 590	26	**i** 28 300	28



goal Continued exploration of the concept "between"; practice with rounding to ten-thousands

page 15 Pupils who have a good grasp of **between** and **rounding** should proceed independently.

Research project. What is the maximum altitude that passenger planes are allowed to fly according to current regulations? Can planes go still higher? What is the record altitude for a piloted plane?

STOP

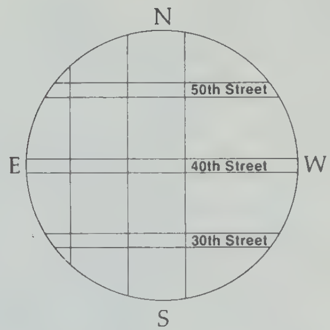
Reinforce the notion of thinking in terms of a number of hundreds. *About how much does a new car cost?* Lead the pupils to answer both 2 thousand 5 hundred dollars and 25 hundred dollars (or whatever number is appropriate). Consider also the number of people expected to attend a coming civic or sports event, the school enrollment, and so on.

lesson Pages 16, 17

goal Application of rounding to real-world situations

memo Pages 16 and 17 are optional. The pupil is required to use what he has learned about rounding.

page 16 There are no new concepts, but this page demands good thinking. This is independent work. (This will give you time to work with the pupils who need your help.)



1. In one part of a city all the east-west streets are numbered. Every tenth street is a four-lane street (30th Street, 40th Street, and so on). When driving east or west, people try to use these streets.

   What is the nearest four-lane street if you are on

   a 38th Street? 40th Street **b** 44th Street? 40th Street **c** 33d Street? 30th Street **d** 45th Street? Either 40th or 50th

   What is the nearest four-lane street if you are on

   e 71st Street? 70th Street **f** 78th Street? 80th Street **g** 76th Street? 80th Street **h** 94th Street? 90th Street

2. The express subway stops at 50th Street, 100th Street, and 150th Street. The following are stops on the local subway. For each, tell which express stop is closest.

   a 128th Street 150th Street **b** 95th Street 100th Street **c** 73d Street 50th Street **d** 60th Street 50th Street

3. A building has observation decks on the 50th and 60th floors. Which observation deck is closer to the following floors?

   a 58th 60th **b** 51st 50th **c** 56th 60th **d** 55th Either 50th or 60th

16

STOP

things for each pupil: graph paper; colored pencil

Have the pupils make their own maps on graph paper and actually show each street. Use colored pencil to indicate the four-lane streets. Ask questions similar to those for problem 1.

Ambitious youngsters might also include north-south avenues on their maps. Locate a store, school, movie, and so on. Have them describe how to get from one location to another most easily.

How Big Is the City?

RECENT CENSUS FIGURES

City	Population
Ottawa	602 510
Winnipeg	540 262
Hamilton	498 523
Edmonton	495 702
Halifax	222 637
Victoria	195 800
Regina	140 734
St. John's	131 814
Saint John	106 744

1. Read the population of each city listed in the table. When do you think the population of Ottawa was exactly the figure given—on the day the census was published? when the census was started? when it was finished? Probably not Probably not Probably not
2. The population of a city changes daily. People move in and out. Babies are born and people die. If we never know exactly what the population is, what does the table tell us? Approximately how many people live in each city
3. Do you think the population of Edmonton is greater than the population of Hamilton today? It could be. At time of the census, yes
4. Is Halifax larger than Victoria? Is it much larger? About how much? About 25 000 No. (Answer depends on definition of "much larger.")
5. Is St. John's larger than Saint John? About how much? Yes About 25 000
6. Is Winnipeg larger than Regina? About how much? Is it twice as large? 400 000 More than twice—almost 4 times

17

goal Application of rounding to real-world situations

page 17 A time to learn about numbers and statistics by sharing ideas. Consider this page for discussion only.

lesson Pages 18, 19

goal Exploration of situations where an estimation, rather than an exact measure, is sufficient

memo This page can be the basis for independent research.

things 2-year calendar covering the current school year

page 18 The calendar, lots of time, and lots of patience are needed for problem 1.

How close is close enough?

1. How long is a school year? Answers depend on your school year.
   10 months? 36 weeks? 180 days? — most comm
   In many places school opens early in September. It closes around the middle of June. So the scho year is 10 months long.

   a Suppose school opens the day after Labor Day and closes on the third Friday in June. Then it's ? weeks long. Is Labor Day always on the same date in September? No
   About 40

   b But classes aren't held every day. There are vacation days. There might be only 180 days of classes in a school year. How many vacatio days does your school have? Answers depend on your school year.

   c Which figure is correct for the length of a school year? All

   d Which is most useful? Answer depends on how accurate you need to be.

2. How old is Jack's dog?
   11 years 135 months 586 weeks 4100 days

   a Which is correct? **b** Which is most useful?
   All; 4100 days would be most accurate. Probably 11 years is sufficient for y

If you're buying rock salt to melt the ice on the sidewalk, you *estimate* how much you'll need. But if you're adding table salt to cookie dough, you measure *accurately*.

18

things 2-year calendar

While the calendar is available, extend the page activity.

How many months until your next birthday? ... How many weeks? ... How many days?

How many months until one year from today? ... How many weeks? ... How many days?

Pick at least one important scheduled event in your community and repeat.

ell whether an estimate or an accurate measurement called for in situation **a** and in situation **b**.

Mass

a When you lift a package estimate

b When you mail a package accurate

2. Time

a To run a 100 m race accurate

b To walk home estimate

Volume

a Lemonade to drink when you're thirsty estimate

b Lemonade to fill a litre jar accurate

4. Number

a Potatoes in a 5 kg bag estimate

b One baked potato for everyone accurate

Length

a The amount of string to fly a kite estimate

b The distance in a race accurate

6. Temperature

a When you're baking a cake accurate

b When you decide which clothes to wear estimate

Mass

a Your mass in the nurse's office accurate

b One more person in a boat estimate

8. Time

a When you leave for school estimate

b When you go fishing estimate

Volume

a Water to fill a swimming pool estimate

b Water for a cake recipe accurate

10. Length

a String to wrap a package estimate

b The record for high jump accurate

alk about this next question.

1. Do you think it is possible to have an exact measurement?

No. You can come closer and closer to an exact measurement, but all measurements are approximate because of the measuring devices that are now in use.

19

goal Deciding whether an ESTIMATE or an ACCURATE MEASURE is needed

memo Let everyone in the group get together to discuss this page. There could be some good arguments. Pupils will have to think carefully to justify their answers.

page 19 In the real world we do not always need to know **exactly** how much – an estimate will do. At other times an exact measure is very important.

That discussion question at the end is a wild one. If some pupils resist the idea that, because of our measuring devices, exact measurements aren't possible, turn the argument away from the measuring devices. Use a dull, broad piece of chalk and mark a 15 cm line segment on the board. (The width of the chalk mark itself could throw the measurement off as much as 1 centimetre.) If the pupils maintain that you should have used a finer line, keep teasing. Wonder what that finer line would look like under a magnifying glass, under a microscope, and so on.

goal **Checkout** – reading and writing numerals that have 1 to 6 digits; using rounding in word-problem situations

page 20 Use the skills shown on the answer key to help pinpoint the pupil's difficulty.

If reading is a serious problem, read each of the questions aloud. Also consider using the alternate form of evaluation found in the Resource Section (page 20a).

CHECKOUT

Skill: Writing the largest or smallest number

1. If you were asked to write the largest number you could, which digit would you use the most? 9

Skills: Rounding to nearest ten, hundred, thousand; estimating problem answer

2. A factory needs 1600 special bolts. They come only in boxes f 1000. How many boxes must be bought? 2

3. The store needs 950 sale notices. The printer will print them only in groups of 100. How many hundreds must the store order to have enough? 10

4. The cafeteria needs 32 kg of sugar. They can buy sugar only in 10 kg bags. How many bags must they order? 4

Skill: Reading or writing any numeral 0 through 999 999

5. The numeral keys on the typewriter wouldn't work. Jack had to write out all the numbers in this note to his friend.

> I will get into the airport at eight o'clock. My flight should be exciting. They say the plane will fly at an altitude of about nine thousand metres and will have a ground speed of about nine hundred twenty kilometres per hour. The plane will carry three hundred forty-three people. The fuel tanks can hold seventeen thousand eight hundred litres of fuel and the total passenger cargo load may be as much as forty-eight thousand kilograms. Wow! See you soon.

Write the numerals to tell –

- **a** how high the plane will fly 9000 m
- **b** how fast 920 km/h
- **c** how many people 343
- **d** how much fuel 17 800 ℓ
- **e** how many kilograms of cargo (including passengers as cargo) 48 000 kg

See activity 5, page 20b.

See activity 6, page 20b.

RESOURCES

another form of evaluation

for Progress Check — page 7

Write a riddle.

1. Write a number with 4 digits — the only one to write;
   Use the digit 5 because no others are in sight. 5555
2. Write a number with 5 digits — the largest you can do;
   Use only a bunch of eights and the digit 2. 88882
3. Write a number with 6 digits — the smallest it can be;
   Use only a bunch of fours and the digit 3. 34444

We have only 10 different digits. The location of each digit tells how to read the number. It also tells how many. Pretend the numbers below tell how many pennies you have. Which would you rather have?

4. 28 or 82	5. 822 or 228
6. 282 or 228	7. 2282 or 2822
8. 8222 or 2822	9. 28,222 or 22,822

Write the numbers in each set in order from least to greatest.

10. 722, 277, 727, 27, 72
    27, 72, 277, 722, 727
11. 6868, 866, 6688, 8668, 8866
    866, 6688, 6868, 8668, 8866
12. 90,543, 53,904, 91,403, 43,910, 19,345
    19,345, 43,910, 53,904, 90,543, 91403

Match the place value listed in column 2 to the value of the digit 7 in column 1.

Column 1		Column 2
13. 222,272	b	a) ones
14. 272,222	e	b) tens
15. 222,227	a	c) hundreds
16. 722,222	f	d) thousands
17. 222,722	c	e) ten-thousands
18. 227,222	d	f) hundred-thousands

for Progress Check — page 13

Use a number line if you need help. What number is —

1. halfway between 20 and 30? 25
2. halfway between 200 and 300? 250
3. halfway between 2000 and 3000? 2500

What number is —

4. halfway between 80 and 90? 85
5. halfway between 800 and 900? 850
6. halfway between 8000 and 9000? 8500

Round each number to the nearest ten.

7. 63 60 8. 85 90 9. 28 30
10. 35 40 11. 96 100

Round each number to the nearest hundred.

12. 687 700 13. 450 500 14. 923 900
15. 805 800 16. 350 400

Round each number to the nearest thousand.

17. 6063 6000 18. 5670 6000 19. 3500 4000
20. 8647 9000 21. 7500 8000

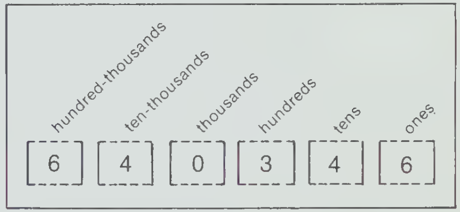
for Checkout — page 20

1. Write the smallest 6-digit number you can. Use the digits 3, 6, and 7 at least once. 333,367
2. The class needs 346 sheets of red construction paper for a special project. It only comes in packages of 100 sheets. How many packages should be ordered? 4
3. A certain book contains 10 words per line. The author wrote a paragraph of 78 words. How many lines long is the paragraph? 8
4. An office needed 4500 paper clips. How many boxes of 1000 clips should the secretary order? 5
5. Write the numeral for each number.
   a) four thousand two hundred fifty-five 4255
   b) six hundred eight 608
   c) seven thousand nine hundred twenty 7920
   d) three thousand four hundred two 3402
   e) twenty-eight thousand 28,000

activities

1. things poster board; paper strips; felt pen

Make a chart and number strips like this.

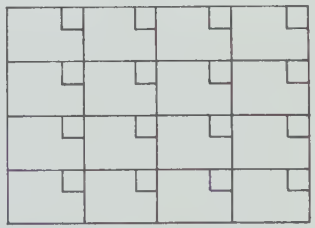


Slide the strips up or down to change the digit in each position.

Types of practice:

- Show the least 2-digit numeral.
- Show the greatest 5-digit numeral.
- Read the numerals.
- Show the number that is 1000 more than 77,592.

2. things spirit master as shown; pencils of two colors



Provide 1 copy for each pair of pupils.

Pupils select and write a different 2-digit number in the large section of each square. Regular pencil, please.

To claim a square the pupils, in turn, must round a number shown to the nearest ten and write the rounded number in the small square in the corner. Each player uses a pencil of a different color. The first to claim a row, column, or diagonal wins.

Variations:
1. Use 3-digit numbers:
   - Round to the nearest tens.
   - Round to the nearest hundred.
2. Use 4-digit numbers:
   - Round to the nearest hundred.
   - Round to the nearest thousand.

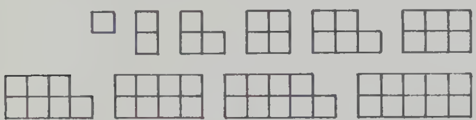
3. things small plastic dog; string

With those who cannot visualize the **nearest ten,** use this technique: Show a number line. The chalkboard, overhead projector, or floor will do.

Tie the string to the dog to serve as a leash. Pretend each mark on the number segment is a street. *Take your dog for a walk. You are tired, so you stop at 34th Street. But your dog wants to keep going. Does he need more leash to get to 30th Street or to 40th Street?* Have the youngsters use the leash to measure.

4. things graph paper

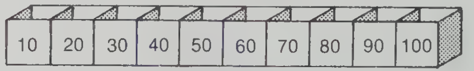
Have the pupil cut out number blocks for 1 through 10 as shown.



Now have him find the numbers that he can name when counting by 2s. Does he notice anything special about these number blocks? about the remaining number blocks? Can he tell what makes a number **even?** other numbers **odd?**

5. things 10 same-size boxes (milk cartons); slips of paper

Show a different multiple of 10 on one face of each box.



On another face show a multiple of 100 (100 through 1000). On a third face show a multiple of 1000 (1000 through 10,000); on the fourth, a multiple of 10,000 (10,000 through 100,000).

For practice in rounding to the nearest ten, turn the boxes to show the multiples of 10. Place them side by side in order. Write 2-digit numerals on the slips of paper, one per slip. The pupil is to select a slip, round the number to the nearest ten, and then place it in the appropriate box.

Change the set of numbers and the position of the boxes to practice rounding to hundreds, thousands, ten-thousands—whatever type of practice is needed.

Save the boxes. You'll use them again.

6. things set of numeral cards 0 through 9

The cards are laid out facedown at random. Pupils play in pairs. Each player selects three cards and arranges a number—not letting his opponent see the number. The object of the game is for the player to determine the number arranged by his opponent. To do this he needs to know:
- The three digits used in the number
- The correct place value of each digit

This information is obtained by questioning. A record of replies as shown below should be kept by each player.
- PV for correct digit in appropriate place-value position
- Digit for naming a digit, but not in the appropriate place-value position

Possible digits
0 1 2 3 4 5 6 7 8 9

Numbers guessed			Replies	
h	t	o	PV	Digits
1	2	5	0	1
6	5	2	1	1
2	5	6	2	0

Suppose player A arranges the numeral 356. Player B might ask, "Is your number 125?" Player A answers, "No, but you have 1 digit."

After player B has asked a question and player A has responded, player A asks a question and player B responds. Play continues until one of the players has determined his opponent's number.

additional learning aids

concept—chapter objectives 1, 3, 4, 5, 6, 7

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
  Spirit Masters: W 1, 2
Mathematics Involvement Program, SRA (1971)
  Cards: 233, 35
Visual Approach to Mathematics, level 3, SRA (1967)
  Visuals: 2, 3, 7

other learning aids (described on page 72d)
  Abacus board, Abacus Spinner Game, Chip Trading, Place Value I and II, Tally Counter

notation—chapter objective 2

SRA products

Diagnosis™: an instructional aid—Mathematics Level A, SRA (1973)
  Probe: L-8

other learning aids—Fundamath, Japanese abacus, Ranko

2 COMPUTATION + AND −

before this chapter the learner has—

1. Rounded 2-digit numbers to the nearest ten or hundred
2. Rounded 3-digit numbers to the nearest ten or hundred
3. Mastered the addition and subtraction facts
4. Added and subtracted two 2-digit numbers
5. Added and subtracted two 3-digit numbers

in chapter 2 the learner is—

1. Mastering estimating and finding the sum or difference of any two 2-digit numbers
2. Mastering estimating and finding the sum or difference of any two 3-digit numbers
3. Finding the sum of three 3-digit numbers
4. Finding the sum of five 2- or 3-digit numbers
5. Estimating and finding the sum of two 4-digit numbers
6. Using data given in a table to solve one-step word problems
7. Deciding when an estimate is sufficient and when an accurate answer is necessary

in later chapters the learner will—

1. Estimate and find the sum of any two 4-digit numbers
2. Estimate and find the sum of any three 4-digit numbers
3. Estimate and find the sum of any four 3-digit numbers
4. Estimate and find the difference of any 3-digit number subtracted from any 4-digit number

Notes & Things

This chapter is a comprehensive review of addition and subtraction. The only assumptions that have been made are that the pupils have knowledge of the addition and subtraction facts and that they have had some experience with adding two 2-digit numbers.

Mastery of the addition and subtraction facts is a must. Any child who does not know them will continue to carry an impossible burden. It's silly to try to get a child to complete 368 + 279 when he doesn't know 8 + 9. Work on the number facts could not be appropriately included in the pupil pages at this level. But there are many ideas in the Resource Section (page 45a) to help you find suitable extra practice for any pupils who need it. This chapter is designed so that you can constantly monitor an individual's skill strengths and weaknesses. Every set of exercises has been carefully controlled. Each sequential skill is examined independently. You will be able to identify specific trouble spots and, with minimum time and effort, give special help to correct the problems.

There is nothing more boring than straight review. The ideas from chapter 1 are used again so that the necessary review can be put into a new context for the learners. The rounding skill is put to work to help make sure that the learners get reasonable answers. It will take a long time before they appreciate that there is no one right way to round a number when it is being used to make an estimate. The whole thing is a very personal affair. An estimate is made by using mental computation—not paper and pencil. Some people are better at mental computation than others. Whatever an individual can process mentally is O.K. as a start, but it is *very* important that the estimates be made mentally. This will require patience, but it will all be worth it.

This chapter will let you establish a good "human" math group. Use peer tutors whenever possible. The idea of helping one another to develop skills can start now. (Unfortunately, *helping* is defined by many children as letting someone copy their answers. Now is your chance to correct this idea.) You can't be everywhere at once. Matter-of-fact acceptance that in this area John needs help, in another area Sara needs help, and so on, can do a great deal to promote a good, healthy, working classroom.

things

real or play money
at least 1 dictionary

For the extra activities you will want to have these things available:
- 2 to 4 beanbags or rubber rings
- 9″ fluted-edge paper plate
- 6″ paper plate

goal Think about and explore ideas through a picture clue.

page 21 The supermarket is the place to find good old-fashioned arithmetic used. Use this page to investigate who knows what about shopping for food. Every child probably has been in a food store but how much did he observe about this specialty store?

You can start off with some direct questioning to get the discussion going. What items do you suppose every family buys in a grocery store? If a store contained only those things that a family needed to survive, would the store look different? How? Why are people so concerned about the price of food? What things does a family have to pay for other than food? Why do stores advertise in newspapers? What foods do they feature in the ads? Why? Could your community get along without a grocery store? Why do stores have cash registers? What does the cash register do? Have you ever looked at the paper tape that comes from a register? What purpose does it have?

From here, you can appreciate the possibilities of further discussion, of some on-location research, of more involvement in this regular family activity. And certainly you have an excellent basis for entering the chapter in the book.

lesson Page 22

goal Survey—ability to estimate, add, and subtract 2- and 3-digit numbers

things real or play money

warm-up Have a pupil pretend he is a customer buying 2 items from you. Sell one item for 98¢ and one for 36¢. Charge \$1.56 (no tax necessary). Don't point out that this amount is incorrect. Play it straight. Act through a similar scene with another pupil. Continue until someone suspects you. You may actually charge the correct amount once or twice. Pupils who can estimate should catch you quickly. Discuss the necessity of estimating costs to avoid expensive errors.

page 22 Pupils are challenged to check on the checkout clerk. Do not expect them to answer the questions quickly and with confidence. Consider this a preview of the skills to be reviewed and mastered in this chapter. Emphasize the learning goal—the goal everyone is striving for.

Pupils who indicate previous mastery of these skills can proceed to the chapter Progress Checks (in sequence) to verify their skills or to identify those skills that require more instruction.

Have you ever thought of checking the checkout clerk?

1. Mr. Ramsey always double-checks when he buys something at the store. He buys a large can of grapefruit juice for 42¢. He also buys a box of crackers for 49¢. First he estimates his bill by rounding.

   42¢ and 49¢ — THINKS → $\begin{array}{r} 40 \\ +\ 50 \\ \hline \end{array}$

   About how much does he owe? 90¢
   Is this exactly what he owes? No
   What is the exact amount? 91¢

2. Mr. Ramsey bought two packages of meat. The chicken was \$1.67. The ground beef cost \$1.04. How much did he owe for meat? \$2.71

3. Mr. Ramsey's bill was \$3.62 for everything. He paid with a five-dollar bill (\$5.00). How much change should he get? \$1.38

Sometimes you have to compute an exact answer. Sometimes you can estimate an answer.

YOUR GOAL is to get good at computation and estimation.

22

lesson Pages 23, 24

goal Rounding two 2-digit numbers and estimating their sum

memo Emphasize again that a number must be rounded before it can be used as an estimate. Estimates are done without paper and pencil.

page 23 Encourage the use of a number line for those pupils who have difficulty in rounding. Focus on the halfway mark. Is the number to be rounded located to the right or to the left of halfway? Let them continue to use the number line as long as is necessary.

Exact answers are not required on problem 3. This will let you have your first check on two skills:

1. Ability to round.
2. Ability to add multiples of 10. (You'll be able to spot those pupils who don't know the easy addition facts.)

The following pages will confirm whether or not the individual has a specific problem in either of these skills.

---

1. Round each number. Then estimate the total for each pair of items.

a	b	c	d
78¢ – sugar	43¢ – mustard	65¢ – cheese	28¢ – beans
51¢ – tea	54¢ – hotdogs	17¢ – toothpicks	37¢ – rye bread
estimate – ? $1.30	estimate – ? 90¢	estimate – ? 90¢	estimate – ? 70¢

e What is the exact total for each pair of items?
a $1.29 b 97¢ c 82¢ d 65¢

f Are these totals different from the estimates? Yes

g If Mr. Ramsey had $5.00, would he have enough money to pay for everything listed in this problem? Yes

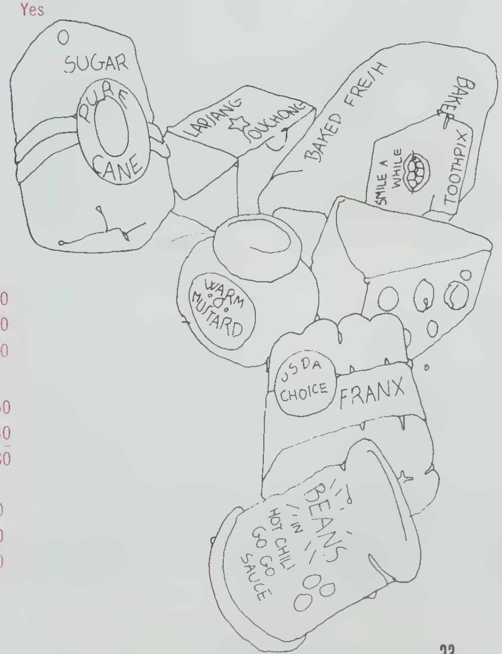
2. A candy bar is 53¢ and a bag of peanuts is 28¢. Mark estimates they will cost about 80¢.

a The clerk said, "79¢, please." Is that close to Mark's estimate? What is the correct total?
Yes 81¢ (plus tax if any)

b How did Mark make his estimate?
By rounding 53 to 50 and 28 to 30 and adding. Sum of estimates = 80¢

3. Round each number. Then *estimate* each sum.

	Problem	Estimate
a	30 + 45	30 + 50 = 80
b	40 + 16	40 + 20 = 60
c	50 + 25	50 + 30 = 80
d	60 + 34	60 + 30 = 90
e	36 + 42	40 + 40 = 80
f	47 + 21	50 + 20 = 70
g	16 + 53	20 + 50 = 70
h	52 + 25	50 + 30 = 80
i	27 + 33	30 + 30 = 60
j	65 + 35	70 + 40 = 110
k	69 + 21	70 + 20 = 90
l	46 + 44	50 + 40 = 90



23

See activity 1, page 45a.

goal Practice in estimating and adding two 2-digit numbers

warm-up (Optional) Discuss inventory management. It is a big job for business. The buyer must decide how many items to buy initially, how many are on hand, and when to reorder. Large companies use computers to do this work. Small companies have people do the figuring. Remind them that mistakes cost money.

page 24 Perhaps some pupils will be able to use mental computation on this page. GREAT! Give praise. But if a mistake is made, ask that pupil to prove his answer with paper and pencil.

In problems 2 and 3, please notice that the estimates may vary from those given in the answer key. 39 + 7 can be thought of as 40 + 7 or 40 + 10. Both are **good** estimates. Notice also that renaming ones is a skill needed to find most of the exact answers. Is this a source of pupil error?

1. The Groove Record Shop has all the top ten records. The shop keeps some records in the bins and extras in the back room. If the total number of copies drops below 50, more are ordered.

Which records should they reorder today?

		Album Title	Bin	Back Room
	a	The Moonstones	23	47
√	b	Too Many Tears	12	31
	c	The Spokesmen	19	36
√	d	Little Lies	25	19
	e	Roof Top Roses	34	20
√	f	Salad Bowl Blues	33	14
√	g	Monkey Shines	25	24
√	h	The Surfs Sing	28	7
	i	Stroodle Doodle	32	22
√	j	Peaches 'n' Cream	19	30

Copy and complete the tables. *Estimate* each sum. Then find the exact sum. Is the estimate close to the exact answer? Estimates may vary.

2.

		Estimate	Exact
a	39 + 7	? 50	? 46
b	41 + 7	? 50	? 48
c	44 + 7	? 50	? 51
d	45 + 7	? 60	? 52
e	46 + 7	? 60	? 53

3.

		Estimate	Exact
a	49 + 17	? 70	? 66
b	51 + 17	? 70	? 68
c	55 + 17	? 80	? 72
d	56 + 17	? 80	? 73
e	60 + 17	? 80	? 77

How are these two problems alike?
The second addend is 7, so it is rounded up.

How are they different?
The first addend varies.
(Listen for other good answers.)

Consider the weaknesses in this type of inventory system. Can your pupils develop a better system?

1. Copy and complete the table. *Estimate* each sum. Then find the exact sum. Is your estimate close to the exact answer?

		Estimate	Exact
a	32 + 46	? 80	? 78
b	52 + 18	? 70	? 70
c	21 + 49	? 70	? 70
d	21 + 27	? 50	? 48
e	33 + 19	? 50	? 52
f	54 + 16	? 70	? 70
g	24 + 56	? 80	? 80

2. When you estimate a sum, can the estimate ever equal the exact sum? Yes (See b, c, f, and g.)

3. Murphysville built a new school to combine two of its elementary schools. It will have 50 teacher's desks. These will come from the two old schools. One old school has 28 desks. The other has 37. Will this be enough desks for the new school? Yes

   Do you need to know the exact number of old desks to be sure of the answer? No

4. George has 80¢. He wants to buy two magazines. One costs 35¢. The other costs 45¢. Estimate the total cost. 90¢

   Do you need to know the exact total cost to be sure that he has enough? Yes

5. Can you think of times that you must find an exact answer? Accept reasonable answers.

25

lesson Pages 25, 26, 27

goal Practice in estimating and adding two 2-digit numbers

page 25 It will take a little discussion before some learners appreciate that many times common sense and an estimate are good enough; but there are other times when exact answers are required. In those cases the exact answer must be correct.

things for each pair of pupils: 1 die

The first player rolls the die and names the faceup number. The other player then turns the die so that any one of the four adjacent sides is up and adds this number to the first number. The first player again turns the die to an adjacent side and adds this number to the previous sum. The game ends when a player reaches 20 or forces his opponent to reach a sum over 20.

goal Examining common renaming errors; practice in adding two 2-digit numbers requiring renaming

page 26 Discuss how estimating can help catch an error in computed answers. Help those who are struggling through the renaming review.

You may want to check answers after the pupil has completed problems **1a** through **1e**. No trouble? Go on to page 27. Pupils who make errors need individual help. This may be the time for a peer tutor.

Four friends handed in their homework. They all had different answers for 28 + 29.

Bob

$$\begin{array}{r} 29 \\ +28 \\ \hline 417 \end{array}$$

John

$$\begin{array}{r} 29 \\ +28 \\ \hline 57 \end{array}$$

Mark

$$\begin{array}{r} 29 \\ +28 \\ \hline 147 \end{array}$$

Ted

$$\begin{array}{r} 29 \\ +28 \\ \hline 47 \end{array}$$

Do you think that Bob and Mark estimated before they added? No

How did Bob add? What did he forget? $9 + 8 = 17$, $2 + 2 = 4$. He forgot to add 1 ten to the 4 tens.

What slip did Mark make? Hard to tell—probably wrote 1 from 17 in the hundreds place instead of adding it to the tens.

What help does Ted need? He needs to practice renaming in addition.

John's answer was correct.
He found the answer in two steps.

Step 1 Step 2

$$\begin{array}{r|c|c|} & \text{tens} & \text{ones} \\ & 2 & 9 \\ + & 2 & 8 \\ \hline & & \end{array} \rightarrow \begin{array}{r|c|c|} & \text{tens} & \text{ones} \\ & \overset{1}{2} & 9 \\ + & 2 & 8 \\ \hline & & 7 \end{array} \rightarrow \begin{array}{r|c|c|} & \text{tens} & \text{ones} \\ & \overset{1}{2} & 9 \\ + & 2 & 8 \\ \hline & 5 & 7 \end{array}$$

What did he do in step 1?
Added ones, renamed 17 as 1 ten, 7 ones.

What did he do in step 2?
Added all the tens and wrote his sum of the tens.

1. Your turn. Try to use the steps that John used.

a $\begin{array}{r} 36 \\ +47 \\ \hline 83 \end{array}$	**b** $\begin{array}{r} 48 \\ +15 \\ \hline 63 \end{array}$	**c** $\begin{array}{r} 65 \\ +29 \\ \hline 94 \end{array}$	**d** $\begin{array}{r} 62 \\ +29 \\ \hline 91 \end{array}$	**e** $\begin{array}{r} 34 \\ +37 \\ \hline 71 \end{array}$
f $\begin{array}{r} 53 \\ +28 \\ \hline 81 \end{array}$	**g** $\begin{array}{r} 46 \\ +36 \\ \hline 82 \end{array}$	**h** $\begin{array}{r} 33 \\ +47 \\ \hline 80 \end{array}$	**i** $\begin{array}{r} 25 \\ +57 \\ \hline 82 \end{array}$	**j** $\begin{array}{r} 57 \\ +34 \\ \hline 91 \end{array}$
k $\begin{array}{r} 79 \\ +19 \\ \hline 98 \end{array}$	**l** $\begin{array}{r} 49 \\ +43 \\ \hline 92 \end{array}$	**m** $\begin{array}{r} 26 \\ +35 \\ \hline 61 \end{array}$	**n** $\begin{array}{r} 47 \\ +27 \\ \hline 74 \end{array}$	**o** $\begin{array}{r} 58 \\ +32 \\ \hline 90 \end{array}$

26

things large piece of heavy paper; 2 to 4 beanbags or rubber rings

Pupils make a calendar for the month on the paper. The calendar now becomes the game board. The rubber rings are tossed onto the calendar to generate problem numbers.

2 rings—add or subtract
3 or 4 rings—add

In each case the answer is the pupil's score.

When 2 rings are used, pupils can practice comparing the numbers.

What does the word *assume* mean?
One dictionary says *assume* means "to take for granted; suppose something to be a fact."
What does your dictionary say about the word?

Can we assume that you know how to add two 2-digit numbers? Why don't you check and make sure.

PROGRESS CHECK

Try these. Skill: Adding two multiples of 10

1. 20 + 50 = 70	2. 60 + 30 = 90	3. 80 + 60 = 140	4. 40 + 70 = 110	5. 90 + 30 = 120	6. 60 + 50 = 110

Next step. Try these. Skill: Adding two 2-digit numbers, no renaming

7. 34 + 45 = 79	8. 75 + 23 = 98	9. 57 + 21 = 78	10. 46 + 33 = 79	11. 81 + 18 = 99	12. 72 + 26 = 98

One more step. Keep adding. Skill: Renaming ones to tens

13. 67 + 25 = 92	14. 49 + 28 = 77	15. 52 + 38 = 90	16. 36 + 48 = 84	17. 79 + 17 = 96	18. 36 + 59 = 95

If you get these right, too, you can assume you know how to add. Skill: Renaming both ones and tens

19. 53 + 49 = 102	20. 76 + 27 = 103	21. 85 + 48 = 133	22. 69 + 66 = 135	23. 87 + 53 = 140	24. 48 + 93 = 141

27

STOP See activity 2, page 45a.

See activity 3, page 45b.

goal Progress Check—adding two 2-digit numbers

things at least 1 dictionary

page 27 You may wish to introduce the **folded-paper** technique for recording answers. Have the pupil place his folded paper directly under the row of problems in the book and record his answers on the paper. He then folds his answers under and places the fold of the paper under the second row, and so on. Those who need to show the renaming step will have to copy the third and fourth rows.

Each row of problems deals with a different skill level in addition. The skills are identified on the answer key. Errors in the first two rows indicate a need for practice with addition facts.

Watch for error patterns such as these in the last two rows:

67 + 25 = 812	The 12 was not renamed. Encourage an estimate.	67 + 25 = 82	Renamed ten was forgotten.

If renaming problems are serious, take a place-value attack. It will take patience, but it works for many youngsters.

	tens	ones	
4 tens 6	4	6	
+ 2 tens 5	+ 2	5	
6 tens 11	1	1	Add the ones.
	6		Add the tens.
	7	1	How many in all?

lesson Pages 28, 29, 30

goal Examining estimation as a way to check the reasonableness of a computed answer; extension of addition skill to 3-digit numbers

memo Only pupils who have demonstrated skill with two 2-digit numbers should go on to this page. The others should continue practice.

page 28 Independent learners are on their own. You'll want to discuss the page with the others. Examine both the advantages and disadvantages of estimation as a check. Emphasize that an estimate tells if the answer is **reasonable**, not if it is correct.

If rounding is still causing a problem, encourage the pupils to continue the use of a number line.

Building skills is like building a house. You start with the foundation and work up. Being able to add 2-digit numbers is the foundation. Adding 3-digit numbers is the next step.

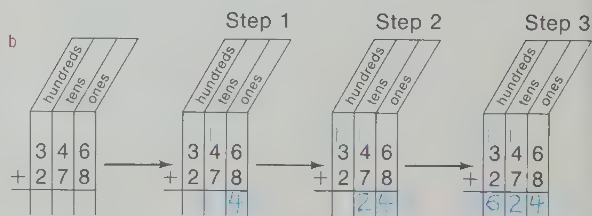
Ralph and Jim did this problem for homework.

Who got the problem right? Jim
Do you think Ralph estimated the answer before he added? No

It is usually a good idea to estimate an answer first. Then find the exact sum. Your estimate will help you tell if your answer is reasonable. Does it tell you that your answer is correct? No!

1. There is more than one way to estimate a sum.
   a Estimate the answer for 346 + 278 by rounding to the nearest ten. 350 + 280 = ? 630
   b Estimate the answer by rounding to the nearest hundred. 300 + 300 = ? 600
   c What is the exact answer for 346 + 278? 624
   Which of your estimates was closer to the exact answer? a
   Which estimate was easier to make? b

Here is how Jim computed the exact sum.



28

See activity 4, page 45b.

1. Find the sums for these problems. Estimate the answer first. Then think through each one the same way Jim did.

a	b	c	d	e
364 + 248	596 + 345	639 + 195	427 + 296	148 + 675
612	941	834	723	823

2. Write as many exact answers as you can without copying the problem.

+	100	
320	420	
421	a	521
514	b	614
227	c	327
368	d	468

+	250	
108	e	358
430	f	680
526	g	776
418	h	668
607	i	857

+	431	
121	j	552
109	k	540
244	l	675
375	m	806
399	n	830

3. The fire department limits the number of people who can be in a theatre. The Roxy Theatre has a limit of 590 people. The ticket taker collected 284 children's tickets and 297 adults' tickets. Is the theatre breaking the fire department's rule? Try to answer by estimating. *

4. An elevator has a sign inside that says: Four people get in the elevator. Another man wants to move a desk to another floor. There is room. But is it safe? The masses of the people and the desk are listed.

Warning: Do not overload

In case of emergency, use the alarm or communicator to summon assistance

10 **Maximum no. of persons** 700 **Maximum kilograms**

14 450 Elevator No. 785 West End Ave. Location

Mr. Larson 94 kg	Miss Small 92 kg	Mr. Ross 103 kg
Mrs. March 83 kg	Mr. Fried 117 kg	desk 191 kg

Rounded to hundreds, 700 kg

a Estimate the total mass. Is this a safe load? Probably

b Would there be any reason to compute the exact mass?

No. The limit is set to allow for some overage.

* If you round to hundreds, your estimate is 600 — the theater is breaking the fire department's rule. If you round to tens, your estimate is 580 — they are not breaking the rule.

29

goal Practice in adding two 3-digit numbers.

page 29 Let the needs and abilities of each pupil determine his assignment. Problems 1 and 2 focus on computation, problems 3 and 4 on applications of the skill.

goal Practice in adding three and five 2- and 3-digit numbers

memo Don't assume that the youngsters know that bowling is a game. Check it, please, or this page won't make much sense.

page 30 Let the pupil's needs and abilities determine his assignment. The columns are long, but the problems are not difficult—no curve balls to watch out for.

Notice again that there is no one correct answer when estimating—there are just good estimates and poor estimates. If the estimate helps a pupil decide the answer is reasonable, it is a good estimate.

(Encourage estimating to the nearest ten.)

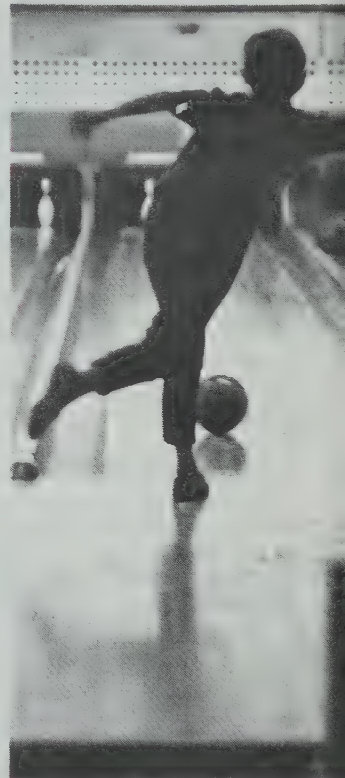
1. Jake loved to bowl. He kept a record of his scores for 6 weeks.

	Week 1	*Week 2*	*Week 3*	*Week 4*	*Week 5*	*Week 6*
	105	114	80	100	95	101
	100	90	100	100	97	106
	110	104	130	110	115	100
To 100's	300	300	300	300	300	300
To 10's	320	300	310	310	320	310

a Estimate the total scores for each week.

b Can you tell his best score without computing? No

c Which three-game score was the highest? Week 1 (315)



2. Jake was on one of the six teams in a bowling tournament. Each team had five members. Prizes are given to the team with the highest team score on a single game. Here are the scores for the players of each team.

	Sox	*Cats*	*Astros*	*Cubs*	*Kings*	*Dukes*
	106	100	96	123	91	80
	112	126	53	114	137	93
	105	91	114	108	102	120
	89	50	109	121	96	83
	78	135	87	73	105	72
To 100's	500	500	500	500	500	500
To 10's	500	510	460	530	540	440

a Estimate the total scores for each of the six teams.

b Which team probably won first prize? If you were on the team that came in second, would you be satisfied with an estimate of the score? Kings

No, not if you know what happens when you round numbers to get estimates.

c Which team got the booby prize? Are you sure?

Dukes Not until you find the exact totals for the Astros and the Dukes.

30

Can we *assume* that you know how to add two 3-digit numbers? It's not that we don't trust you, but wouldn't it be better for you to say, "I know I can add two 3-digit numbers!"

PROGRESS CHECK

Try these. Skill: Adding two 3-digit numbers, no renaming

1. $500 + 200$ (700)	2. $400 + 307$ (707)	3. $610 + 201$ (811)	4. $720 + 132$ (852)	5. $546 + 231$ (777)

Now try these. Skill: Renaming ones

6. $524 + 357$ (881)	7. $675 + 216$ (891)	8. $463 + 528$ (991)	9. $388 + 407$ (795)	10. $209 + 107$ (316)

This row will prove you know. Keep on adding. Skill: Renaming ones, tens, and hundreds

11. $756 + 164$ (920)	12. $825 + 198$ (1023)	13. $297 + 703$ (1000)	14. $639 + 584$ (1223)	15. $786 + 867$ (1653)

When you can get all problems like the ones on this page right, you deserve

CONGRATULATIONS

If you need more practice, now is the time to get it. First find out the kind of errors you made. Do you know the facts? Did you estimate before you started computing? Did you make an error in renaming?

31

See activity 5, page 45b.

Follow suggestions given in the guide column; then use activity 5, page 45b.

goal **Progress Check**—adding two 3-digit numbers

page 31 Each row of problems focuses on a different skill level. These skills are identified on the answer key. Use this format to help you prescribe additional necessary help.

Errors in the first row signal the need for more practice with the addition facts.

Watch out for these types of errors:

$$\begin{array}{r} {}^{1}524 \\ +357 \\ \hline 971 \end{array}$$ Renamed to the wrong position

$$\begin{array}{r} {}^{1}209 \\ +107 \\ \hline 406 \end{array}$$

$$\begin{array}{r} {}^{1}529 \\ +326 \\ \hline 5616 \end{array}$$ Not aligned

If you have used the place-value approach, you will want to continue.

$$\begin{array}{r} t\ o \\ 4\ 8 \\ +\ 2\ 9 \\ \hline \end{array}$$ Add 8 + 9. Add 40 + 20. How many in all?

$$\begin{array}{r} h\ t\ o \\ 6\ 3\ 7 \\ +\ 2\ 4\ 6 \\ \hline \end{array}$$ Add 7 + 6 Add 30 + 40. Add 600 + 200 How many in all?

How is the first problem different from the second? How are they the same?

Try to make the transition to the short form by continuing to use the same place-value clues.

$$\begin{array}{rl} h\ t\ o & \\ 3\ 4\ 8 & \\ +\ 1\ 9\ 4 & \\ \hline 1\ 2 & \text{Add ones.} \\ 1\ 3\ & \text{Add tens.} \\ 4\ \ \ & \text{Add hundreds.} \\ \hline 5\ 4\ 2 & \text{How many in all?} \end{array}$$

$$\begin{array}{rl} h\ t\ o & \\ {}^{1}3\ {}^{1}4\ 8 & \\ +\ 1\ 9\ 4 & \\ \hline 5\ 4\ 2 & \text{Add ones. Add tens. Add hundreds.} \end{array}$$

lesson Page 32

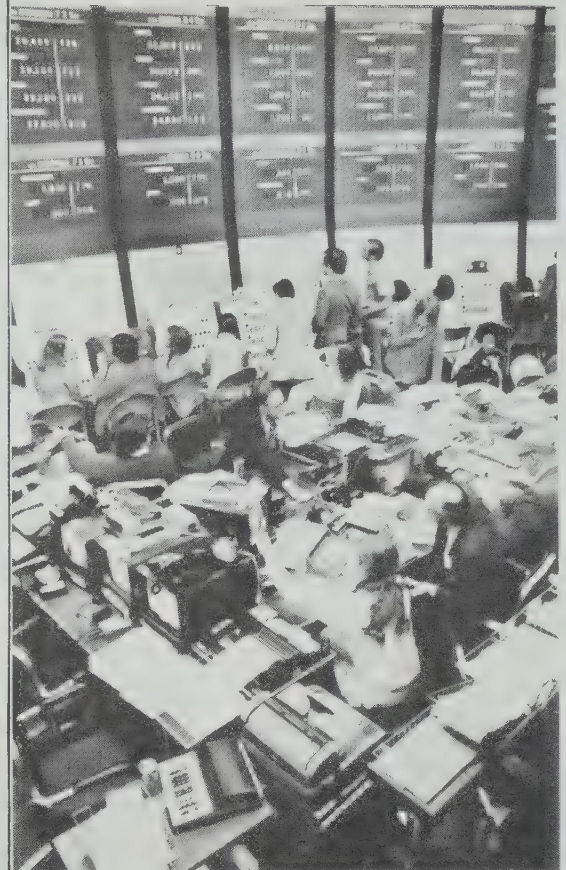
goal Review of the use of rounded numbers in decision-making.

memo Use only with pupils who do not need special help on skills on the last Progress Check.

page 32 This page is for discussion only. Some statistics for the federal election of 1972 might help in discussing 1a.

Total votes polled	9 966 148
Liberal votes	3 679 021
Conservative votes	3 357 094

Relate the discussion for problem 2 to the pupils' world. If you want to buy only 1 item and it is 1 cent cheaper across town, would you spend the bus fare to go? If you were planning to buy 100 of those items, would you spend bus fare to go? (It depends on the bus fare.) How many would you have to buy before you really saved money by going to the other store?



1. Have you ever heard someone say that an election is "a close race"? What does that mean?

   a Millions of people vote when there is an election. Imagine you are a TV reporter giving election results in a close race. Would you round votes to the nearest ten? to the nearest hundred? to the nearest thousand? to the nearest million? Explain. It depends on how many votes were cast and how close the votes are.

   b If you heard someone say "13 million people voted today," what would you think? Did exactly 13 million people vote? Explain.
   The number is an estimate. Exact number could be between 12 500 000 and 13 500 000.

   c Sometimes a man with 20 thousand votes will give up before all the votes are counted. Is that a silly thing to do? Why?
   No He may be way behind.

When you solve a problem, ask yourself, "What kind of answer do I need? Must I have an exact number? Would an estimate be just as good?"

2. It is 210 km from Nutown to Farley by road. It is 212 km from Nutown to Nero by road.

   a Someone asks you, "Which is farther from Nutown—Farley or Nero?" What would you answer? Does the 2 km difference matter much to a person driving to Farley or to Nero? No About the same dist[ance]

   b If you were in the highway planning department, would the 2 km difference matter? (Fact: It could cost over a million dollars to build 1 km of road.) Yes

32

See activity 6, page 45b.

Estimating helps prevent mistakes when you do subtraction too.

1. *Estimate* the answer for each problem. (Exact answer in parentheses.)

	a	b	c	d	e
	43 − 30	59 − 41	30 − 15	71 − 16	36 − 14
	10 (13)	20 (18)	10 (15)	50 (55)	30 (22)

	f	g	h	i	j
	76 − 19	43 − 24	68 − 30	95 − 46	87 − 38
	60 (57)	20 (19)	40 (38)	50 (49)	50 (49)

Compare your estimates with someone else's.
Were there any disagreements? There shouldn't be if both rounded to tens for their estimates.

Estimating can't tell you if your answer is right.

$$\begin{array}{rl r} 51 & \text{rounds to} & 50 \\ -25 & \text{rounds to} & -30 \\ \hline 26 & & 20 \end{array} \qquad \begin{array}{rl r} 35 & \text{rounds to} & 40 \\ -11 & \text{rounds to} & -10 \\ \hline 24 & & 30 \end{array}$$

is not equal to
But you know the
answer is reasonable.

is not equal to
But you know the
answer is reasonable.

2. Go back to problem 1 and find the exact answer to any five problems.

Complete.

3.

Problem	Estimate	Exact
49 − 18	a 30	b 31
88 − 18	c 70	d 70
37 − 18	e 20	f 19
76 − 18	g 60	h 58
55 − 18	i 40	j 37
94 − 18	k 70	l 76

4.

Problem	Estimate	Exact
69 − 8	a 60	b 61
68 − 18	c 50	d 50
67 − 28	e 40	f 39
66 − 38	g 30	h 28
65 − 48	i 20	j 17
64 − 58	k 0	l 6

For which problems is the exact answer the same as the estimate?
3 c, d
4 c, d

33

STOP See activity 7, page 45c.

lesson Pages 33, 34

goal Extension of estimation to subtraction; practice in estimating and subtracting two 2-digit numbers

memo This is another critical page that will help you find those pupils who need careful monitoring.

page 33 Carefully note the directions for problems 1 and 2. Watch for errors due to renaming. Page 34 will help these students. The 100-percenters should go directly to page 35.

goal Practice in subtracting two 2-digit numbers requiring renaming

memo Look also at page 35 so that you can determine the scope of the skill diagnosis.

page 34 Some pupils will simply need to be reminded of the steps involved in this skill. The page provides them sufficient guidance to operate independently.

Those who are in real trouble will need more help. *Where did the 17 come from? . . . Why did the 9 change to 8?* Listen to their reasoning. Try to determine where and why they are confused.

Adjust assignments to meet the learner's needs.

Jerry's basketball team scored 97 points in the first game. In the second game it scored only 68 points. The captain said, "That's awful. We dropped 39 points."

Jerry thought a minute. 97 points—that's almost 100, and 68 points is almost 70. 100 − 70 = 30

They hadn't dropped that many points. Did the captain subtract wrong? Yes

Jerry computed just to make sure. This is what his work looked like:

$$\begin{array}{r} \overset{8}{\cancel{9}}\,{}^{1}7 \\ -\,68 \\ \hline 29 \end{array}$$

Here is how he thought.

Step 1: Jerry renamed 97. Why? To get enough ones to subtract 8.

Step 2: Then he was ready to subtract.

Where did the 9 come from? 17 − 8 = 9

And how did the 2 get in the tens column? 8 − 6 = 2

Is this the way you think when you do a problem like this?

tens	ones
9	7
− 6	8

Step 1 Rename

tens	ones
$\overset{8}{\cancel{9}}$	${}^{1}7$
− 6	8

Step 2 Subtract

tens	ones
$\overset{8}{\cancel{9}}$	${}^{1}7$
− 6	8
2	9

1. Copy and subtract.
Think the way Jerry did.

a $\begin{array}{r}32\\-17\\\hline 15\end{array}$	b $\begin{array}{r}65\\-36\\\hline 29\end{array}$	c $\begin{array}{r}84\\-29\\\hline 55\end{array}$	d $\begin{array}{r}53\\-15\\\hline 38\end{array}$	e $\begin{array}{r}70\\-42\\\hline 28\end{array}$
f $\begin{array}{r}41\\-17\\\hline 24\end{array}$	g $\begin{array}{r}79\\-4\\\hline 75\end{array}$	h $\begin{array}{r}90\\-39\\\hline 51\end{array}$	i $\begin{array}{r}98\\-79\\\hline 19\end{array}$	j $\begin{array}{r}57\\-29\\\hline 28\end{array}$

34

STOP See activity 8, page 45c.

Bet you can solve a subtraction problem with two 2-digit numbers. If you can, you can solve problems with 3-digit numbers.

1. Let's prove it. Find the differences.

a	b	c	d	e
$\begin{array}{r} 90 \\ -\ 40 \\ \hline 50 \end{array}$	$\begin{array}{r} 700 \\ -\ 300 \\ \hline 400 \end{array}$	$\begin{array}{r} 540 \\ -\ 130 \\ \hline 410 \end{array}$	$\begin{array}{r} 785 \\ -\ 384 \\ \hline 401 \end{array}$	$\begin{array}{r} 365 \\ -\ 165 \\ \hline 200 \end{array}$

2. You're on your own. Find the differences.

a	b	c	d	e
$\begin{array}{r} 308 \\ -\ 205 \\ \hline 103 \end{array}$	$\begin{array}{r} 999 \\ -\ 333 \\ \hline 666 \end{array}$	$\begin{array}{r} 706 \\ -\ 502 \\ \hline 204 \end{array}$	$\begin{array}{r} 881 \\ -\ 551 \\ \hline 330 \end{array}$	$\begin{array}{r} 666 \\ -\ 345 \\ \hline 321 \end{array}$

f	g	h	i	j
$\begin{array}{r} 385 \\ -\ 166 \\ \hline 219 \end{array}$	$\begin{array}{r} 592 \\ -\ 283 \\ \hline 309 \end{array}$	$\begin{array}{r} 825 \\ -\ 644 \\ \hline 181 \end{array}$	$\begin{array}{r} 650 \\ -\ 419 \\ \hline 231 \end{array}$	$\begin{array}{r} 921 \\ -\ 478 \\ \hline 443 \end{array}$

Time to look at some more problems like the last row
Did you use a way like this?

Step 1	Step 2	Step 3
$\begin{array}{r} {\scriptstyle 4\ 11} \\ 7\cancel{5}\cancel{1} \\ -\ 164 \\ \hline 7 \end{array}$	$\begin{array}{r} {\scriptstyle 6\ 14\ 11} \\ \cancel{7}\cancel{5}\cancel{1} \\ -\ 164 \\ \hline 87 \end{array}$	$\begin{array}{r} {\scriptstyle 6\ 14\ 11} \\ \cancel{7}\cancel{5}\cancel{1} \\ -\ 164 \\ \hline 587 \end{array}$

Or did you do all the renaming first?

Step 1: Rename.	Step 2: Subtract.
$\begin{array}{r} {\scriptstyle 6\ 14\ 11} \\ \cancel{7}\cancel{5}\cancel{1} \\ -\ 164 \\ \hline \end{array}$	$\begin{array}{r} {\scriptstyle 6\ 14\ 11} \\ \cancel{7}\cancel{5}\cancel{1} \\ -\ 164 \\ \hline 587 \end{array}$

Either way is O.K.

lesson Pages 35, 36

goal Identification of ability in subtracting two 3-digit numbers requiring renaming

page 35 Exercises 1 and 2 are independent work. They will signal the additional kinds of help the pupil needs.

No renaming is required in the first two rows. So errors signal the need for more practice of subtraction facts.

The third row (f through j) will identify those pupils who need help with renaming.

Watch out for this error pattern. It's common but hard to spot.

$\begin{array}{r} {\scriptstyle 5\ 17\ } \\ \cancel{6}\cancel{7}8 \\ -\ 199 \\ \hline \end{array}$ Everything looks O.K. But that 17 should be 16.

And this one—**Ugh!**

$\begin{array}{r} {\scriptstyle 3\ 13\ } \\ \cancel{4}\cancel{3}5 \\ -\ 176 \\ \hline 261 \end{array}$ That's subtracting in reverse.

With these pupils, examine the two methods for renaming shown on the page—step by step; rename first, and then subtract.

See activity 9, page 45c.

goal **Progress Check** – subtracting two 3-digit numbers requiring renaming

page 36 Use the top of the page for discussion with your group of strugglers. Listen carefully for faulty reasoning. Follow through by correcting the errors during the discussion.

The Progress Check is independent work. These problems are finely sequenced. No two problems are exactly alike:

1. No renaming is needed.
2. Subtract from 0 in the ones place.
3. Subtract from 0 in the ones place; there's a 0 in the answer.
4. Rename in the tens.
5. Renaming in the tens results in a 2-digit answer.
6. Rename in the ones and in the tens.
7. Rename in the ones and in the tens, with 0 in the tens.
8. Rename all the way!

Learners who need additional help with renaming continue on to page 37. Those who have mastered this skill skip to page 38.

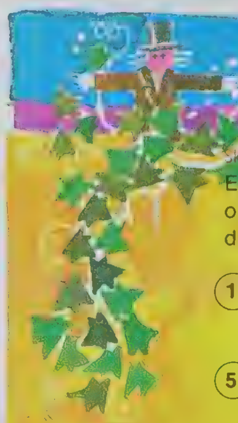
Dan was having trouble with subtraction.
Look at the three problems he did. Where did he make errors?

$$\begin{array}{r} 490 \\ -\,156 \\ \hline 346 \end{array}\ \times$$
Subtracted 0 from 6.

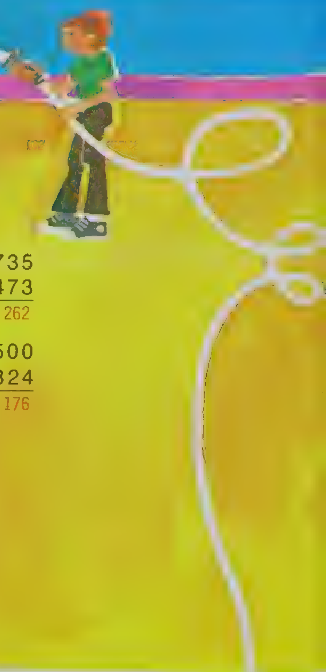
$$\begin{array}{r} {\scriptstyle 11}\\ 5\cancel{2}6 \\ -\,278 \\ \hline 348 \end{array}\ \times$$
Forgot to rename 5 hundreds.

$$\begin{array}{r} {\scriptstyle 2}\\ \cancel{3}0{\scriptstyle 1}2 \\ -\,178 \\ \hline 174 \end{array}\ \times$$
Forgot to rename tens.

If it was your job to help him, what would you do? Show him again when and how to rename tens and hundreds



PROGRESS CHECK



Skill: finding the difference between two 3-digit numbers

Each one of these problems is a bit different from the others. Copy each problem. Think about what you are doing as you subtract each problem.

1. $\begin{array}{r} 375 \\ -\,261 \\ \hline 114 \end{array}$	2. $\begin{array}{r} 390 \\ -\,152 \\ \hline 238 \end{array}$	3. $\begin{array}{r} 370 \\ -\,164 \\ \hline 206 \end{array}$	4. $\begin{array}{r} 735 \\ -\,473 \\ \hline 262 \end{array}$
5. $\begin{array}{r} 648 \\ -\,577 \\ \hline 71 \end{array}$	6. $\begin{array}{r} 827 \\ -\,568 \\ \hline 259 \end{array}$	7. $\begin{array}{r} 402 \\ -\,397 \\ \hline 5 \end{array}$	8. $\begin{array}{r} 500 \\ -\,324 \\ \hline 176 \end{array}$

If you can spot how each one is different from the others, you deserve an award. If you got them all right, you deserve an even bigger award.

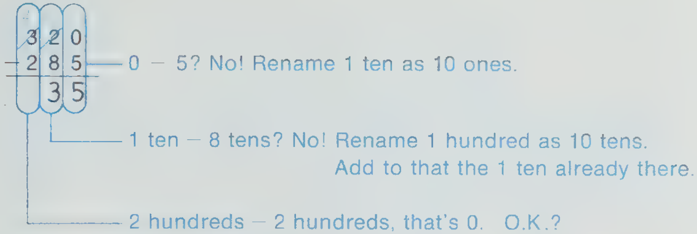
Don't be discouraged if you made some errors. The next page will help.

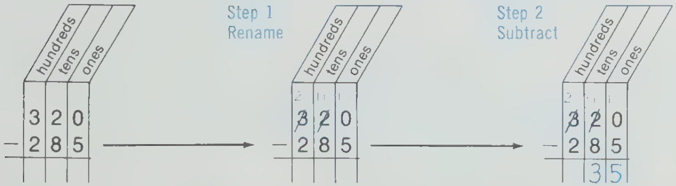
36

STOP Continue to text page 37 as directed.

Continue to text page 38 as directed.

Ken's father wants to trade in his old car. One used-car dealer has offered $320 for the car. Another dealer has offered $285. How much difference?





1. Copy each problem. Estimate each answer by rounding to the nearest hundred. Then find the exact answer. (Exact answer in parentheses.)

a 665 − 534	**b** 714 − 506	**c** 286 − 179	**d** 485 − 326	**e** 532 − 289	**f** 893 − 499
200 (131)	200 (208)	100 (107)	200 (159)	200 (243)	400 (394)

2. Subtract. Think carefully each time you rename a number.

a 934 − 228	**b** 628 − 179	**c** 605 − 417	**d** 211 − 19	**e** 456 − 148	**f** 751 − 164
706	449	188	192	308	587
g 305 − 198	**h** 600 − 456	**i** 740 − 473	**j** 807 − 398	**k** 700 − 623	**l** 900 − 891
107	144	267	409	77	9

37

lesson Pages 37, 38

goal Practice in subtracting two 3-digit numbers requiring renaming

memo Use this page only with those pupils who need additional help. This may be another good time to use peer tutors.

page 37 Pupils who need only a review can work on their own. The strugglers will need your help. Ask them to explain their work to you. Listen for faulty thinking. Then **lead** them through the sequence of steps.

Adjust the assignments to the pupil's needs. Don't risk turning off the pupil with unnecessary practice.

goal Examining real-world situations and determining when a small difference makes a big difference

memo Use this page only with pupils who have mastered subtracting two 3-digit numbers.

page 38 You may want to group these pupils to provide each group with at least one independent reader. The groups may operate on their own in a seminar. Each situation opens the door to

TALK

1. A company that makes hair spray is trying to figure out what to charge for its product. Another company charges $1.24. Should the new company charge the same or charge an even $1.25? It depends. What does it cost to make the new hair spray?
   - **a** If you were buying a can of hairspray, would a 1¢ difference matter to you? Have students give the reason for their answer.
   - **b** Pretend you are the president of the company. You expect to sell 2 million cans a year. Would the 1¢ difference in price matter? Yes ($20,000 difference)

2. Walter would like to buy a desk. It is $30\frac{1}{2}$ inches wide.
   - **a** Would it matter to you that the width of the desk is $30\frac{1}{2}$ inches? Would you be just as happy with a desk that is only 30 inches wide? No Yes
   - **b** Could the exact width of the desk make a difference? What if the door to Walter's room is narrow? Yes Maybe the desk could not be moved through the door.

3. A newspaper reporter writes a story that is 851 words long. Another reporter writes a story 793 words long. Estimates: 900 800
   - **a** Round to the nearest hundred. Then estimate the difference in the lengths of the stories. If you were reading the stories, would this difference matter much to you? 100 w Have students give the reason for their answer.
   - **b** Suppose you work for the newspaper. You need to know which story will fit best on page 2. Would you estimate or find the exact difference? (Words vary in length. Exact count wouldn't help much.)

4. Tell about a situation where it would be important to know the exact difference between two numbers. Report a situation where an estimate would be O.K. Answers will vary Examples: You need to know the exact difference when making ch in a store. You can estimate to find out if you have enough milk f the weekend.

38

When you add, how can you tell if your answer is right? An estimate will help you know your answer is reasonable. One way to check your answer is to reverse the order of the numbers to be added.

	286 + 904	**Estimate** →	300 + 900 = 1200
Exact ↓	286 + 904 = 1190	**Check** →	904 + 286 = 1190

Subtraction undoes addition. So you can also check addition by subtracting.

$$\begin{array}{r} 286 \\ +904 \\ \hline 1190 \end{array} \quad \textbf{Check} \quad \begin{array}{r} 1190 \\ -\ 904 \\ \hline ? \end{array} \quad \text{or} \quad \textbf{Check} \quad \begin{array}{r} 1190 \\ -\ 286 \\ \hline ? \end{array}$$

Where did this number come from?

1. Estimate, add, and check by reversing the order of the numbers added. (This method won't help much if you don't know your addition facts.) (Estimated answer in parentheses)

a	b	c	d	e	f
513 + 711	307 + 299	921 + 387	4185 + 4577	2649 + 5420	5876 + 3048
(1200) 1224	(600) 606	(1300) 1308	(9000) 8762	(8000) 8069	(9000) 8924

2. Estimate, add, and use subtraction to check. (Estimated answer in parentheses)

a	b	c	d	e	f
341 + 586	411 + 579	366 + 485	2935 + 4508	7928 + 2197	8047 + 3609
(900) 927	(1000) 990	(900) 851	(8000) 7443	(10 000) 10 125	(12 000) 11 656

3. Can you figure out a way to check with subtraction without recopying the whole problem? Write one of the addends under the sum and subtract

39

See activity 10, page 45c.

lesson Pages 39, 40

goal Examining the methods for checking accuracy in addition computation

memo Discussion is needed to kick off this new idea.

page 39 Each of the two methods of checking presented has advantages and disadvantages. When reversing addends, have pupils cover their original addition. Repeating the same error will not check the work. Discuss when one of these methods is more appropriate than estimation as a check. When is it really important to be accurate?

Checking addition with subtraction eliminates nearly any chance of error. If a youngster checks out an incorrect problem as correct, it normally means that (1) he needs a review of addition and subtraction facts; (2) he really didn't check the problem; or (3) he made an error in renaming.

Encourage pupils to find their own errors by genuine checking. You might have them pretend that they are tax collectors checking for errors.

Note that problems **1d–f** and **3d–f** involve 4-digit numbers. Pupils should not view working with these as being any different from working with 3-digit numbers.

goal Examining the methods for checking accuracy in subtraction computation

memo More discussion. This page has another new idea.

page 40 A good tax collector must be able to check subtraction also! Discuss the example at the top of the page. Then the independent learners are on their own.

Note that problem 2 again includes 4-digit numbers. The pupils' degree of success on page 39 will indicate which pupils are able to handle this one independently and which will require your guidance.

You can check an answer to a subtraction problem too.

When you subtract, estimate the answer and then compute the difference. Check your answer by adding it to the number you subtracted.

What number should you get? The number you subtracted from

	$\begin{array}{r} 415 \\ -297 \\ \hline \end{array}$	**estimate** →	$\begin{array}{r} 400 \\ -300 \\ \hline 100 \end{array}$
exact ↓			
	$\begin{array}{r} 415 \\ -297 \\ \hline 118 \end{array}$	**check** →	$\begin{array}{r} 118 \\ +297 \\ \hline 415 \end{array}$

1. Estimate, subtract, and check by addition. (Estimated answer in parentheses)

a	b	c	d	e	f
$\begin{array}{r} 587 \\ -392 \\ \hline \end{array}$	$\begin{array}{r} 714 \\ -228 \\ \hline \end{array}$	$\begin{array}{r} 350 \\ -234 \\ \hline \end{array}$	$\begin{array}{r} 625 \\ -428 \\ \hline \end{array}$	$\begin{array}{r} 831 \\ -179 \\ \hline \end{array}$	$\begin{array}{r} 902 \\ -563 \\ \hline \end{array}$
(200) 195	(500) 486	(200) 116	(200) 197	(600) 652	(300) 339

2. The chart shows the length of four rivers.

River	Length (kilometres)
Saint John	669
Ottawa	1114
Fraser	1360
Nile	6699

   a How much longer is the Ottawa River than the Saint John River? 445 km

   b How much longer is the Fraser than the Ottawa? than the Saint John? 246 km 691 km

   c How much longer is the Nile than the Saint John? than the Ottawa? than the Fraser?
   6030 km 5585 km 5339 km

3. Do you think that the length of each river is *exactly* what is stated in the table? Probably not

   a How do you think these lengths were determined? Answers may vary.
   Examples: from a topographic survey; from a photographic survey

   b What makes a fact a fact? Answers will vary.
   "When it appears in print" is not acceptable. Examples of acceptable answers: when experts agree; when more than one source of information shows the same information; when it can be demonstrated that it is true

40

EXTRA PRACTICE AND REVIEW

Plan with your teacher. Decide what to do with these pages.

1 Add. Check your answers. Make sure things are reasonable.

a 68 + 21 = 89	**b** 47 + 23 = 70	**c** 58 + 91 = 149	**d** 97 + 23 = 120	**e** 86 + 57 = 143
f 31 + 74 = 105	**g** 85 + 19 = 104	**h** 814 + 33 = 847	**i** 754 + 28 = 782	**j** 685 + 56 = 741
k 168 + 681 = 849	**l** 477 + 918 = 1395	**m** 605 + 409 = 1014	**n** 1297 + 3916 = 5213	**o** 9118 + 882 = 10,000

2 Use the table to answer these questions. Estimate the answers.

Accept answers from 300 to 1000 sq. mi.

a How much larger is Vermont than New Hampshire? What is the area of both states together? 19,000 sq. mi.

b What is the combined area of Massachusetts, Rhode Island, and Connecticut? 14,000 sq. mi.

c Which is larger—Maine or the rest of New England? Round to hundreds How much larger? What is the area of all 200 sq. mi. New England? Accept 66,000 to 66,600 sq. mi.

Area of New England States

State	Area (sq. mi.)
Maine	33,215
New Hampshire	9,304
Vermont	9,609
Massachusetts	8,257
Rhode Island	1,214
Connecticut	5,009

41

See activity 11, page 45d.

lesson Pages 41, 42, 43, 44

goal More practice in adding 2-, 3-, and 4-digit numbers

memo You will want to skim pages 41 through 44, since they work together to provide for a wide range of pupil abilities. Skip any pages that are not applicable. You may decide to assign specific pages to specific students or choose to give each pupil the option of selecting the kind of work he thinks he needs.

page 41 Use only with pupils who need more addition practice. You can be even more specific—assign only the type of practice needed.

goal More practice in subtracting 2- and 3-digit numbers

page 42 Use this page with pupils who need more subtraction practice. Adjust the assignment to the individual pupil's needs.

Subtract. Check your answers.

a $93 - 72$ = 21	b $46 - 27$ = 19	c $591 - 99$ = 492	d $235 - 107$ = 128	e $614 - 307$ = 307
f $734 - 34$ = 700	g $102 - 39$ = 63	h $200 - 52$ = 148	i $500 - 499$ = 1	j $192 - 93$ = 99
k $178 - 89$ = 89	l $823 - 76$ = 747	m $768 - 427$ = 341	n $267 - 159$ = 108	o $405 - 396$ = 9

Señor Sanchez is an artist.

a He sold three paintings last month for $50, $133, and $185. About how many dollars did he get? Accept answers from $370 to $400

b The month before he sold four paintings. One sold for $75, another for $50, a small one for $12, and another for $68. About how many dollars did he get for his paintings that month? About $210

c The señor had a lot of bills to pay:

Rent	$100
Art supplies	$ 56
Dentist bill	$ 86
Clothing bill	$ 95
Groceries	$ 37

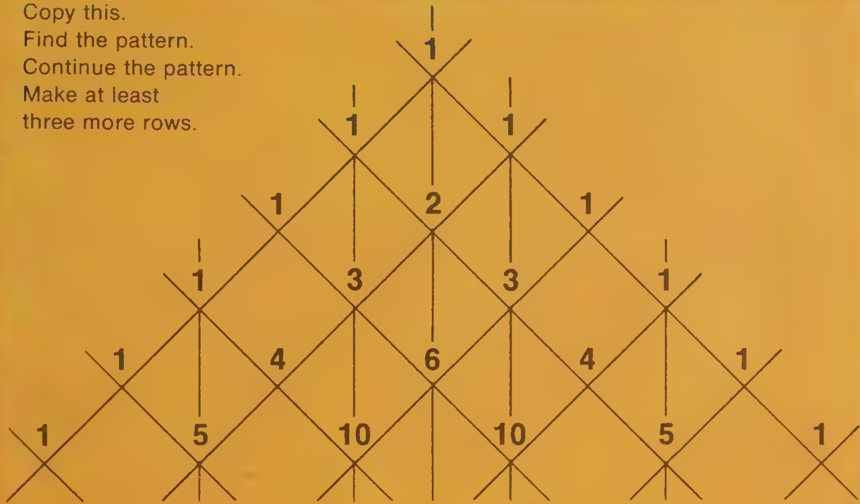
Estimate whether the señor earned enough in the two months to pay his bills. He did. Earnings — $580
Bills — $390

42

See variation of activity 11, page 45d.

Just for fun

Copy this.
Find the pattern.
Continue the pattern.
Make at least
three more rows.



1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

How many patterns can you find?
Look for them in each row.
Look for patterns on each diagonal.
Are there patterns in each column?
What is the sum of the numbers in each row?

43

goal Discovering and continuing number patterns

page 43 Use this page now or consider it as a page that pupils are free to come back to whenever time permits.

goal Providing an opportunity for decision making

page 44 The page lends itself to small-group, independent work. It's really a great problem-solving situation where there is no right answer.

Billy wants to buy a pet but he's not sure whether to buy a dog or a cat. He knows these facts:

- **a** Dogs have to have licenses. The license costs \$10.00.
- **b** Dogs have to have rabies and distemper shots. The shots cost \$25.00.
- **c** House cats do not have to have shots, but his mother would want a cat to have distemper shots and pneumonitis shots. These shots would cost \$15.00.
- **d** House cats use up a bag of cat litter a week. Litter costs 50¢ a bag.
- **e** Dogs eat one can of food a day. Dog food costs 19¢ a can.
- **f** Cats eat one can of cat food a day. Cat food costs 11¢ a can.

Answers wil

1. Would there be any other costs to consider?

Examples: Would you have to build a pen? Pet shop, food bowls, leash, etc.

2. Which of the five animals would be the cheapest to buy and own? Stray kitten

Which animal would you choose? Answers wil

What other things might influence your decision?

Examples: What kind of pets are allowed where you live?
Do people in your family have allergies to cats or dogs?
What kind of pet do you want?

44

1. Add 2-digit numbers. Skill: Adding two 2-digit numbers

a 24 + 35 = 59	b 76 + 17 = 93	c 53 + 17 = 70	d 94 + 63 = 157	e 80 + 37 = 117
f 97 + 15 = 112	g 86 + 73 = 159	h 54 + 68 = 122	i 37 + 18 = 55	j 76 + 45 = 121

2. Add 3-digit numbers. Skill: Adding two 3-digit numbers

a 462 + 425 = 887	b 270 + 404 = 674	c 545 + 407 = 952	d 766 + 708 = 1474	e 465 + 269 = 734
f 721 + 695 = 1416	g 818 + 519 = 1337	h 961 + 498 = 1459	i 639 + 169 = 808	j 882 + 267 = 1149

3. Subtract. You will have to copy these problems. Skill: Subtracting 3-digit numbers

No renaming

a 76 − 53 = 23	b 48 − 14 = 34	c 366 − 253 = 113	d 858 − 237 = 621	e 849 − 345 = 504

Renaming

f 64 − 45 = 19	g 75 − 47 = 28	h 861 − 164 = 697	i 930 − 335 = 595	j 700 − 672 = 28
k 437 − 381 = 56	l 604 − 137 = 467	m 595 − 196 = 399	n 558 − 486 = 72	o 586 − 289 = 297

4. Complete. **WATCH OUT!** Skill: Adding and subtracting two 3-digit numbers

a 578 + 422 = 1000	b 500 − 499 = 1	c 378 − 269 = 109	d 562 + 537 = 1099	e 802 − 713 = 89

45

See activity 12, page 45d.

STOP Follow suggestions given in the guide column; then use activity 9, page 45c.

lesson Page 45

goal **Checkout** – adding and subtracting 2- and 3-digit numbers

page 45 Specific skills being checked are identified on the answer key. There certainly are more problems than necessary to identify learner ability. Why not assign just one row from each section initially? Give any additional help that may be needed. Then use the remaining problems to recheck the learner.

A good chapter-closing activity would be to have each pupil make a personal, private record of the types of problems he should go back and review. It will just take a minute of your time to individually discuss his assessment to see if it agrees with yours. You then can continue to give meaningful support and guidance.

RESOURCES

another form of evaluation

for Progress check—page 27

Try these.

1.	2.	3.	4.	5.
70	80	50	60	70
+20	+70	+90	+40	+50
90	150	140	100	120

Next step. Try these.

6.	7.	8.	9.	10.
42	35	67	61	24
+46	+24	+12	+16	+53
88	59	79	77	77

One more step. Keep adding.

11.	12.	13.	14.	15.
36	18	57	39	23
+27	+54	+23	+56	+68
63	72	80	95	91

If you get these right too, you can assume you know how to add.

16.	17.	18.	19.	20.
49	13	54	68	76
+84	+98	+66	+32	+89
133	111	120	100	165

for Progress Check—page 31

Try these.

1.	2.	3.	4.	5.
400	600	560	350	348
+300	+309	+108	+249	+321
700	909	668	599	669

Now try these.

6.	7.	8.	9.	10.
516	429	248	607	354
+247	+435	+705	+204	+416
763	864	953	811	770

This row will prove you know. Keep on adding.

11.	12.	13.	14.	15.
374	453	305	968	417
+169	+287	+596	+358	+999
543	740	901	1326	1416

for Progress Check—page 36

Subtract.

1.	2.	3.	4.
679	490	650	927
−342	−256	−440	−643
337	234	210	284

5.	6.	7.	8.
738	523	806	600
−654	−267	−489	−273
84	256	317	327

for Checkout—page 45

1. Add 2-digit numbers.

a)	b)	c)	d)	e)
43	27	55	96	90
+24	+38	+35	+23	+85
67	65	90	119	175

f)	g)	h)	i)	j)
34	52	98	87	76
+97	+78	+67	+34	+68
131	130	165	121	144

2. Add 3-digit numbers.

a)	b)	c)	d)	e)
465	303	269	863	374
+432	+680	+502	+408	+286
897	983	771	1271	660

f)	g)	h)	i)	j)
961	738	893	425	576
+793	+648	+564	+876	+795
1754	1386	1457	1301	1371

3. Subtract.

a)	b)	c)	d)	e)
94	78	669	585	839
−63	−45	−237	−422	−235
31	33	432	163	604

f)	g)	h)	i)	j)
37	76	545	465	900
−18	−17	−407	−269	−384
19	59	138	196	516

k)	l)	m)	n)	o)
623	374	498	768	563
−495	−198	−399	−589	−257
128	176	99	179	306

4. Complete. Watch out!

a)	b)	c)	d)	e)
687	567	700	234	504
+313	−488	−699	+765	−415
1000	79	1	999	89

activities

1. things 10 same-size boxes (milk cartons); index cards

On each card write a problem for the type of practice needed.

Use those boxes you saved from activity 5 on page 20b (or use those directions for making a set for this activity). Place the boxes side by side in order, with the multiples of 10 showing.

Shuffle the problem cards. The youngster is to take a card, round appropriately, mentally estimate the answer, and then place the card in the box labeled with the estimated answer. For example:

37	rounds to:	40	Card is placed
+29		+30	in the 70 box.

Order the multiples of 100 shown on another face of the boxes to accommodate problem cards that require rounding and estimating hundreds.

2. things bundles of 10; single counters; rubber bands

To help the youngster who seems totally lost, bring out those bundles of 10 and use a manipulative approach. Mastery of addition facts is prerequisite. Have the youngster show 37, then 29 more. Reinforce the algorithm by having him add ones first.

Do you have more than 10 ones? Make a bundle of 10. Add tens. How many ones? tens? How many in all? Repeat for several examples.

Next combine manipulatives with recording.

tens	ones
4	6
+3	9
1	5
7	

Record ones. Record tens. Where do you see this 1 ten in your collection of counters? (Make a bundle of 10.) Complete the answer.

Repeat for several examples. Then have a child try to complete a problem without manipulatives. Let him operate with the long algorithm until he feels confident. Don't push now for the shorter computational form.

3. Make a spirit master with the following words and illustration: Be a number detective. Find 3 boxes that touch in some way and whose numbers add up to 50. Shade these boxes the same color.

Can you find more than one set? Shade each set a different color.

33	29	9	27	5	1
30	10	18	4	16	26
9	17	28	25	20	13
7	14	8	11	19	2
32	23	21	6	3	7
16	31	17	9	30	12

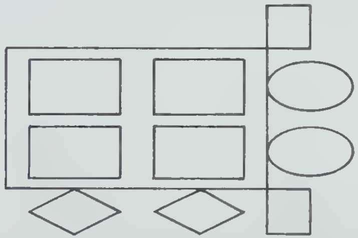
4. things 4 sets of numeral cards 0 through 9

Group game for 2, 3, or 4 players. Shuffle the cards and completely deal them facedown. The cards remain facedown in a stack before each player. Each player turns over the 2 top cards in his stack and gives the sum, going once around the group. The player with the greatest sum wins and takes all the cards used in the round. Should there be a tie for the greatest sum, the cards remain and only those who tied play another round. The winner for this round collects all cards that are faceup.

Sums may be challenged. If the challenged player did give an incorrect sum, his 2 cards automatically go to the challenger. If the sum is correct, however, the challenger gives his cards to the player he challenged.

5. Individual activity (Reproduce the directions for the pupil.)

things spirit master of the following diagram



1. Write a 2- or 3-digit number in each ▭
2. Add across. Write the sums in the ⬭s.
3. Add down. Write the sums in the ◇s.
4. Add the numbers diagonally. Write these sums in the □ s.
5. Find the sum of the numbers in the □ s.
6. Find the sum of the numbers in the ⬭s.
7. Find the sum of the numbers in the ◇ s.
8. Is there anything **special** about the sums for 5, 6, and 7? (They should all be the same.)

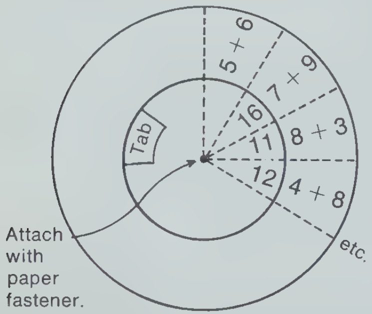
6. Here's a challenge for the sharpies. Record these figures on the chalkboard.

Year	Candidate	Number of Votes	Electoral
1860	Abraham Lincoln	1,866,352	180
	Stephen A. Douglas	1,375,157	12
	John C. Breckinridge	845,763	72
	John Bell	589,581	39
1888	Benjamin Harrison	5,444,337	233
	Grover Cleveland	5,540,050	168
1968	Richard M. Nixon	31,785,480	301
	Hubert H. Humphrey	31,275,166	191
	George C. Wallace	9,906,473	46
1972	Richard M. Nixon	45,767,218	521
	George S. McGovern	28,357,668	17

In each year, who received the most votes? Who won the election? You may want to introduce the electoral college here. *Which place values do you need to compare to find the largest number?*

7. things 9″ fluted-edge paper plate; 6″ paper plate; felt pen; paper fastener

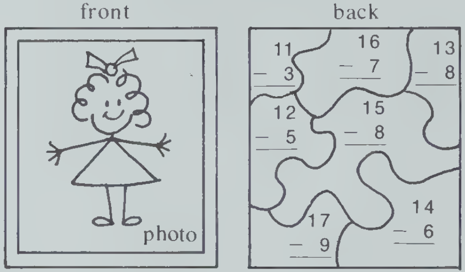
Mark the center of the 9″ plate. Around the edge of this plate write facts that a pupil has not quite mastered. Move in 4 inches from the edge and write the answer. These answers should be on an imaginary line from the center of the plate to the problem. Cut a tab in the rim of the 6″ plate and leave it attached to form a hinge. Center the 6″ plate over the 9″ plate and attach with a paper fastener. Turn the small plate so that the tab is opposite a problem. The tab is lifed to verify answers. Adapt for any type of fact practice necessary.



8. things magazines; newspapers; shirt cardboard; paste; small boxes; scissors

Pupils hunt to find and cut out a picture they like—even their own photograph if they have a spare. Paste the pictures on the cardboard. When the paste is dry, turn over, outline puzzle pieces, and cut apart. Write a problem (any type of practice needed) on each piece. Store each puzzle in a small box. Hosiery boxes are great!

Pupils work with a tutor. The tutor holds the puzzle pieces. Each time the pupil gives the correct answer, he receives the puzzle piece. When he has all the pieces, he can put the puzzle together.

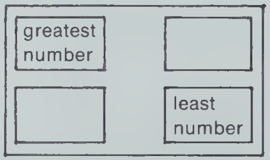


9. Individual activity (Reproduce these directions for the pupil.)

things spirit master from activity 5

1. Write a 2-digit number in each □

   Be sure to follow this rule:

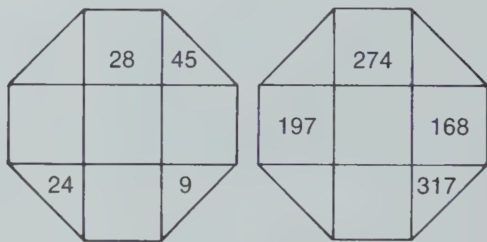


2. Subtract across.
   Write the differences in the ◯s.
3. Subtract down.
   Write the differences in the ◇s.
4. Add diagonally.
   Write the sums in the □s.
5. Subtract the numbers in the ◯s.
6. Subtract the numbers in the ◇s.
7. Subtract the numbers in the □s.
8. Is there anthing **special** about the answers for 5, 6, and 7? (They should all be the same.)

10. Prepare a spirit master of figures as shown. Fill in just enough numbers so that the pupil will need to use both additon and subtraction.

Directions: Add any 2 numbers separated by a triangle and put the sum in the triangle between the 2 numbers.

Find the sum of the numbers written in the 4 outside squares and write this sum in the center square.



11. things game board; 25 small cards

Prepare a game board as shown. The addends can be any 2- or 3-digit numbers. You decide whether to include renaming, depending on the abilities of your pupils. You may want to write the numbers in order, to simplify locating them.

+	35	26	50	72	93
25					
30					
42					
29					
60					

Write one of the sums on each small card. These cards (tiles) should be the same size as the empty boxes on the game board.
Turn the answer tiles facedown in random order. Each player selects 4 tiles. The first player places an answer tile on any box for which the numeral on the tile is the sum. (Players may need to be reminded of their work with the addition and multiplication tables where they wrote in the answer.) Each succeeding player than attempts to place a tile on the board so that it is a correct sum and touches (vertically or horizontally) the tile played previously by another player. If a player does not have a tile he can play, he draws from the facedown tiles until he finds one he can use. The winner is the first player to use all his tiles.

Variation: Adapt the game for subtraction practice by preparing the game board as shown below. Make the appropriate answer tiles.

These numbers must be greater than those across the top.

−	33	45	23	51	26
98					
64					
87					
75					
68					

12. things 40 index cards

Write a 1-, 2-, or 3-digit numeral on each card. These should be numerals that can be easily paired to obtain the sum 9, 99, or 999. For example: 4, 5, 0, 9, 45, 54, 333, 666, and so on.

Two to four persons can play the game. A player deals 5 cards to each person. The remaining cards are placed in a stack facedown. The top card is turned faceup to start the discard pile. Players, in turn, draw the top card from either pile, try to form a pair of cards whose sum is 9, 99, or 999, and discard one card. The winner is the first player who can lay down 3 pairs of cards, each pair totaling either 9, 99, or 999.

additional learning aids

concept—chapter objectives 6, 7

SRA products

Mathematics Involvement Program, SRA (1971)
Cards: 104, 124, 284, 155, 165, 26
Visual Approach to Mathematics, level 3, SRA (1967)
Visuals: 7, 8, 9, 10, 11

operation—chapter objectives 1, 2, 3, 4, 5

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: P 1, 9
W 3, 12
Arithmetic Fact Kit, SRA (1969)
All addition and subtraction cards
Computapes, SRA (1972)
Module 1, Lesson: AS 19
Module 2, Lessons: AS 21, 23, 24, 25, 26, 34, 35
Computational Skills Development Kit, SRA (1969)
Addition cards: 11, 12
Subtraction cards: 2, 4, 5, 6, 7, 8
Cross-Number Puzzles (Whole Numbers), SRA (1966)
Addition cards: 1, 2, 3, 5, 12, 13, 14
Subtraction cards: 1, 2, 3, 4, 5, 6
Diagnosis: an instructional aid—Mathematics Level A, SRA (1973)
Probes: L-1, 2
Skill Modes in Mathematics, SRA (1974)
Level I, Molecule: A
Skill through Patterns, level 4, SRA (1974)
Masters: 1, 2, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 26, 27, 30, 36, 37, 38, 42, 45, 62, 63, 67, 68, 72

other learning aids (described on page 72d)

Chip Trading, Dial-a-Matic* Adding Machine, I Win (sets 1 and 2), Numo

* Registered trademark of Sigma Scientific, Inc.

3 × AND ÷ FACTS

before this chapter the learner has –

Worked with the multiplication facts

in chapter 3 the learner is –

1. Telling the factors and product shown by an array
2. Mastering the multiplication facts
3. Finding an unknown factor in an open multiplication sentence
4. Identifying the factors and product of a multiplication sentence
5. Finding the quotient for a division fact
6. Writing a related division fact for a given multiplication fact

in later chapters the learner will –

1. Master the division facts
2. Estimate and find the product of any two 2-digit numbers
3. Estimate and find the quotient and remainder (if any) for any 3-digit number and any 1-digit number
4. Show that the order in which any two 2-digit numbers are multiplied does not affect their product

Notes & Things

Multiplication facts are something that no pupil will claim to remember at this level. A good healthy review in a new context will help that. And this chapter starts from scratch on the division concept. It will give a lot of practice with division, but please notice that mastery of division facts is not expected until later.

Multiplication review is presented in the context of real-life situations. Arrays are not just a bunch of dots in rows and columns. Finding how many in all when one has many sets with the same number of like objects makes multiplication a valid problem-solving tool. Do take time to develop some of the rich discussion possibilities. You will learn still more about your pupils' thinking and will also have a chance to open their eyes and their minds to the mathematics that everyone needs and uses.

The broad concept of division is studied in detail in a later chapter. The emphasis there will be on repeated subtraction. For now, the exploration of the operation is the outgrowth of a missing-factor multiplication problem. This approach helps to reinforce the multiplication facts and yet allows the division symbol and sentence to be introduced. Multiplication and division reinforce each other as the pupil learns the basic facts.

You will want to use every gimmick in your bag of tricks to stimulate practice and the resulting memorization of multiplication facts. You'll also have to use wisdom, because too much pressure can be harmful. If each child will accept the mastery of facts as his own personal goal, then competition along with fun gimmicks will get the job done in no time at all. If learning multiplication facts turns out to be fun, then perhaps the mastery of division facts will come earlier than expected too. Keep your fingers crossed.

things

graph paper
several kinds of empty boxes
spirit master of a blank multiplication table
index cards
paper bags
same-size boxes
counters or small wood blocks

For the extra activities you will want to have these things available:

- deck of playing cards
- snap-type clothespins
- 6-by-6 array game board

goal Think about and explore ideas through a picture clue

page 46 The supermarket will continue to serve as a source of ideas. The photograph shows empty shopping carts. (They represent an excellent example of congruent parts, but you will want to save that observation for later.) Once again some direct questioning is needed to get the discussion going. What kind of stores provide shopping carts for you to use? What kind don't? Are there many salespeople around to help you in a store that provides shopping carts? How do the jobs of people differ in the two kinds of stores? How are their jobs alike?

Now for the entry into the chapter itself. Pretend you filled one of those shopping carts full of packages. What things could you buy that come packaged so that they can be divided? If you bought five things that were alike, would each one be the same price? How could you figure out how much all five would cost?

You know how to add and subtract. Find out how good you are with multiplication and division. Your GOAL POST is to get really good.

1. Roller coaster
   4 cars – 8 seats each
   How many seats in all?
   - **a** Can you add to find the answer? Yes
   - **b** Can you multiply to find the answer? Yes
   - **c** What is the answer? 32

2. Toy train
   Engine – 10 wheels
   Freight car – 8 wheels
   Boxcar – 8 wheels
   How many wheels?
   - **a** Can you add to find the answer? Yes
   - **b** Can you multiply to find the answer? No
   - **c** What is the answer? 26

3. Buttons
   6 on a jacket
   6 on a shirt
   6 on a sweater
   How many buttons?
   - **a** Can you add to find the answer? Yes
   - **b** Can you multiply to find the answer? Yes
   - **c** What is the answer? 18

4. Can you always add rather than multiply? Yes

5. Can you always multiply rather than add? No

6. Decide whether it is best to add or multiply to find an answer.
   - **a** 7 apples, 7 pears, 7 plums, 7 oranges. How many? Multiply
   - **b** 9 carrots, 9 potatoes, 9 onions. How many? Multiply
   - **c** 6 colas, 6 orange sodas, 5 root beers. How many? Add
   - **d** 8 red, 8 blue, 8 yellow, 8 green. How many? Multiply

7. What operation do you use to find these answers?
   - **a** 54 chairs in all. There were 6 rows. How many chairs in each row? Division
   - **b** 72 stamps in the book in all. 9 on each page. How many pages? Division

47

lesson Page 47

goal **Survey** – knowledge of multiplication and division facts

memo Introduce the words OPERATE and OPERATION as they apply to mathematics. Use common English usage to help develop understanding.

The doctor operates on her patient.
The carpenter operates the electric saw.
The chef operates the food mixer.

What is there in common? Each person is doing some work. Can you operate with numbers? What can you do with numbers? (Add, subtract, and so on) The notion that the pupil operates with numbers should be appealing. Defining addition, subtraction, multiplication, and division as arithmetical operations with numbers should be easy.

page 47 Handle this page as discussion if the group is insecure.

Problems 1 through 6 focus on understanding when addition and multiplication are parallel. Problems 1, 3, and 6 check ability with multiplication facts. Problem 7 checks knowledge of division.

Reassure pupils who meet with little or no success that the following pages will help them learn these skills. Establish the learning goal in a positive way.

lesson Pages 48, 49, 50, 51

goal Development of readiness for rectangular arrays and multiplication facts

memo Pages 48 through 50 are important concept pages that are the key to later success. Please treat them as discussion and group-activity pages.

things graph paper
3 empty boxes

page 48 Graph paper will make the box designs go more quickly and probably more accurately. Challenge the pupils to design more than one box—if they can—for each quantity of glasses given.

In order to understand the machine problem presented, have some empty boxes taken apart very carefully. Many adults as well as children are surprised to see what a box looks like before it is assembled.

The notion of standard size and lower costs on large quantities are easily understood ideas that are well worth discussing. What other things are cheaper when buying in quantity? Why might these things be cheaper?

Is it your job to carry out the trash? There are usually a lot of boxes. There are big boxes, little boxes, sturdy boxes, thin boxes. Each had a purpose. In fact, much time was spent in planning shapes and sizes.

Today it is your job to plan boxes for drinking glasses. A box to hold 4 glasses would probably look like this:

Draw a box to hold 6 glasses (1 by 6) 2 by 3
8 glasses (1 by 8) 2 by 4
9 glasses How many rows in each box? (1 by 9) 3 b
10 glasses How many glasses in each row? (1 by 10
12 glasses (1 by 12) 2 by 6, 3 by 4
16 glasses (1 by 16) 2 by 8, 4 by 4

Most boxes are made by machine. Every time a different-sized box has to be made, the machine has to be changed. Boxmakers try to get their customers to use a standard-size box. A shoe box is an example of a standard-size box. Size-12 shoes and size-6 shoes both fit into the same-size box. Why would a standard-size box be cheaper to buy?

Each time the boxmaking machine has to be changed, the cost goes up.

48

Here is a picture of a box that games are packed in.
It measures how many units wide? 10 how many units long? 10

1. a Could you put a game board that measures 3 units by 4 units in the box? Yes
   b How many square units would it cover? 12

2. a Can a 5-by-10 game board go in? Yes
   b How many square units would it cover? 50

3. a Can you put an 8-by-9 game board in? Yes
   b How many square units would it cover? 72

4. a How many square units would a 7-by-6 game board cover? 42
   b How many for a 6-by-6 board? 36 for a 10-by-10 board? 100

Games often come in boxes that are larger than needed. It's because the manufacturer wants to use a standard box size. This keeps the cost of the box lower. He then puts in a stuffer to fill up the empty space. How would a standard box size for a set of games be determined?

It would be the smallest size needed to hold the largest game.

goal Development of readiness for rectangular arrays and multiplication facts

things graph paper

page 49 Make sure that pupils understand that the words **10-by-10** mean 10 units long, 10 units wide. The unit itself can be any measure — 1 inch, $\frac{1}{2}$ inch, 5 inches, or 10 inches.

Graph paper will help the pupils who have difficulty with problems 1 through 4. After marking off the game board (array), encourage them to skip-count, rather than to count individual squares, to find the total number of squares covered.

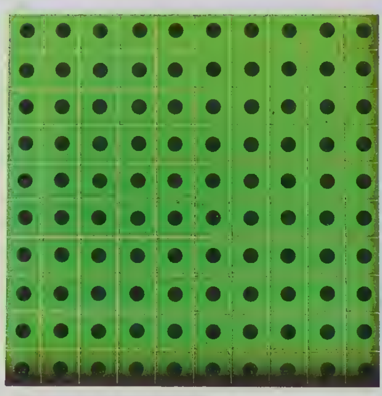
Challenge the learners to compute the number of squares that would be covered by the stuffer for each of the game boards. Don't suggest how this computation can be done. Let them figure it out.

Determining the size of a box for a set of games will help develop a basic problem-solving tool. Think about these real-world situations: The sign maker — the longest line of copy determines the size of the lettering; the typist — the longest entry in one column determines the amount of space left for the other columns; the housewife — the maximum number of people coming to dinner determines the amount of food to prepare; the expected distance of a baseball hit is a major consideration for the position that both infield and outfield players take. This list could go on and on.

goal Finding all possible arrays for a given number

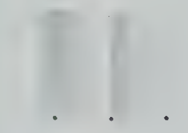
page 50 Have patience! Allow plenty of time to experiment with the mask and 10-by-10 grid: For now, consider an array of 2 rows with 7 dots in each row to be different from an array of 7 rows with 2 dots in each row—even though the total number of dots in each is the same.

You'll find an overhead projector, a 10-by-10 grid, and a similar mask helpful if these materials are available to you.

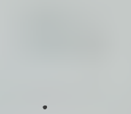


How many rows of dots in the grid? 10
How many dots in each row? 10
How many dots in all? 100

Take a sheet of paper. Fold it in half.



Fold it in half again.

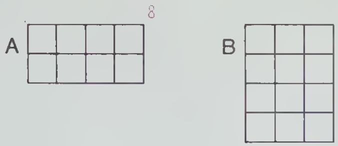


Open the paper. Tear out one fourth.

Use this paper as a mask to show arrays of dots in the grid above.

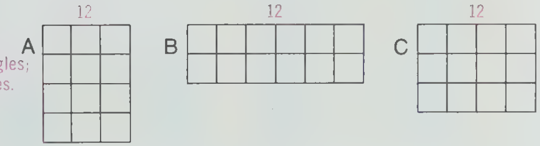
Place the mask over the grid so only 14 dots show.
How many rows show? How many dots in each row?
Can you show an array of 14 dots by placing the mask in another position? 2 rows of 7, 7 rows of 2

1. Show 12 dots. How many rows? How many dots in each row?
   How many ways can you show 12 dots? 4
   2 rows of 6, 3 rows of 4, 4 rows of 3, 6 rows of 2
2. Show 16 dots. How many rows? How many dots in each row?
   How many ways can you show 16 dots? 3
   2 rows of 8, 4 rows of 4, 8 rows of 2
3. Look out for this one!
   Show 24 dots. How many rows? How many dots in each row?
   How many ways can you show 24 dots? 4
   4 rows of 6, 3 rows of 8, 6 rows of 4, 8 rows of 3

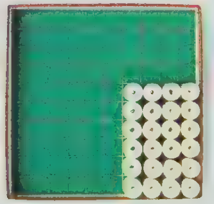


1. How many glasses would fill each box?
   How are the boxes alike? They're rectangles; length is the same.
   How are they different? Different widths
   Could these be pictures of the same box? No

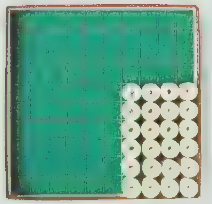
2. How many glasses would fit in each of these boxes? How are these boxes alike? They're rectangles; hold 12 glasses.
   How are they different? Their shapes
   Could any two of these be pictures of the same box? Yes—A and C



3. It was inventory time in the dime store. The manager hired extra people to help count all the things he had in the store. Bill and Tim started counting the number of spools of thread.



Bill thought
6 + 6 + 6 + 6
and wrote 24
on the top of
the box.



Tim thought
4 × 6
and wrote 24
on the top of
the box.

- **a** Were there the same number of spools in each box? Yes
- **b** Were there the same number of rows? Yes
- **c** Were there the same number of spools in each row? Yes
- **d** Who do you think might be able to work faster? Tim

51

goal More exploration of boxes to show multiplication as repeated addition

page 51 Answers to problems 1 and 2 serve as a clue to the pupil's background in geometry and measurement.

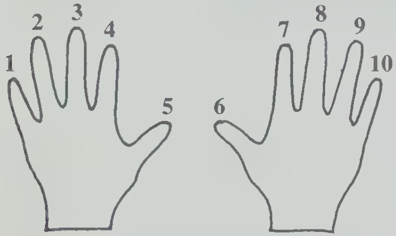
Problem 3 could trigger a contest. Use the overhead, a grid, and the mask to show an array. One team is to add to find the total number while the other team is to multiply. Which is faster? Alternate the tasks and repeat.

lesson Pages 52, 53

goal Relating addition and multiplication

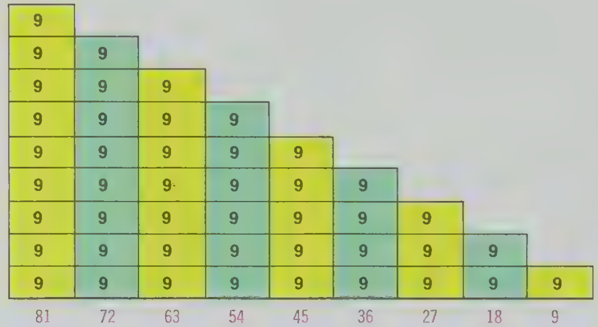
page 52 Repeated addition is tedious. Some youngsters may recall rote skip-counting experiences and use this technique. That's O.K., but emphasize that knowing the multiplication facts is even faster.

Time for fun with the nine-facts. Have pupils assign a numeral from 1 through 10 to each finger as shown.



What's 2×9? Bend down the finger number 2.
How many fingers to the left of the bent finger? 1
How many fingers to the right of the bent finger? 8
$18 = 2 \times 9$

Does it work for any other nine-facts? Will it work for any number facts other than 9?



There is no doubt about it, you can add *or* multiply if you have one or more dots or objects in sets AND if you have the same number in each set. Here are stacks of boxes. Each box contains 9. Add or multiply to find how many in each stack.

That first stack is a tall one! How many in all in that stack? 81

READY FOR MULTIPLICATION PRACTICE?

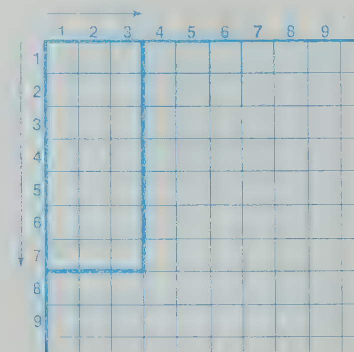
Have you forgotten the multiplication facts?
Here is a good way to review. It is the same idea as arrays.

The only thing that's different is that the grid is marked with numbers.

Let's say you have forgotten the product of 7×3. Look at the grid. Read 7 down. Read 3 across. You could just count the number of squares in all.

But it is faster to skip-count.

You can skip-count by 3s or skip-count by 7s to find how many in all.



1. How good are you at skip-counting? Find out.

a Count by 2s to 18. Why 18?
2, 4, 6, 8, 10, 12, 14, 16, 18 ($2 \cdot 9 = 18$)

b Count by 3s to 27. Why 27?
3, 6, 9, 12, 15, 18, 21, 24, 27 ($3 \cdot 9 = 27$)

c Count by 4s to 36. Why 36?
4, 8, 12, 16, 20, 24, 28, 32, 36 ($4 \cdot 9 = 36$)

d Count by 5s to 45. Why 45?
5, 10, 15, 20, 25, 30, 35, 40, 45 ($5 \cdot 9 = 45$)

e Count by 6s to 54. You know the question.
6, 12, 18, 24, 30, 36, 42, 48, 54 ($6 \times 9 = 54$)

f Count by 7s to 63. Why?
7, 14, 21, 28, 35, 42, 49, 56, 63 ($7 \times 9 = 63$)

g Count by 8s to 72. You'd better know why by now.
8, 16, 24, 32, 40, 48, 56, 64, 72 ($8 \cdot 9 = 72$)

h Count by 9s to 81.
9, 18, 27, 36, 45, 54, 63, 72, 81 ($9 \cdot 9 = 81$)

2. Skip-counting can be just as hard as adding sometimes. So get ready to tackle this problem—the easy way.

a Copy and complete this table. Put a little mark in any box where you had to stop and think before you could write the answer. If you're honest with yourself, you'll know which facts need practice.

×	1	2	3	4	5	6	7	8	9
1	? 1	2	? 3	? 4	? 5	? 6	? 7	? 8	? 9
2	? 2	? 4	? 6	8	? 10	? 12	14	? 16	? 18
3	? 3	? 6	9	? 12	? 15	? 18	? 21	? 24	27
4	4	? 8	? 12	? 16	? 20	24	? 28	? 32	? 36
5	? 5	? 10	? 15	20	? 25	? 30	? 35	40	? 45
6	? 6	12	? 18	? 24	30	? 36	? 42	? 48	? 54
7	7	? 14	? 21	? 28	? 35	? 42	49	? 56	? 63
8	? 8	? 16	24	? 32	? 40	? 48	? 56	64	? 72
9	? 9	? 18	? 27	? 36	? 45	54	? 63	? 72	81

b Look for patterns in your table. Look at just the 5-column. Do you see your skip-counting numbers? Now look at the 5-row. What do you see? Yes
The same skip-counting numbers

c On your table draw a diagonal line from the top left corner of the table to the bottom right corner, like this: What do you see in the two parts?
The same numbers on both sides of the line

53

goal Relating skip-counting and multiplication

things spirit master of a blank multiplication table (same form as on pupil page)

page 53 Less capable students will probably need a crutch for some time. They may need to practice skip-counting over a period of several days or weeks.

You may want to duplicate the multiplication table on a spirit master, rather than have pupils copy the chart. Pupils should save their own copy of the table for personal record-keeping purposes. It can be used to identify those facts that have been mastered and those on which more practice is needed.

See activity 1, page 72b.

See activity 2, page 72b.

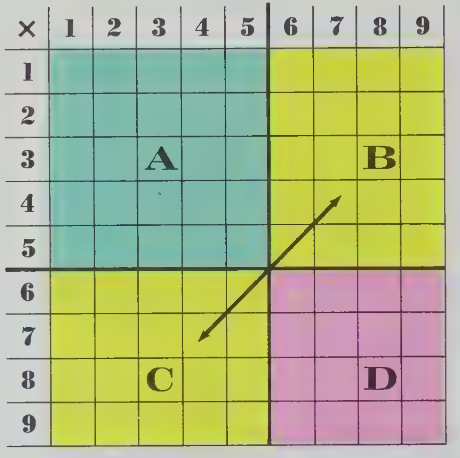
lesson Pages 54, 55

goal Identifying learner ability with multiplication facts

memo Fact practice on this page is limited to factors of 5 or less (section **A** in table).

page 54 Take time to discuss the top of the page. What do the pupils notice about sections **B** and **C**? Later have them locate their errors on their own multiplication table. Are all the errors located in one section?

Accuracy first—then speed. This is a good opportunity to use the folded-paper technique. Place the paper under the row, record the answers, fold the answers under, place the fold under the next row, and so on. If only answers are to be written and if the group is confident enough, consider making this a **1-minute game.** Call a stop at the end of 1 minute, have pupils ring the answer just written, and then have them finish the set. Each pupil's goal will be to see how many more he will be able to finish the next time he plays the 1-minute game. Competition with others **is not** the point. The competition is with one's self. "I know I can do better. Just watch me!" is strong, positive motivation.



This blank table is like the one that you have.

1. Most people think the facts in the part of the table labeled **A** are the easiest. Do you agree? (Ask why.)

2. Most people think the facts in section **D** are the hardest. Do you agree? (Ask why.)

3. You do know the secret of sections **B** and **C**, don't you? They are average. Same numbers in both sections.

Same multiplication fact

$4 \times 5 = 20$ $\quad$ $\begin{array}{r}5\\ \times 4\\ \hline 20\end{array}$

How are these alike?
How are they different?

Written in two ways

Multiplication is a skill. To be able to multiply quickly you need practice. You know how to write the problem. BUT do you know the answers?

Time to check out section **A** on your table. Put a sheet of paper under the first row. Write only the answer on the paper. Keep going.

	a	b	c	d	e	f	g
4.	$\begin{array}{r}2\\ \times 1\\ \hline 2\end{array}$	$\begin{array}{r}3\\ \times 3\\ \hline 9\end{array}$	$\begin{array}{r}1\\ \times 4\\ \hline 4\end{array}$	$\begin{array}{r}4\\ \times 3\\ \hline 12\end{array}$	$\begin{array}{r}5\\ \times 5\\ \hline 25\end{array}$	$\begin{array}{r}2\\ \times 2\\ \hline 4\end{array}$	$\begin{array}{r}5\\ \times 4\\ \hline 20\end{array}$
5.	$\begin{array}{r}3\\ \times 2\\ \hline 6\end{array}$	$\begin{array}{r}5\\ \times 3\\ \hline 15\end{array}$	$\begin{array}{r}1\\ \times 2\\ \hline 2\end{array}$	$\begin{array}{r}2\\ \times 5\\ \hline 10\end{array}$	$\begin{array}{r}2\\ \times 4\\ \hline 8\end{array}$	$\begin{array}{r}3\\ \times 4\\ \hline 12\end{array}$	$\begin{array}{r}4\\ \times 4\\ \hline 16\end{array}$
6.	$\begin{array}{r}4\\ \times 1\\ \hline 4\end{array}$	$\begin{array}{r}1\\ \times 1\\ \hline 1\end{array}$	$\begin{array}{r}3\\ \times 1\\ \hline 3\end{array}$	$\begin{array}{r}4\\ \times 5\\ \hline 20\end{array}$	$\begin{array}{r}2\\ \times 3\\ \hline 6\end{array}$	$\begin{array}{r}5\\ \times 1\\ \hline 5\end{array}$	$\begin{array}{r}4\\ \times 2\\ \hline 8\end{array}$

Now, that wasn't so bad. Are you operating as fast as you would like to?

54

Now to check out sections **B** and **C** facts.

	a	b	c	d	e	f	g
1.	9 × 2 18	7 × 4 28	6 × 5 30	8 × 3 24	7 × 1 7	6 × 2 12	8 × 5 40
2.	2 × 8 16	5 × 9 45	2 × 7 14	1 × 6 6	4 × 8 32	5 × 7 35	4 × 9 36
3.	4 × 6 24	7 × 3 21	8 × 1 8	3 × 9 27	6 × 3 18	1 × 9 9	You're free! Guess why.

If you know 4 × 6, you also know 6 × 4. You found the product of one of each pair of facts in sections **B** and **C**.

One more section to go. Be sure to remember those facts that give you a hard time.

	a	b	c	d	e	f	g
4.	6 × 6 36	8 × 8 64	8 × 6 48	9 × 7 63	7 × 8 56	6 × 7 42	9 × 6 54
5.	7 × 6 42	8 × 9 72	6 × 9 54	8 × 7 56	7 × 9 63	6 × 8 48	7 × 7 49

There were only 2 nine-facts that you didn't do. 9 × 8 and 9 × 9. Time out to look at the nine-facts all by themselves. There is a great way to remember them if you have trouble. Do the facts in order. Then take a good look at the digits in the product.

6.	1 × 9 9	2 × 9 18	3 × 9 27	4 × 9 36	5 × 9 45	6 × 9 54	7 × 9 63	8 × 9 72	9 × 9 81

Sum of the digits in the product is 9. (Someone might discover that when written in order, the digit in the ones place decreases by 1 and the digit in the tens place increases by 1.)

55

goal Identifying learner ability with multiplication facts

things index cards

page 55 Let pupil ability determine the speed with which this page is completed. Independent learners probably will breeze right through and should not be made to practice material that has been mastered. On the other hand, those who are struggling should not be confronted with so much to practice that the task seems

endless

You may want to use the 1-minute game again with confident pupils. Have pupils make flash cards (on index cards) for facts missed. Peer tutors can help those who need additional practice. As the fact is mastered, eliminate the card. Keep the learning goal one that the learner can attain.

Use the nine-facts again to revitalize interest. (They already know the finger pattern.) Now look for patterns in the product: The sum of the digits in each product is 9. When written in order, the digit in the ones place decreases by 1 and the digit in the tens place increases by 1.

lesson Pages 56, 57

goal Examining the commutative property of multiplication as related to real-world situations

page 56 This page requires discussion. Your pupils probably have a good intuitive understanding of the commutative property of multiplication. The questions on the page are intended to focus the child's critical thinking on the property as well as on the multiplication sentence. When one operates with numbers, $3 \times 5 = 5 \times 3$ is a valuable fact to know. You can depend on this property for any pair of numbers. **But**—some situations are not obviously commutative. Here order does make a difference in the action even though the resulting number is the same.

Extend the three situations given on the page as follows:

Problem 1—Would it make a difference if each item weighed 10 pounds?

Problem 2—If you were selling tickets for the seats, would you collect the same amount of money?

Problem 3—Would it make a difference to your eyes? Would it make a difference to the monthly electric bill?

The remainder of the page is independent work.

Computing numbers may be one thing. Figuring out real situations involving numbers may be another.

1. Think carefully. You know 5×8 is the same as 8×5. But are 5 items that cost 8¢ apiece the same as 8 items that cost 5¢ apiece? No
Would it make a difference in the amount of money you pay? Would it make a No
difference in the number of items you carry? Yes

2. You know 4×9 is the same as 9×4. Think about seats at a puppet show. Are 4 rows with 9 seats the same as 9 rows with 4 seats?*
But how many seats in all? 36
*Same number of seats but different arrangement

3. Is 3×8 the same as 8×3? Think about watching TV. Is 3 hours each day for 8 days the same as 8 hours each day for 3 days?*
But how many hours of watching in all? 24
*Same number of hours, but it is *not* the same schedule.

Record your answers.

4. **6**
$\times 6 =$? 36
$\times 7 =$? 42
$\times 8 =$? 48
$\times 9 =$? 54

5. **9**
$\times 6 =$? 54
$\times 7 =$? 63
$\times 8 =$? 72
$\times 9 =$? 81

6. **8**
$\times 6 =$? 48
$\times 7 =$? 56
$\times 8 =$? 64
$\times 9 =$? 72

7. **7**
$\times 6 =$? 42
$\times 7 =$? 49
$\times 8 =$? 56
$\times 9 =$? 63

56

PROGRESS CHECK

Try these. See how well you do. Multiply. Skill: Multiplication facts

1. 5×3 = 15	2. 1×1 = 1	3. 4×5 = 20	4. 3×1 = 3	5. 6×2 = 12	6. 5×7 = 35	7. 8×4 = 32
8. 9×5 = 45	9. 4×7 = 28	10. 7×6 = 42	11. 6×8 = 48	12. 9×6 = 54	13. 8×7 = 56	14. 9×9 = 81

Answer the questions. Skill: Solving problems with multiplication

15. 7 nickels in your pocket.
5 cents each.
How many cents is this? 35¢

16. 8 cookies on the tray.
8 trays of cookies.
How many cookies in all? 64

17. 8 girls in the club.
7 projects for each girl.
How many projects in all? 56

18. 8 boys in the club.
6 projects for each boy.
How many projects in all? 48

19. 9 blocks to the pool.
2 minutes to walk a block.
How many minutes of walking? 18

20. 7 pieces in one pack.
7 packs sold.
How many pieces sold in all? 49

21. 6 cars in each row.
7 rows of parked cars.
How many cars in all? 42

22. 9 hours of sleep each day.
7 days a week.
How many hours of sleep in all? 63

goal Progress Check – skill with multiplication facts

page 57 Use the folded-paper technique for problems 1 through 14. Answers are sufficient. Observe how quickly pupils are working, but do not time this skill check. Look for accuracy first. Speed will follow with more practice. Don't let poor reading skills keep a child from succeeding on problems 15 through 22.

This Progress Check should help you to identify three distinct groups. Pupils who made no more than three errors and who worked with speed are independent for the present.

Pupils who worked accurately but slowly need to work for speed. Use any individualized practice kits you have available. Games are suggested in the Resource Section for this chapter.

Pupils who did not work accurately will perhaps need some more basic instruction on the concept of multiplication, followed by practice a few minutes each day over a period of time.

When you can pinpoint the practice necessary, don't hesitate to call on parents. But be specific about what help the child needs and how they can give it.

See activity 3, page 72b.

See activity 4, page 72b.

lesson Pages 58, 59, 60, 61

goal Exploration of the concept of division

things for each group:
paper bag
36 objects
4 same-size boxes

page 58 This page is an overview of what is to come in the rest of the chapter. New ideas are explored and small-group activity will be the most effective way to develop understanding. Have each group enact the action for problem 1. Someone should record the results.

Next, increase the number of objects to 36. Have them remove 2 at a time, then repeat with 3 at a time, then 4, then 6, then 9. Finally, have them remove 5 each time. Division frequently involves a number that remains. A remainder serves to reinforce the total concept of division.

Problems 2 and 3 will be more difficult for the children. Four same-size boxes for problem 3 will really help. The boxes can extend the exploration even more. Use only 3 boxes. The label still says 24. How many should be in each box? Make the label say 18. Use only 2 boxes. How many should be in each box? Continue with the boxes as long as there is high interest.

1. The label said 24 balls. It looks as if the bag has been opened. I had better make sure there are 24. I can hold only 3 balls at a time. How many handfuls can I take out if there are 24? 8

You get a bag. Put 24 of any object in the bag. Take out 3 each time until the bag is empty. How many times do you take out a set of 3? 8 If you took 4 each time, how many times would you have taken a handful? 6 If you took 6 each time, how many times would you have taken a handful? 4 If you took 8 each time, how many times would you have taken a handful? 3

How many?

2. Here's a carton that holds 12 eggs. I can see that there are only 2 rows. How many eggs should fit in each row? 6

How many?

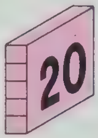
3. This package of blocks is wrapped so that I can see only 4 blocks. The label says 24 blocks. How many blocks should there be in each of the 4 rows? 6

58

See 3 rows?
18 in all.
How many in each row?

2 in each row?	$3 \times 2 = 6$	No.
4 in each row?	$3 \times 4 = 12$	No.
5 in each row?	$3 \times 5 = 15$	No, but it's closer.
6 in each row?	$3 \times 6 = 18$	Yes.

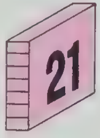
Answer each question.

1. 

5 rows.
20 in all.
How many in each row? 4
$5 \times \underline{?} = 20$
4

2. 

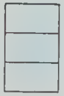
6 rows.
48 in all.
How many in each row? 8
$6 \times \underline{?} = 48$
8

3. 

How many rows? 7
How many in all? 21
How many in each row? 3
Write the sentence.
$7 \times 3 = 21$

4. 

How many rows? 4
How many in all? 32
How many in each row? 8
Write the sentence.
$4 \times 8 = 32$

5. 
12 cubes in all.
3 stacked boxes.
? in each box. 4

6. 
36 glasses in all.
4 stacked boxes.
? in each. 9

7. 
30 things in all.
5 stacked boxes.
? in each box. 6

8. 
54 in all.
6 stacked boxes.
? in each box. 9

goal Finding an unknown factor, using real-world situations

things small wood blocks may be necessary

page 59 Independent learners are on their own. You'll want to help the others get started. If any youngsters have difficulty, have them use wood blocks. Once they catch on, they should be able to work independently.

goal Practice in finding an unknown factor

things counters or small wood blocks

page 60 Sharpies will sail through this page. Strugglers may profit by working in teams of 2 or 3 with manipulatives available to use if necessary.

Complete each sentence.

1. 36 buttons.
   9 cards of buttons.
   How many on each card?
   9 × ? = 36   4

2. 24 in all.
   6 rows.
   How many in each row?
   6 × ? = 24   4

3. 30 players.
   6 on each team.
   How many teams?
   ? × 6 = 30   5

4. 27 needed.
   9 in each box.
   How many boxes?
   ? × 9 = 27   3

5. 24 chairs.
   8 rows.
   How many in each row?
   8 × ? = 24   3

6. 32 want tickets.
   4 ticket windows.
   How many in each row?
   4 × ? = 32   8

7. 42 in all.
   7 rows.
   How many in each row?
   7 × ? = 42   6

8. 48 problems.
   8 problems on each page.
   How many pages?
   ? × 8 = 48   6

Write the number that should be in place of the ?.

9. 6 × ? = 24 4	**10.** 9 × ? = 36 4	**11.** 5 × ? = 35 7
12. ? × 8 = 24 3	**13.** ? × 9 = 45 5	**14.** ? × 6 = 18 3
15. 9 × ? = 18 2	**16.** 6 × ? = 36 6	**17.** 8 × ? = 32 4
18. 7 × ? = 63 9	**19.** ? × 8 = 72 9	**20.** 9 × ? = 81 9

See activity 5, page 72b.

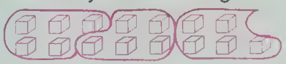
Write the number that should be in place of the ?.

1. $6 \times ? = 42$ 7
2. $7 \times ? = 35$ 5
3. $8 \times ? = 48$ 6
4. $? \times 5 = 45$ 9
5. $? \times 3 = 24$ 8
6. $7 \times ? = 42$ 6
7. $8 \times ? = 56$ 7
8. $9 \times ? = 72$ 8
9. $? \times 4 = 36$ 9
10. $6 \times ? = 54$ 9
11. $? \times 7 = 49$ 7
12. $? \times 9 = 63$ 7

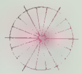
Accept other appropriate pictures.

13. Draw a picture to show each of the situations.

a There are 15 boxes in all.
Divide them equally for 3 people.
How many does each get?



b There are 18 pieces of pizza.
6 people are hungry.
Divide the pizza pieces equally.
How many does each get?



c There are 10 chicken legs. 5 people are coming for dinner. Divide equally.
How many for each?

d There are 16 quarters. 4 people need money.
Divide the quarters equally.
How many can each person get?

Too many words? Too much work to draw pictures? Let's use symbols.

$$16 \div 4 = ?$$

14. Go back to your first picture. Use symbols to show the situation. $15 \div 3 = ?$
What sentence can represent situation **b**? situation **c**? $10 \div 5 = ?$
$18 \div 6 = ?$

15. Think carefully about this one. What sentence can represent 0 dollars being divided equally among 4 people? $0 \div 4 = ?$

61

goal Relating division to finding an unknown factor

memo The division symbol may be new to some pupils. Watch out!

page 61 Sharpies are strictly on their own, but you'll want to introduce the division sentence.

Those pictures for problem 13 are very important to your strugglers. Observe them as they work and give any necessary help. You'll want to give this group lots of time as you develop the division sentence with them.

things box; index cards

Challenge those who are ahead to show what great authors they are. Have them write problems for a word-problem box. One problem to a card, with the answer on the back.

These cards can then be used at any time as an independent activity. Anyone can select a card from the box, try to solve the problem, and then check the answer.

lesson Pages 62, 63, 64, 65

goal Relating repeated subtraction to finding an unknown factor

things paper bag
28 counters

page 62 We're going to fit all the pieces together, so let's bring the groups together too. This idea was touched on page 58. This time the exploration goes further. Simulate the problems on the page and have someone record the results. Complete the page with *Why do we care about finding a faster way?*

Go right on to page 63.

Let's change from boxes to bags.

The label says 28.
Take out 7 at a time.

28 to start with.

Take away 7.	$28 - 7 = 21$
Take away 7 more.	$21 - 7 = 14$
Take away 7 more.	$14 - 7 = 7$
Take away 7 more.	$7 - 7 = 0$

None left in the bag.

How many times was 7 subtracted? 4
Does $4 \times 7 = 28$? Yes

The label says 25¢.
Each piece should cost 5¢.
How many pieces?
You could subtract to find out.

25 to start with.

Take away 5.	$25 - 5 = 20$
Take away 5 more.	$20 - 5 = 15$
Take away 5 more.	$15 - 5 = 10$
Take away 5 more.	$10 - 5 = 5$
Take away 5 more.	$5 - 5 = 0$

How many times was 5 subtracted? 5
Does $5 \times 5 = 25$? Yes

Is there a faster way to find the answer?
Yes—divide 25 by 5.

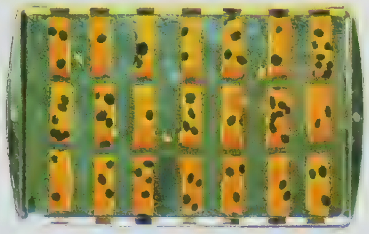
62

You can always subtract rather than divide.
Use subtraction to find this answer. ⟶

24 cookies.
6 in each row.
How many rows? 4

$24 - 6 = 18$
$18 - 6 = 12$
$12 - 6 = 6$
$6 - 6 = 0$

Multiplication is related to division too.



The multiplication sentence for this array is $3 \times 7 = \underline{?}$
This sentence would help you find how many in all.

Pretend now that you already know there are 21.
You know there are 3 rows.
But you don't know how many in each row.

You can use multiplication ⟶ $3 \times \underline{?} = 21$ 7
or division ⟶ $21 \div 3 = \underline{?}$ 7

Now pretend you know how many in each row.
But you don't know how many rows.

You can use multiplication ⟶ $\underline{?} \times 7 = 21$ 3
or division ⟶ $21 \div 7 = \underline{?}$ 3

$21 \div 3 = 7$ because $7 \times 3 = 21$ $21 \div 7 = 3$ because $3 \times 7 = 21$

Try some on your own.

	a	b
1.	$24 \div 8 = \underline{?}$ because $\underline{?} \times 8 = 24$ 3	$32 \div 4 = \underline{?}$ because $\underline{?} \times \underline{?} = \underline{?}$ $8 \times 4 = 32$
2.	$45 \div 5 = \underline{?}$ because $\underline{?} \times \underline{?} = \underline{?}$ $9 \times 5 = 45$	$36 \div 6 = \underline{?}$ because $\underline{?} \times \underline{?} = \underline{?}$ $6 \times 6 = 36$
3.	$18 \div 2 = \underline{?}$ because $\underline{?} \times \underline{?} = \underline{?}$ $9 \times 2 = 18$	$27 \div 3 = \underline{?}$ because $\underline{?} \times \underline{?} = \underline{?}$ $9 \times 3 = 27$
4.	$54 \div 9 = \underline{?}$ because $\underline{?} \times \underline{?} = \underline{?}$ $6 \times 9 = 54$	$42 \div 7 = \underline{?}$ because $\underline{?} \times \underline{?} = \underline{?}$ $6 \times 7 = 42$
5.	$63 \div 7 = \underline{?}$ because $\underline{?} \times \underline{?} = \underline{?}$ $9 \times 7 = 63$	$81 \div 9 = \underline{?}$ because $9 \times 9 = 81$ 9

63

goal Relating multiplication to division

memo The operation of division traditionally is difficult for pupils. No one knows for sure why it is hard. These next pages will encourage the pupil to see division as another way to look at multiplication. There will be no fancy talk of inverse relationships but rather a down-to-earth "golly, gee whiz, these two operations are related—knowing one will help the other" approach. A positive attitude is the most important outgrowth of these pages. There's a lot of division to learn before the child finishes school. It's too early to hate division.

page 63 Stay together for now. Discuss both types of unknown-factor sentences. Point out what each factor indicates:

number of rows × number in each row = number in all

Division will tell either the number of rows or the number in each row.

The remainder of the page is independent work for everyone.

goal Introduction to the terms **factor** and **product** as they relate to multiplication and division

page 64 All groups work together. You're about to develop two important vocabulary words—factor and product—that take care of both multiplication and division. How neat! **And** only one table is necessary for both operations!

Although pupils need to know the multiplication facts and the division facts, they rely most often on multiplication.

We need some math words to help us talk about multiplication and division.

$3 \times 4 = 12$

3 and 4 are called factors.
12 is called the product.

$3 \times \underline{?} = 12$

3 is the known factor. ? is the unknown factor. 12 is still the product. We can use the same names when we think about division.

$12 \div 4 = 3$

12 is still the product.
3 and 4 are still factors.

$12 \div 4 = \underline{?}$

12 is the product.
4 is the known factor.
? is the unknown factor.

Look at part of the multiplication table.

×	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

3, 4, and 12 are marked.

How are these numbers related?

$3 \times 4 = 12$

$4 \times 3 = 12$

$12 \div 3 = 4$

$12 \div 4 = 3$

The factors are on the left side and across the top of the table.

1. Find 20.
   What are its factors? 4, 5
2. Is 20 in more than one place in the table? Now what are the factors? Ye 5, 4
3. Find 8.
   Is there more than one? Y
   What are the factors? 2, 4
4. Find 15.
   Is there more than one? Y
   What are the factors? 3, 5

This table can help you with multiplication *and* division.

Find out how

Remember the multiplication facts for 9?
Write them in order on your paper.
Then use that set of facts to help you
complete a list of division facts for 9.

1x9=9 9÷9=1

2x9=18 18÷9=2

3x9=27 27÷9=3

× 9 = 9	9 ÷ 9 = 1
× 9 = 18	18 ÷ 9 = 2
× 9 = 27	27 ÷ 9 = 3
× 9 = 36	36 ÷ 9 = 4
× 9 = 45	45 ÷ 9 = 5
× 9 = 54	54 ÷ 9 = 6
× 9 = 63	63 ÷ 9 = 7
× 9 = 72	72 ÷ 9 = 8
× 9 = 81	81 ÷ 9 = 9

Start with the problems below and
complete each pattern of facts.

54÷6=
48÷6=
42÷6
36÷6
30÷

63÷7=
56÷7=
49÷7=
42÷7
35÷

72÷8=
64÷8=
56÷8=
48÷8
40÷
32

54 ÷ 6 = 9	63 ÷ 7 = 9	72 ÷ 8 = 9
48 ÷ 6 = 8	56 ÷ 7 = 8	64 ÷ 8 = 8
42 ÷ 6 = 7	49 ÷ 7 = 7	56 ÷ 8 = 7
36 ÷ 6 = 6	42 ÷ 7 = 6	48 ÷ 8 = 6
30 ÷ 6 = 5	35 ÷ 7 = 5	40 ÷ 8 = 5
24 ÷ 6 = 4	28 ÷ 7 = 4	32 ÷ 8 = 4
18 ÷ 6 = 3	21 ÷ 7 = 3	24 ÷ 8 = 3
12 ÷ 6 = 2	14 ÷ 7 = 2	16 ÷ 8 = 2
6 ÷ 6 = 1	7 ÷ 7 = 1	8 ÷ 8 = 1

goal Relating multiplication to division

page 65 Everyone on his own. Help with directions where necessary.

lesson Pages 66, 67

goal Practice with division facts

page 66 The emphasis is accuracy first, then speed. Require answers only.

Watch for anyone struggling with the reading at the bottom of this page. An able-reader buddy can be a big help. (They read together but **think** for themselves.)

To be really

FAST

you have to practice. Put your paper next to the first row. Write the answers. Fold the paper each time you finish a row.

1. $27 \div 3$	9	**2.** $36 \div 4$	9	**3.** $45 \div 5$	9
$24 \div 3$	8	$32 \div 4$	8	$40 \div 5$	8
$21 \div 3$	7	$28 \div 4$	7	$35 \div 5$	7
$18 \div 3$	6	$24 \div 4$	6	$30 \div 5$	6
$15 \div 3$	5	$20 \div 4$	5	$25 \div 5$	5
$12 \div 3$	4	$16 \div 4$	4	$20 \div 5$	4
$9 \div 3$	3	$12 \div 4$	3	$15 \div 5$	3
$6 \div 3$	2	$8 \div 4$	2	$10 \div 5$	2
$3 \div 3$	1	$4 \div 4$	1	$5 \div 5$	1

Patterns can sometimes help you. Once in a while they can hurt you. You can't be sure of your speed and accuracy using division until you "mix up" the combinations listed above.

Here are some people problems. Give each person an equal number.

4. 24 cookies.
4 people want some.
How many cookies for each? 6

5. 28 free tickets.
4 people want some.
How many tickets for each? 7

6. 35 pennies.
5 people want some
How many pennies for each? 7

7. 21 sour balls.
3 people want some.
How many sour balls for each? 7

8. 12 sticks of gum.
3 people want some.
How many sticks for each? 4

9. 40 marbles.
5 people want some.
How many marbles for each? 8

66

THERE ARE NO PATTERNS HERE. FIND THE ANSWERS.
LET THE MULTIPLICATION FACTS HELP YOU NOW.
USE THE FOLDED-PAPER IDEA TO SAVE TIME.

1.	2.	3.	4.	5.
1 ÷ 1 1	32 ÷ 8	45 ÷ 9 5	9 ÷ 9	25 ÷ 5
63 ÷ 7 9	4 ÷ 2	42 ÷ 6 7	56 ÷ 8	24 ÷ 8
72 ÷ 9	56 ÷ 7 8	64 ÷ 8	42 ÷ 7	9 ÷ 9
7 ÷ 7 1	36 ÷ 6	49 ÷ 7	36 ÷ 6	54 ÷ 6
48 ÷ 6 8	25 ÷ 5	72 ÷ 8	63 ÷ 9	30 ÷ 6
40 ÷ 5 8	18 ÷ 9 2	14 ÷ 7	16 ÷ 8	20 ÷ 5
49 ÷ 7 7	48 ÷ 8 6	54 ÷ 9	35 ÷ 7	45 ÷ 5
30 ÷ 5 6	35 ÷ 5 7	6 ÷ 6	12 ÷ 6	15 ÷ 5
10 ÷ 5 2	15 ÷ 5 3	21 ÷ 7	27 ÷ 9	28 ÷ 7
0 ÷ 9 0	81 ÷ 9 9	40 ÷ 8	8 ÷ 8	5 ÷ 5

Find the number that should replace each ?

6. **12**
÷ 3 = ?
÷ 4 = ?
÷ 6 = ?
÷ 2 = ?

7. **18**
÷ 2 = ?
÷ 6 = ?
÷ 3 = ?
÷ 9 = ?

8. **24**
÷ 6 = ?
÷ 8 = ?
÷ 4 = ?
÷ 3 = ?

67

goal Practice with division facts

page 67 Sharpies record answers only, working for accuracy and speed.

This is a demanding page even for average students. You may want to take two days.

Nice and easy with those who are struggling along. You don't want to lose them. You may want to skip columns 1 through 5 and substitute flash cards and oral drill (10 cards at a time). Practice with multiplication facts will help too.

lesson Pages 68, 69

goal Progress Check – division facts

page 68 Adapt the folded-paper technique for use with columns and have the pupils record answers only.

The problems are separated into levels of difficulty. Part 1 is the easiest. Parts 2 and 3 have the same level of difficulty. Part 4 is the hardest. You may want to assign only the first two or three parts to those who are struggling.

Unless the majority of your pupils are operating confidently, don't use a 1-minute time limit now.

Be sure to follow up on errors. Flash cards should be made and used.

PROGRESS CHECK

Skill: Division facts

Divide

Part 1	Part 2	Part 3	Part 4
$8 \div 4$ 2	$30 \div 5$ 6	$30 \div 6$ 5	$72 \div 8$ 9
$20 \div 5$ 4	$18 \div 3$ 6	$18 \div 2$ 9	$63 \div 7$ 9
$15 \div 3$ 5	$24 \div 8$ 3	$24 \div 4$ 6	$54 \div 9$ 6
$12 \div 4$ 3	$18 \div 9$ 2	$36 \div 9$ 4	$48 \div 6$ 8
$6 \div 2$ 3	$35 \div 7$ 5	$21 \div 7$ 3	$63 \div 9$ 7
$12 \div 3$ 4	$28 \div 4$ 7	$35 \div 5$ 7	$49 \div 7$ 7
$10 \div 5$ 2	$27 \div 9$ 3	$27 \div 3$ 9	$54 \div 6$ 9
$16 \div 4$ 4	$40 \div 8$ 5	$28 \div 7$ 4	$42 \div 7$ 6
$15 \div 5$ 3	$24 \div 6$ 4	$45 \div 9$ 5	$56 \div 8$ 7
$25 \div 5$ 5	$32 \div 4$ 8	$32 \div 4$ 8	$72 \div 9$ 8

5. 40 in all. 5 in each car. How many cars? 8

6. 42 in all. 6 in each box. How many boxes? 7

7. 32 in all. 8 at each table. How many tables? 4

8. 63 in all. 7 in each set. How many sets? 9

68

STOP Use the same raw materials page 72b. Switch to division facts.

See activity 6, page 72c.

A doctor makes a diagnosis when he tries to find out how to make a person feel better.

The last page was a diagnosis of the division facts. You might need more practice to compute better.

Get or make a complete multiplication table.

Only part of the table is pictured below. Look at your errors on the last page. Can you locate the fact in the table?

Part 1 facts come from here.

Part 2 and Part 3 facts come from here.

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Part 2 and Part 3 facts come from here.

Part 4 facts come from here.

Maybe the diagnosis can help you decide what facts you have to study the most.

69

goal Diagnosis of division-fact difficulties

things for each pupil:
multiplication table from page 53

page 69 **Diagnosing** one's own troubles is a new idea. Better talk about this together. Are the division facts that are causing the pupil trouble related to multiplication facts that he has not yet mastered? Use page 69 to help these youngsters identify their problems.

lesson Pages 70, 71

goal Examining the number 0 in basic operations in real-world situations

memo Strictly for discussion!

page 70 Any number that answers the question How many? is a **whole number.** *Is zero a whole number?* Take advantage of these problem situations to reinforce this concept. You can have fun with this page.

We have ignored an important number.

The number 0 tells how many.

Bill is broke. He has no money at all. *How many* cents does Bill have? 0
The cookie jar was empty. *How many* cookies in the cookie jar? 0
She didn't make any mistakes. *How many* mistakes did she make? 0

1. He had 0.
She had 3.
How many in all? 3

2. She had 0.
She lost 0.
How many remain? 0

3. Mr. Billings promised to put money in each of his children's banks. His promise was to put enough money in to make the amount three times as much. Here's what each child thought.

a You compute the new amount in each bank.

Earl thought $3 \times \$2 = \underline{?}$ $6

Jean thought $3 \times \$.50 = \underline{?}$ $1.50

Jerry thought $3 \times \$3.50 = \underline{?}$ $10.50

b Murl's bank was empty. Write the multiplication sentence that tells what Murl thought. $3 \times \$0 = \$0$

c How much did Mr. Billings put in Murl's bank? $0

4. The prizes were planned as a joke. There were 3 bags. Each bag was empty. The winner was to get the prizes in two of the bags. The runner-up got the prizes in the other bag. The winner opened his bags and caught on to the joke. The winner said to the runner-up, "Oh boy! I got twice as many prizes as you did." How many prizes did each get? Did the winner get twice as many?
0 Yes but $2 \times 0 = 0$

Compute. Replace the ? with the correct whole number.
It might help to think of a story with a paper bag.
Or maybe thinking about the number line would help.

1. $2 + 0 = ?$	2	**2.** $2 - 0 = ?$	2	**3.** $2 \times 0 = ?$	0
$0 + 9 = ?$	9	$0 - 9 = ?$	Can't be done	$0 \times 9 = ?$	0
$10 + 0 = ?$	10	$10 - 0 = ?$	10	$10 \times 0 = ?$	0
$0 + 2 = ?$	2	$0 - 2 = ?$	Can't be done	$0 \times 2 = ?$	0

What about the number 1? That's an important number also.

4. $2 + 1 = ?$	3	**5.** $2 - 1 = ?$	1	**6.** $2 \times 1 = ?$	2	**7.** $9 \div 1 = ?$	9
$1 + 9 = ?$	10	$1 - 1 = ?$	0	$1 \times 9 = ?$	9	$1 \div 1 = ?$	1
$10 + 1 = ?$	11	$10 - 1 = ?$	9	$10 \times 1 = ?$	10	$4 \div 1 = ?$	4
$0 + 1 = ?$	1	$5 - 1 = ?$	4	$0 \times 1 = ?$	0	$10 \div 1 = ?$	10

Compute. WATCH OUT!

You might have to add or subtract or multiply.

8. $0 + 0 = ?$	0	**9.** $5 \times 0 = ?$	0	**10.** $5 + 1 = ?$	6
$0 \times 0 = ?$	0	$1 + 3 = ?$	4	$4 - 0 = ?$	4
$1 \times 0 = ?$	0	$3 - 1 = ?$	2	$3 + 1 = ?$	4
$0 \times 1 = ?$	0	$0 \times 3 = ?$	0	$0 \times 4 = ?$	0
$0 + 1 = ?$	1	$4 + 1 = ?$	5	$1 + 4 = ?$	5
$1 + 1 = ?$	2	$5 - 1 = ?$	4	$4 - 1 = ?$	3
$1 - 0 = ?$	1	$3 - 0 = ?$	3	$3 \times 0 = ?$	0

11. What would happen if everybody had forgotten about zero? What changes would we have to make if we didn't have that number at all? Answers will vary.

71

goal Examining the numbers 0 and 1 as they relate to the basic number operations

page 71 A great page for the folded-paper technique. Answers only are sufficient.

Careful! Your independent learners may consider this such a simple page that they make careless errors. Have them go back and correct their own errors.

Your average learners will plod along. If they see the pattern in each case, their speed will pick up.

Nice and easy with those who are struggling along. You don't want them to give up and write whatever comes into their heads. If they can just understand these generalizations, they will have taken a giant step forward.

some number + 0 = that same number
some number − 0 = that same number
some number × 0 = 0
0 × some number = 0

Help them to see this with numbers, rather than with words. When these are down pat, work on the number 1.

lesson Page 72

goal **Checkout**—multiplication and division facts

page 72 Use the folded-paper technique. Answers only are sufficient. Again, skills are identified on the answer key to help you diagnose the learner's trouble spots. For more extensive diagnosis, you may want to reuse pages 54, 55 (multiplication), and 68 (division).

praise those who have shown progress!

CHECKOUT

Multiply. Skill: Multiplication facts

	a	b	c	d	e	f
1.	5×6 = 30	8×4 = 32	9×7 = 63	6×3 = 18	7×4 = 28	8×8 = 64
2.	2×8 = 16	9×9 = 81	8×6 = 48	4×3 = 12	5×4 = 20	7×6 = 42

Divide. Skill: Division facts

	a	b	c	d
3.	$42 \div 7 =$? 6	$36 \div 4 =$? 9	$21 \div 7 =$? 3	$54 \div 9 =$?
4.	$56 \div 7 =$? 8	$27 \div 3 =$? 9	$25 \div 5 =$? 5	$63 \div 9 =$?
5.	$54 \div 6 =$? 9	$35 \div 5 =$? 7	$36 \div 9 =$? 4	$24 \div 3 =$?

Skill: Solving problems with multiplication

6. a 6 oaks, 6 elms, 6 maples, 6 pines, 6 poplars. How many? 30

   b 7 pears, 7 plums, 7 peaches, 7 bananas. How many? 28

   c 9 baskets, 9 bags, 9 boxes, 9 cages. How many? 36

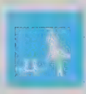
Skill: Solving problems with division

7. a 36 quarts of milk. There are 4 quarts in a gallon. How many gallons of milk? 9

   b $48 was earned. 6 people helped earn it. How much should each person get? $8

   c The cloth was 27 feet long. There are 3 feet in each yard. How many yards long? 9

72



See activity 7, page 72c.

See activity 8, page 72c.

RESOURCES

another form of evaluation

for Progress Check—page 57

Try these. See how well you do.
Multiply.

1.	2.	3.	4.	5.
5	4	6	2	7
× 4	× 8	× 1	× 3	× 4
20	32	6	6	28

6.	7.	8.	9.	10.
2	8	9	6	4
× 7	× 3	× 7	× 7	× 6
14	24	63	42	24

11.	12.	13.	14.	15.
9	7	5	8	1
× 8	× 8	× 9	× 6	× 0
72	56	45	48	0

Answer the questions.

16. 6 hours in school each day. 5 days a week. How many hours of school in all? 30
17. 8 friends at the party. 2 cupcakes for each. How many cupcakes in all? 16
18. 7 cars going to the game. 4 kids in each. How many kids in all? 28
19. 6 candy bars in a package. 8 packages bought. How many candy bars in all? 48
20. 5 cakes baked. 8 pieces in each. How many pieces in all? 40
21. 5 desks in a row. 5 rows in the room. How many desks in all? 25
22. 4 teams. 9 players on each team. How many players in all? 36
23. 8 pop bottles in a carton. 3 cartons returned. How many bottles returned in all? 24

for Progress Check—page 68

Divide.

Part 1.		Part 2.	
$10 \div 2$	5	$16 \div 8$	2
$20 \div 5$	4	$35 \div 5$	7
$9 \div 3$	3	$36 \div 4$	9
$20 \div 4$	5	$12 \div 6$	2
$8 \div 2$	4	$14 \div 7$	2
$6 \div 3$	2	$21 \div 3$	7
$12 \div 4$	3	$32 \div 4$	8
$15 \div 5$	3	$18 \div 9$	2
$4 \div 1$	4	$45 \div 5$	9
$12 \div 3$	4	$24 \div 4$	6

Part 3.		Part 4.	
$40 \div 5$	8	$64 \div 8$	8
$14 \div 2$	7	$56 \div 7$	8
$24 \div 8$	3	$81 \div 9$	9
$30 \div 5$	6	$42 \div 6$	7
$28 \div 4$	7	$48 \div 8$	6
$24 \div 3$	8	$72 \div 8$	9
$36 \div 9$	4	$36 \div 6$	6
$16 \div 2$	8	$63 \div 7$	9
$27 \div 9$	3	$49 \div 7$	7
$35 \div 7$	5	$54 \div 6$	9

5. 56 in all. 8 in each case. How many cases? 7
6. 42 in all. 7 in each bag. How many bags? 6
7. 72 in all. 9 in each set. How many sets? 8
8. 36 in all. 6 in each row. How many rows? 6

for Checkout—page 72

Multiply.

	(a)	(b)	(c)	(d)	(e)	(f)
1.	5	6	8	7	9	6
	× 3	× 4	× 5	× 3	× 6	× 8
	15	24	40	21	54	48
2.	8	6	9	7	5	9
	× 7	× 5	× 4	× 9	× 7	× 8
	56	30	36	63	35	72

Divide.

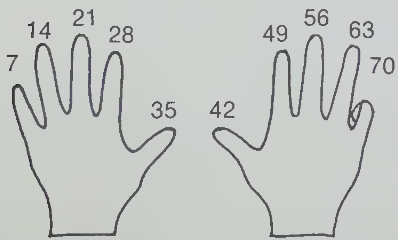
3. (a) $28 \div 7 = \underline{?}$ 4 (b) $48 \div 6 = \underline{?}$ 8
   (c) $56 \div 8 = \underline{?}$ 7 (d) $72 \div 9 = \underline{?}$ 8
4. (a) $72 \div 8 = \underline{?}$ 9 (b) $63 \div 7 = \underline{?}$ 9
   (c) $42 \div 7 = \underline{?}$ 6 (d) $36 \div 6 = \underline{?}$ 6
5. (a) $54 \div 9 = \underline{?}$ 6 (b) $81 \div 9 = \underline{?}$ 9
   (c) $45 \div 5 = \underline{?}$ 9 (d) $64 \div 8 = \underline{?}$ 8
6. What's the answer?
   a) 8 fish, 8 birds, 8 dogs, 8 cats. How many? 32
   b) 7 potatoes, 7 turnips, 7 pears, 7 tomatoes, 7 carrots. How many? 35
   c) 5 kites, 5 balls, 5 bats, 5 jump ropes, 5 planes. How many? 25
7. a) 42 fish in the pet shop. There are 6 in each tank. How many tanks? 7
   b) 28 glasses of milk. There are 4 glasses in a quart. How many quarts? 7
   c) 36 children in the class. The class has 4 teams. How many on each team? 9

activities

1. things markers; index cards; small box

Have pupils fold a piece of paper into 16 spaces, select 16 products from the multiplication table (page 53), and write one product in each space. Then have them write a multiplication fact on each index card. There are 45 different facts. The cards are mixed in the box. One pupil serves as caller, selecting a fact from the box and calling it out. If the product appears on their sheet, the others cover it with some sort of marker. A row across, a column down, or a diagonal covered wins the game.

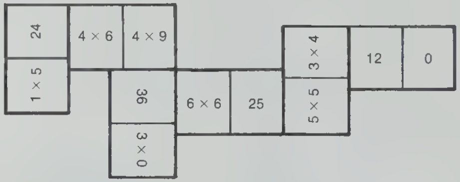
2. Fingers are always available to help with computation. But these kids need a system that will help. Use fingers for skip-counting practice. Make fists. As each number is said, a finger goes up.



How many times 7 is 63? How many fingers are up?

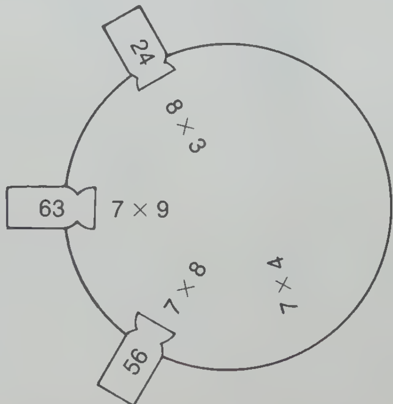
3. things index cards

Draw lines with a ruler dividing the cards into halves. Write either a multiplication fact or a product on each half. These are placed facedown in a random arrangement. One card is turned faceup. Each player in turn selects a card and turns it faceup. Job: Match each product with an appropriate fact; match each fact with an appropriate product.



4. things cardboard; snap-type clothespins; container

Cut circular regions from cardboard. (Pizza boards are great.) Write the facts that an individual student needs to practice around the outside of the region and the product of each fact on a clothespin. The clothespins are mixed in a container. The youngster matches each product to the appropriate multiplication sentence.

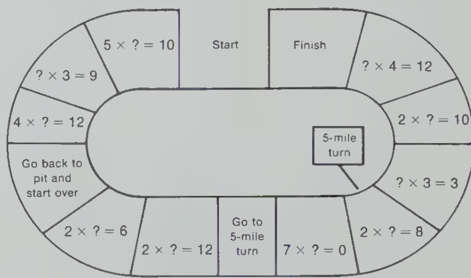


5. things large sheets of paper; dice; variety of markers

Group pupils. Have each group design its own Mathopolis Speedway and section off parts of the track. Provide a list of unknown factor sentences. Let the pupils choose from the list and fill in the sections of the track. They may want to include some special directions in some of the sections.

We're ready to play the game. One die is rolled to determine the number of sections of track to move. A correct answer enables the player to leave his marker (a minicar makes a great one) in the space; an incorrect answer forces him to move back to his previous position. Remember to follow any special instruction given on the track.

Variations:
1. Use another theme for the game board.
2. Change the type of sentence used on the board.



6. things game board; 36 small cards

The game board consists of a 6-by-6 array of 2-inch squares. Write a division fact on one card, its quotient on another. Make 18 pairs of cards. The cards are mixed and placed facedown on the board. The first player turns over any two cards. If the cards match, showing a fact and its quotient, the cards are removed from the board by the player. If the cards do not match, they are again turned facedown. The next player takes his turn. Play continues until all the cards have been removed from the board. The player with the most cards wins.

7. things for each group: deck of playing cards

Remove all face cards. Change the aces to 1s and the jokers to 0s. The cards are shuffled and placed in a stack facedown. The first player draws 2 cards. If he can give the correct product for the 2 numbers shown, he keeps his cards. If the product is incorrect, the cards go to the bottom of the deck. The play continues until all cards are gone. The player with the most cards wins.

8. things spirit master: 3-by-3 arrays of squares; pencils of two different colors

Make the squares large enough so that pupils can write in facts on which they need additional practice. Pair pupils. Each pupil uses a different-colored pencil. To claim a square, he must write the correct product. Pupils alternate. The winning combination: three products of the same color in a row, column, or diagonal.

9×4	9×10	9×6
9×9	9×2	9×3
9×7	9×5	9×8

Variation: Practice division facts.

additional learning aids

concept—chapter objectives 1, 3, 6

SRA products
Mathematics Involvement Program, SRA (1971)
Cards: 143, 14, 24, 64, 144, 125, 275, 285
Visual Approach to Mathematics, level 3, SRA (1967)
Visuals: 13, 14, 15, 16

other learning aids (described on page 72d)
Counting chips, Cuisenaire* rods

notation—chapter objective 4

other learning aids—Multifax & Quotient

operation—chapter objectives 2, 5

SRA products
Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: W 5, 6, 13
Arithmetic Fact Kit, SRA (1969)
All multiplication cards
Computapes, SRA (1972)
Module 3, Lessons: MD 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 15, 16
Module 4, Lesson: MD 28
Diagnosis: an instructional aid—Mathematics Level A, SRA (1973)
Probes: L-12, 13
Skill through Patterns, level 4, SRA (1974)
Spirit masters: 21, 23, 24, 25, 26, 27, 28, 29, 35, 36, 38, 42, 59, 68

other learning aids—I Win (sets 2 and 3), Mathfacts Games** (multiplication-division levels I-IV), Motivator Activity Cards (multiplication facts), Multiplying Machine, Orbiting the Earth (multiplication), Ting, Veri-Tech Senior (multiplication books), Winning Touch

*Registered trademark of Cuisenaire Co. of America, Inc.
**Trademark of Milton Bradley Co.

Other Learning Aids

whole-number concepts

Abacus board (Creative Publications) Counting board useful for teaching place value

Abacus Spinner Game (Math Shop) Game designed to provide practice in recognizing and understanding place value

Chip Trading (Scott Scientific) Game to develop an understanding of place value

Counting Chips (Creative Publications) Plastic chips useful for teaching simple computations

Cuisenaire® Rods (Cuisenaire) Centimetre rods designed to provide practice with arrays

Fundamath (Ideal) Boards with beads that demonstrate addition and subtraction

Japanese Abacus (Creative Publications) An abacus designed to teach place value and basic number operations

Multifax & Quotient (Math Shop) Game in which number sentences based on the multiplication facts are formed

Place Value I and II (Creative Publications) Self-correcting cards to provide practice in reading numbers through hundred millions

Ranko (Math Shop) Game designed to provide practice in ordering numbers

Tally Counter (Creative Publications) Adding device to help children understand place value

whole-number operations

Dial-A-Matic® Adding Machine (Sigma Scientific) A simple calculator for practice in addition and subtraction

I Win (sets 1, 2 and 3) (Scott, Foresman) Card game to provide practice in four basic operations with whole numbers

Mathfacts Games™ (Milton Bradley) Self-instructional and self-checking games that deal with basic multiplication and division facts

Motivator Activity Cards—Multiplication Facts (Singer/SVE) Laminated fact cards for practice with multiplication facts

Multiplying Machine (Math Shop) Self-checking machine to be used for practice with the multiplication facts

Napier's Rods (Sigma Scientific) Rods designed to provide practice in multiplication

Numble™ (Sigma Scientific) Crossword-type number game to reinforce the four basic operations with whole numbers

Numo (Math Shop) A bingo-type game to provide practice in addition and subtraction

Orbiting The Earth (Scott, Foresman) [multiplication and division] Game with vinyl playing field to provide practice in multiplication and division

Rally with Remainders (Math Shop) A self-correcting game providing division practice

Ting (SEE) A jigsaw puzzle for reinforcement of multiplication facts

Veri-Tech Senior (ETA) [addition, subtraction, division, and multiplication books] A self-checking device that provides practice with whole-number operations

Winning Touch (Ideal) Game that provides for reinforcement of multiplication facts

fractional-number concepts

Action Fraction Games (Constructive Playthings) Game designed to develop concepts and increase skills with fractional numbers

Experiments in Fractions (Math Shop) Activities that help the student understand, develop, and use vocabulary, notation, and operations

Fraction Bars Student Activity Book (Creative Publications) Games and activities designed to teach specific objectives for fractions

Fraction Dominoes (SEE) Game involving matching a fractional numeral with its model

Fraction Line Set (Sigma Scientific) A learning activity designed to help students visualize operations by computing with fraction strips

Fraction Wheel (Ideal) Circle showing fractions and their relationships by revolving disks

Student Fraction Sets (ETA) [Circular, Square] Activities that provide experiences in matching a unit with fractional parts and vice versa

geometry

Geoboards and Motion Geometry Resource Book and Activities (Scott, Foresman) Activities dealing with congruence, coordinates, transformations, and area

Geometry Figures and Solids (Creative Publications) Forms to facilitate understanding of basic geometric concepts

Learn to Fold—Fold to Learn (Lyons & Carnahan) Workbook that presents a variety of paper-folding activities

Mira (Creative Publications) An aid for investigating properties of plane geometry

Mira Math for Elementary School (Creative Publications) Activities to be used with the Mira

Polyhedra Model Kit (Creative Publications) Kit to make five basic polyhedra models

Soap-film Shapes (SEE) Wire shapes that are dipped in soapsuds to make six different geometric-shaped soap bubbles

Shape Tracers (Math Shop) Geometric shapes for tracing and practice in shape recognition

measurement

Easy Money (Milton Bradley) Game that provides practice in dealing with money

Good Time Mathematics (Holt, Rinehart & Winston) A multimedia program designed to give activity-based learning experiences

Learning About Measurement (Lyons & Carnahan) A workbook with activities using the metric and customary systems

Money Matters (Math Shop) Book of puzzles providing practice in problem solving

Pay the Cashier Game (Garrard Publishing) Board game centered around counting, adding, and subtracting money

Spin-A-Coin (Math Shop) Game dealing with place value and operations involving money

Spin-A-Yard (Math Shop) Game providing practice with linear measurement

statistics and probability

Block Graph (ESA) Demonstration set for introducing bar graphs, averages, and so on

Histogram Board (ESA) Board for making bar diagrams—to be used with Stern Unit Cubes

Probability Maze (ESA) A board to illustrate probability and statistics

problem solving and applications

Heads Up™ (Creative Publications) Game providing practice with equations

Number Sentence Games (Creative Publications) Booklet of worksheets that provide practice with operations and number combinations

True or False (ESA) A game for deciding true or false statements

4 MEASUREMENT

before this chapter the learner has—

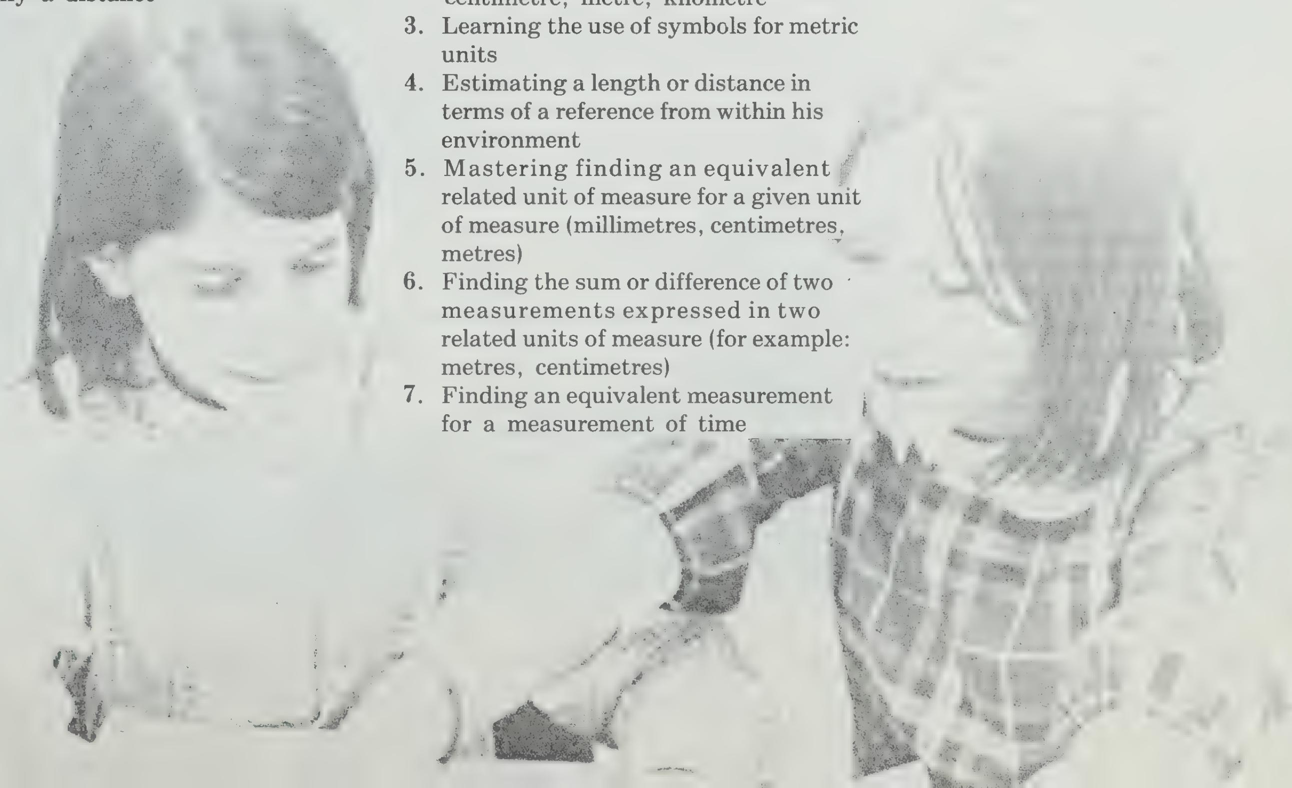
1. Experienced measuring with a variety of units of measure
2. Added and subtracted two measurements with no renaming
3. Selected an appropriate unit of measure from the metric system to quantify a distance

in chapter 4 the learner is—

1. Mastering selecting an appropriate metric unit of measure for a measurement situation
2. Mastering naming an object that can be appropriately measured with any of the following units of measure: centimetre, metre, kilometre
3. Learning the use of symbols for metric units
4. Estimating a length or distance in terms of a reference from within his environment
5. Mastering finding an equivalent related unit of measure for a given unit of measure (millimetres, centimetres, metres)
6. Finding the sum or difference of two measurements expressed in two related units of measure (for example: metres, centimetres)
7. Finding an equivalent measurement for a measurement of time

in later chapters the learner will—

Master adding or subtracting two measurements in like units without renaming and then label the answer with the appropriate unit



Notes & Things

This is a "talk and do" chapter. It is intended to develop the concept of measure and measurement with emphasis on length and time. It is hoped each learner will understand that the unit of measure used can be arbitrary. But if we want to communicate with people about the measurement of an object, we probably should use an agreed-on (standard) unit.

It is an obligation to emphasize the International System of measurement—not because it is easier to compute but because it is the system that each pupil will be using the rest of his life. It is up to you whether or not you want to investigate which countries presently use SI, its use in North America, or the controversy about the problems created and solved by switching to it. These would, however, make great research projects.

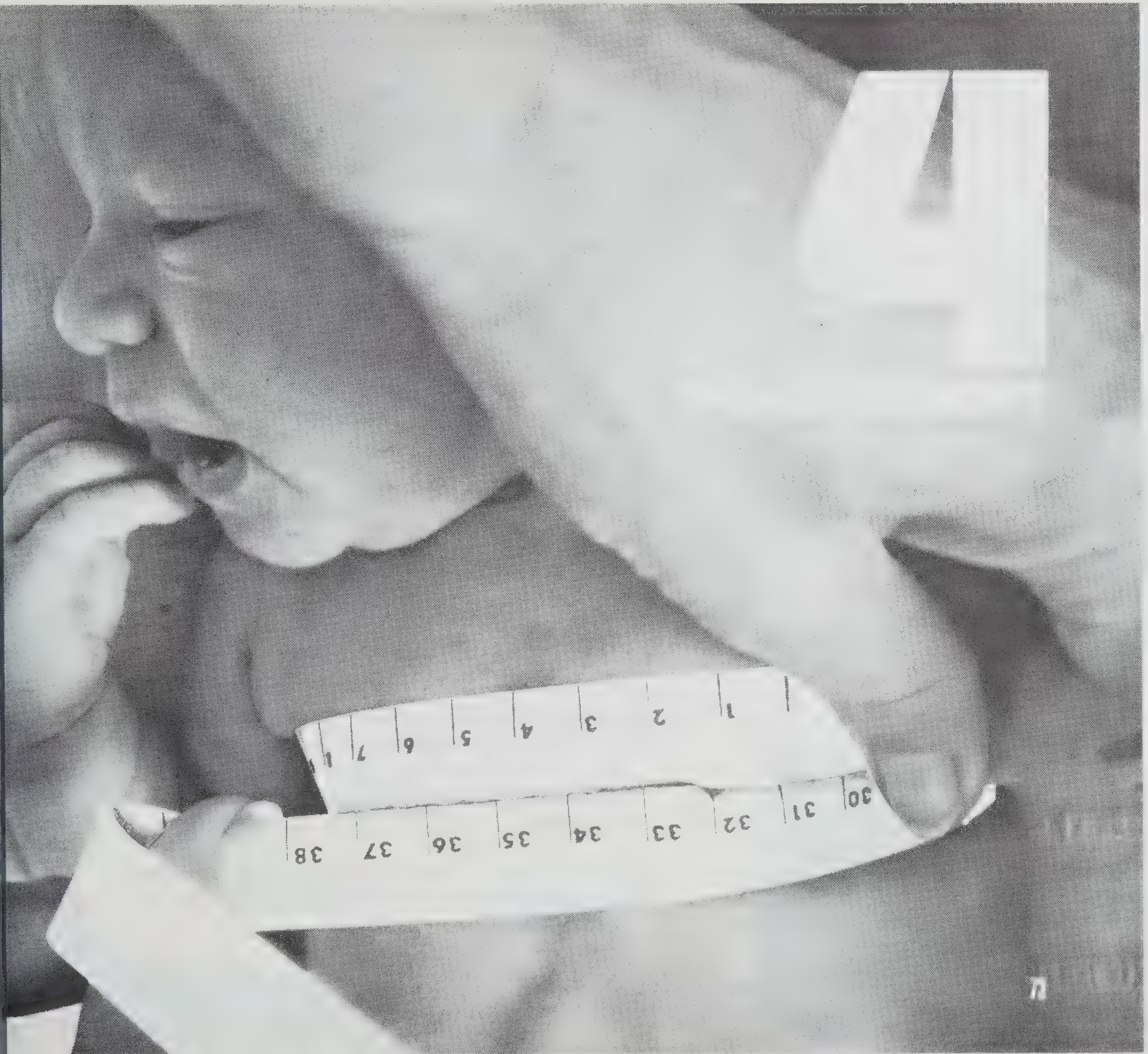
For pupils who have had little work with the International System of measures, this chapter provides a great deal of practice in measuring, estimating, and in general becoming familiar with the SI units. There is no substitute for actually feeling and using rulers and metresticks and measuring real objects. Pupils who are already familiar with the SI units will not need so much work, but it is up to you to decide how much they need.

The pupils should remember that no measurement is absolutely accurate. It will always be limited by the precision of the measuring device used. There is no need to make a big deal out of this. But do be aware that when someone says that the length of a book is equal to 25 cm, for example, he is really stretching the mathematical definition of *equal.* Avoid this situation whenever possible.

The theme of estimation is put in a new context in this chapter. Judging the length of a pencil certainly involves a different set of thoughts than rounding numbers so that one can estimate an answer to a problem. Please be aware that estimation means something a bit different now.

things

centimetre rulers for each pupil
metresticks
globe, if available
flashlight
watch with second hand
tape measures
dictionary for each pupil



goal Think about and explore ideas through a picture clue

page 73 This photograph will serve you in many ways. Most important will be your chance to find out how much the children know about centimetres.

Direct questioning will help get the discussion going in such a way that the youngsters will want to do some independent research.

Is this baby a year old? Six months? How do you know? It looks as if someone is measuring the baby's chest. How many centimetres around does the tape show? How big were you when you were born? Can you find out how big you were when you were one year old?

If your pupils become interested in the task of finding out how big they were at birth, you have a perfect ongoing activity. Have them decide how they could compile the information for the class. *Was length related to mass? Is the tallest person in class also the person who was longest at birth? What about the shortest?*

Here are more measuring opportunities. Measure the entire family and relate age to height. Are the males that were measured taller than the females in the same age bracket?

That would be a good opening for the pupils' work on measurement.

lesson Pages 74, 75

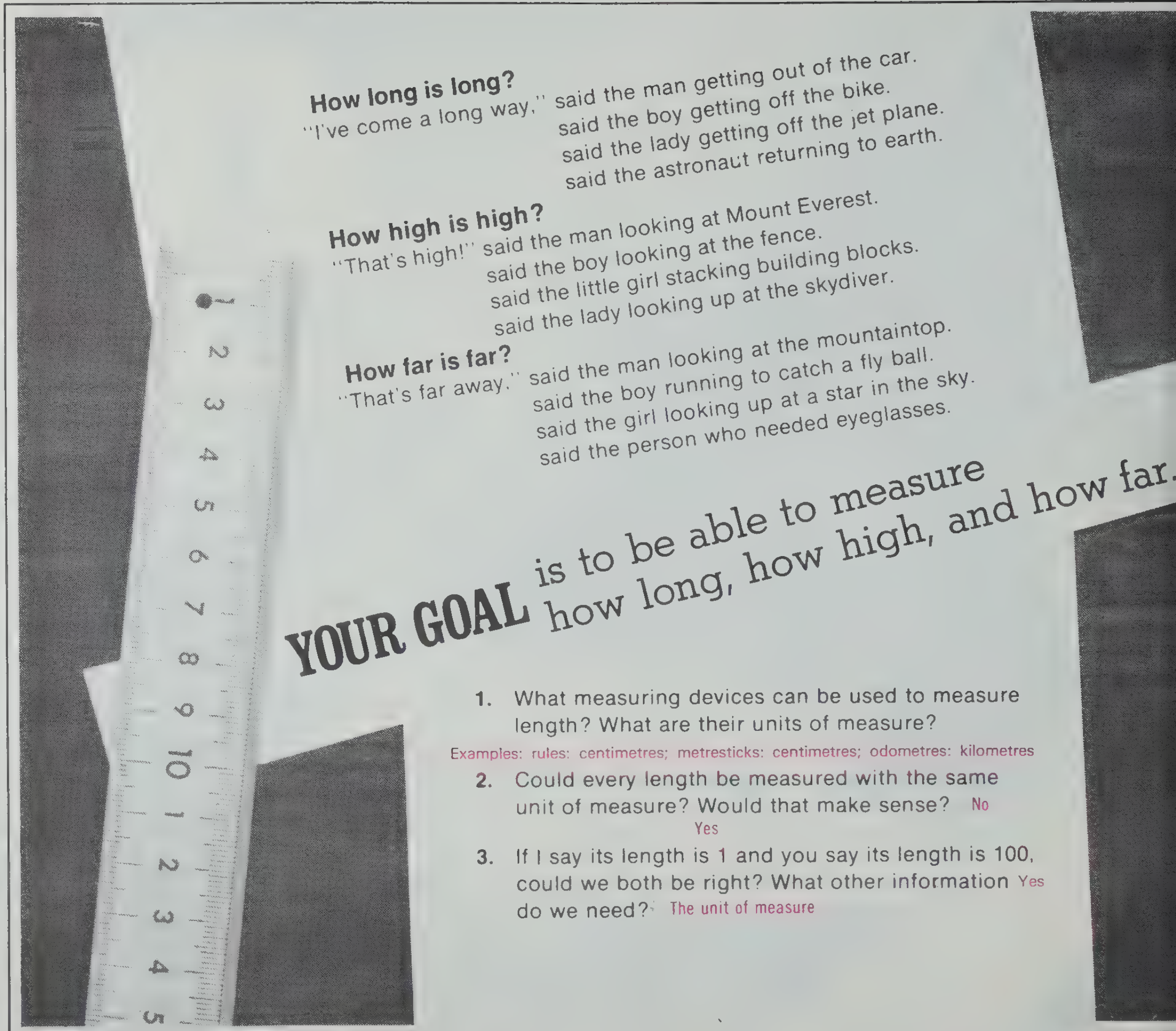
goal **Survey**—knowledge of measuring devices and units of measure

warm-up Have pupils jot down their answers. *Name an object that you think is short. . . long. . . fast. . . big.* Discuss the range of answers. (For example: A stock car is fast, a rocket is faster, and the speed of light is faster yet.) Challenge the pupils to explain the different answers. Comparison is the key idea. Even a jet plane is slow when compared with the speed of light.

page 74 Discuss the *long, high, far* questions together. Analyze each notion. Name appropriate units of length for each situation. The references to metric units of measure will indicate the extent of your pupils' acquaintance with metric measurement. This discussion is an informal survey – listen carefully.

Let the abilities of your class determine whether problems 1 through 3 should be discussed together or written independently.

There are some good questions here. The goal sounds a bit vague, but the kick-off discussion should have made it considerably more valid.



How long is long?

"I've come a long way," said the man getting out of the car.
said the boy getting off the bike.
said the lady getting off the jet plane.
said the astronaut returning to earth.

How high is high?

"That's high!" said the man looking at Mount Everest.
said the boy looking at the fence.
said the little girl stacking building blocks.
said the lady looking up at the skydiver.

How far is far?

"That's far away." said the man looking at the mountaintop.
said the boy running to catch a fly ball.
said the girl looking up at a star in the sky.
said the person who needed eyeglasses.

YOUR GOAL is to be able to measure how long, how high, and how far.

1. What measuring devices can be used to measure length? What are their units of measure?
   Examples: rules: centimetres; metresticks: centimetres; odometres: kilometres
2. Could every length be measured with the same unit of measure? Would that make sense? No Yes
3. If I say its length is 1 and you say its length is 100, could we both be right? What other information do we need? Yes The unit of measure

How long?
How high?
How far?

These are all questions about length.

To answer these questions, you must measure.

BUT before you measure—
Are you sure you know what you're going to measure?

What unit of measure should you use?
(Would you measure the distance from home to school with a metrestick?)

After you have answered these questions, you can measure.
Sometimes your measurement must be very accurate.
Sometimes a rough estimate will do.
Often you will have to compute.

Think of some distances to measure—some long and some short.

1. From home to school—
   a What large unit of measure could be used? Guess how many. kilometres
   b What small unit of measure? metres
   Guess how many.

2. From floor to top of desk—
   a What large unit of measure could be used? Guess how many. metres
   b What small unit of measure? centimetres
   Guess how many.

3. Compute.

a

4 m	8 cm
+ 3 m	2 cm
7 m	10 cm

b

2 km	4 m
− 1 km	1 m
1 km	3 m

c

1 m	
+	1 cm
1 m	1 cm

75

See activity 1, page 96a.

goal **Survey** [*continued*]—selecting an appropriate unit of measure; adding and subtracting measurements

page 75 Back to discussion for the top part of the page. Expand on the question "Are you sure you know what you're going to measure?" *If a football coach asks you how big you are, what measurement is he probably looking for?* (Mass or height) *If a passenger asks a cab driver how far to the airport, what measurement is he probably looking for?* (Time or maybe the amount of fare)

Pupils must realize that before a task of measuring is begun, two questions must be resolved: *What am I going to measure?* (mass, time, rate, distance, height, width, and so on) and *What unit of measure is most appropriate to use?*

Once again, you decide how best to handle exercises 1 through 3, depending on the abilities of your pupils.

In general, mixed units are to be avoided in metric notation. 408 cm or 4.08 m would be accepted usage. However, until the children have been introduced to decimal notation, it may be thought of as 8 cm more than 4 m .

It is very important that pupils do not become discouraged because they don't know the answers to these questions. It's O.K. The questions give a hint about the chapter activities to come. Remind them of the learning goal. They will know all these answers and a lot more in no time at all.

lesson page 76

goal Measuring with centimetres; introduction to millimetres

page 76 Let students begin work independently. Then form a group of those who are finding it difficult. Ask questions that will guide learning: *Does the point of the pen come nearer to the 14 cm mark or to the 15 cm mark?* (14 cm) *Is the clip longer than 2 cm? than 3 cm?* (Yes) *Is it longer than 4 cm?* (No) *Which is it nearer to?* (3 cm)

After millimetres have been discussed, ask for the measurement of the clip to the nearest millimetre.

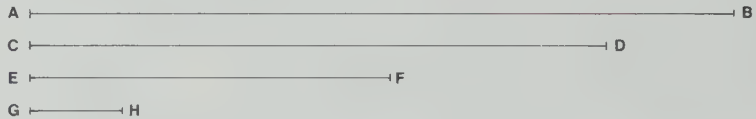
This rule has centimetres marked on it.
What is the length of this rule? 15 cm

About how long is the paperclip? 3 cm

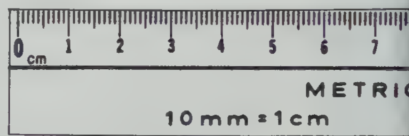
About how long is the pen? 14 cm

Look below.

1. Is line segment AB closer to 13 cm or 14 cm? 14 cm
2. Is line segment CD closer to 11 cm or 12 cm? 11 cm
3. Is line segment EF closer to 7 cm or 8 cm? 7 cm
4. Is line segment GH closer to 1 cm or 2 cm? 2 cm



There are lots of marks between the numbers on this rule.
Do you know what they show?



5. They show millimetres. How many millimetres are there in 1 cm? (Count them.)
   1 cm = ? mm 10
6. How many millimetres are there in 2 cm? in 5 cm? 20 50

76

Measure your handspan in centimetres on this rule.

Measure the width of 2 fingers in centimetres.

Knowing the length of your handspan and the width of 2 fingers can be useful.
Imagine you don't have a rule and you want to measure something. You could use your handspan or your two fingers. This would give you a rough estimate.
Try it.

Estimate the width of your desk.

object	estimate
width of desk	. . . . cm

Now measure it with a rule.

object	estimate	measure
width of desk	. . . . cm	. . . . cm

What is the difference between your estimate and your measurement with a rule? Answers will vary

object	estimate	measure	difference
width of desk	. . . . cm	. . . . cm	. . . . cm

Try these. See how close you can make your estimates. Answers will vary
(Pick some objects of your own choice to add to the list.)

pencil			
shoe length			
height of desk			

lesson Page 77

goal Estimating the length of objects in centimetres; measuring objects in centimetres and comparing with estimates

things for each pupil: centimetre rule

page 77 Discuss the meaning of handspan; where it begins, where it ends, whether long finger-nails are included. After the initial discussion, this is a learn-by-doing page. Give minimal directions on how to use the measuring devices (handspan, two fingers, rule) so that each pupil is making his own discoveries. If pupils work in pairs, each partner should measure the objects so that results can be compared. Problems can be handled during the discussion period.

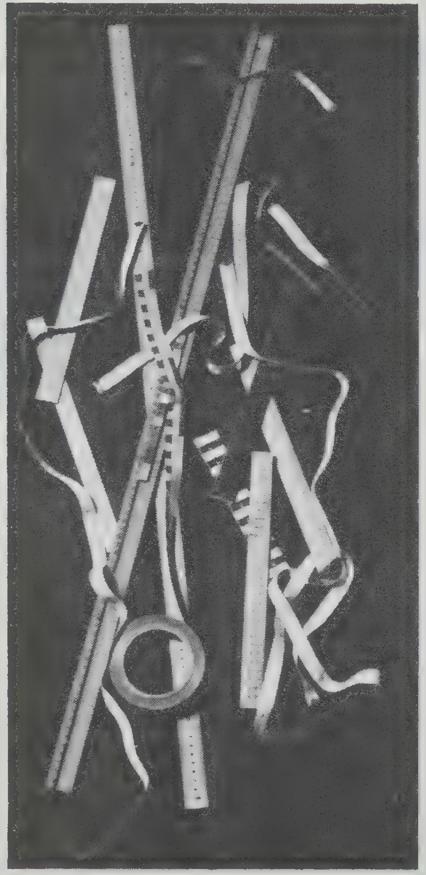
Each pupil keeps his own record. When the estimates and measuring tasks are finished, bring the class together for discussion. Make a chart on the board to record the range of differences (from 1 cm up) between estimates and measures of the desks. Show also the number of pupils in each section. If pupils chose identical objects in the last exercise, let them compare findings.

goal Learning about measuring devices; selecting appropriate measurements for various objects

things metresticks
measuring tapes
for each pupil:
centimetre rules

page 78 Actual hands-on experience with the centimetre rules, metresticks and measuring tapes would be useful, to give a feel for the relationship of centimetre rules to a metre length. The pupils should appreciate the flexibility of the measuring tape.

Form small groups and let the pupils measure several items for exercises 1 and 2, and research others. When the measuring tasks are completed, call the groups together to compare the results. *Why do you have different measures? Were different units chosen? Were some measurements more accurate than others?* No measurement is exact. Think about a mismarked or crooked rule, a metrestick that slipped (just a little), an overlapping, or a gap formed when moving the measuring device.



How long is your rule? What units are marked on it?

There are lots of different kinds of rules. Some may be 20 cm long. Some may be 40 cm long. Have you ever seen one that is 1 m long? (Look at the rules in the picture.)

A rule that is 1 metre long is called a metrestick.

$$100 \text{ cm} = 1 \text{ m}$$

So a metrestick is 100 cm long.

What's the longest measuring tape you have ever seen? When might you use a measuring tape rather than a metrestick or a rule? Why? When you are measuring something that is not a straight line.

1. Get a rule that is marked in centimetres.
   - **a** Is your foot one metre long? No
   - **b** About how many centimetres is it?
   - **c** About how many centimetres long is your hand?
   - **d** How wide is your hand? Answers
   - **e** Which is longer, your hand or your foot? By how many centimetres is it longer?

2. Which of the measurements below do you think would be about right for the things in the box?

6 m	c	**a**	a tall building
3 mm	d	**b**	your height
100 m	[illegible]	**c**	the length of a car
130 cm	b	**d**	the length of a fly's wing
1 mm	f	**e**	your middle finger
7 cm	[illegible]	**f**	the thickness of a piece of wire

78

How many millimetres are there in a centimetre? 10

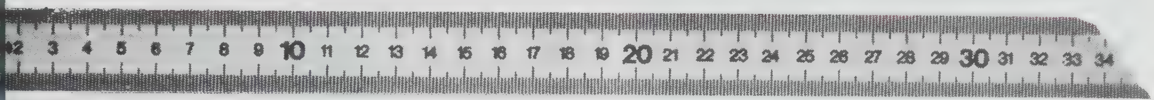
How many centimetres are there in a metre? 100

You may need to look at a metrestick to help you with this one. How many millimetres are there in a metre? 1000

4. Copy and complete these tables:

cm	mm
1	10
2	? 20
3	? 30
5 ?	50
7 ?	70
10	? 100
100	? 1000

m	cm
1	100
2	? 200
3 ?	300
5 ?	500
7	? 700
10	? 1000
100	? 10 000



Measure 6 different things. (Don't pick things longer than your arm.) Measure two of them in centimetres. Measure two of them in millimetres. Measure two of them in centimetres first and then in millimetres. Make a table like this one to record your work.

thing	cm	mm

Was it easier to use one unit of measure than the other?
Longer units are better for longer distances.

When you measured two things first in centimetres and then in millimetres were the measurements equal? No

Which unit (centimetres or millimetres) lets you measure more closely? Millimetres

Measure 4 more things. You decide if you should measure them in centimetres or in millimetres. (If you have run out of ideas, here are a few: the width of a book; a finger nail; the length of your thumb; the width of your belt . . .)

79

lesson Page 79

goal Exploring millimetres; selecting the most appropriate unit of measure

memo You may want to prepare a spirit master of the table on the page to help pupils in recording their measurements.

things as many as possible:
centimetre rules
metresticks

page 79 After discussion of exercises 1 to 3, exercise 4 should be tackled independently.

Use small groups for exercise 6. Be prepared for a noisy classroom; the learning experience will be well worth it.

When the measuring and recording activities are completed, share the results. Focus on the appropriateness of the unit selected. A very sophisticated conclusion could result from these activities: different measures (number of units) for the same thing measured are equivalent. Let the pupils discover this for themselves. Praise any who do.

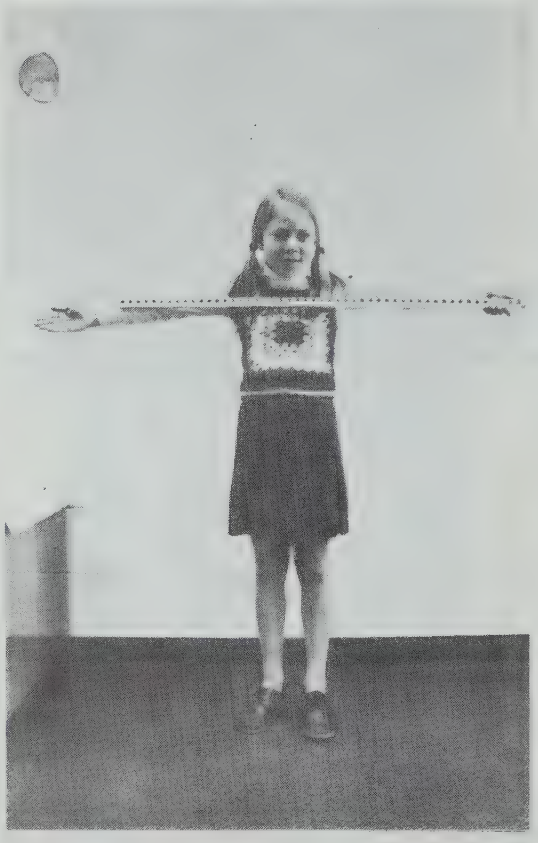
lesson Page 80

goal Estimating and measuring in metres

memo Again, you may wish to prepare a spirit master of the table on the page to help students in recording their measurements.

things metre sticks

page 80 This is a learn-by-doing page. Keep everyone involved in small group activity. Variations in arm length and paces again emphasize the need for standard units of measure.



Try estimating some distances in metres.
When you were estimating in centimetres you used handspan and finger widths to help you. What can you use to help you estimate in metres?

Try an arm length as a metre estimate (to the next shoulder o elbow). Try a long pace. Try a double pace.

Practise first. If you are going to practise paces you could do by measuring off 10 m with a metrestick on the floor. Mark off the metres on the floor with tape. Practise your paces between these marks.

1. Do the measurements in the table below. Estimate first. Then measure. Then work out the difference between yo estimate and your measurement. (Copy the table to keep record of your work.)

Distance	Estimate in m	Measure in m	Difference in m
length of classroom			
width of classroom			
height of door			

Pick three more distances of your own. Add them to the tabl Complete the table.

2. Can you always use paces to estimate? No.

3. When is it easy to make estimates in arm lengths?

When the distance you are measuring is longer than your arm.

80

easure the distance around your school. How many
nes do you have to walk around your school to walk
00 m? Answers will vary

kilometre is a unit for measuring long distances.

1 km = 1000 m

n you imagine a distance of 1000 m down the road?
you can, that will give you an idea of how long 1 km

hen would you use kilometres as a unit of measure?
For the distance from one city to another

e of the tallest buildings in the world is about 550 m
gh. Is it more or less than 1 km high? How many metres is
more or less than 1 km? Less; 450 m less

e highest mountain in the world is Mount Everest. It is
40 m high. About how many kilometres high is it?
About 9 km

e of the biggest steel arch bridges in the world is the
dney Harbour Bridge in Australia. It is about 1150 m
ng. Is it longer or shorter than 1 km? How many metres
nger or shorter is it than 1 km? Longer; 150 m longer

e Bernstein family went on a three-day trip. The first
y they drove 150 km to town A. The second day they
ove 68 km to town B. The third day they drove 42 km
town C. They then drove 140 km back to their home
wn. How many kilometres did they travel on the
hole trip? 400 km

81

lesson Page 81

goal Exploring kilometres as a unit of measure

things metre tape

page 81 Let the children actually experience the problem in Exercise 1. Different teams may do the walking and the measuring. The distances in the following exercises may then be paced off or estimated in terms of number of times around the school.

Each student does exercise 6 on his own.

goal Introduction to symbols for units of measure

page 82 This is a good time to discuss the rationale of the International System of Units.

If students use the terms **abbreviation** or **short form**, or use a period, stress that an abbreviation is a shortened word, and as such always has a period at the end, but that a symbol is something different, and does not take a period. A symbol is chosen by agreement consistently to represent a specific word. Show some familiar symbols, such as ¢, $, +, x, =, ☠. Can anyone think of some more? (Shorthand is a classic example. It is a complete system of writing using special symbols.)

Can you write the numbers from one to ten so that someone who spoke only Spanish or Greek or Russian would understand them?

Yes, you can. Like this: 1 2 3 4 5 6 7 8 9 10

These are symbols. They are not part of any language. Over most of the world they are written in the same way. It is easier and faster to write symbols than words.

We use symbols for our units of measurement as well. These symbols are also the same all over the world.

The symbol **cm** is the easy way to write the word centimetre.

To write five centimetres in symbols you would write:

5 cm

People all over the world have agreed to use the same rules when they write the measurement symbols. These rules are part of the International System of Units. Here are some of the rules.

5 cm

- Don't write a period here.
- Don't write an **s** here.
- Don't use a capital letter here. (Except in some special cases.)
- Leave a space here.

1. Can you spot the mistakes? (Watch out! Some have more than one mistake.)

a 4 Km 4 km	**b** 15cm. 15 cm	**c** 34 cm. 34 cm
d 6 mm ok	**e** 85 cms. 85 cm	**f** 13 kms 13 km

2. Write these in symbols:

a six centimetres 6 cm	**b** fourteen metres 14 m	**c** eight kilometres 8 km
d nine millimetres 9mm	**e** four millilitres 4 ml	**f** three grams 3 g
g one hundred fifty-nine kilograms 159 kg		

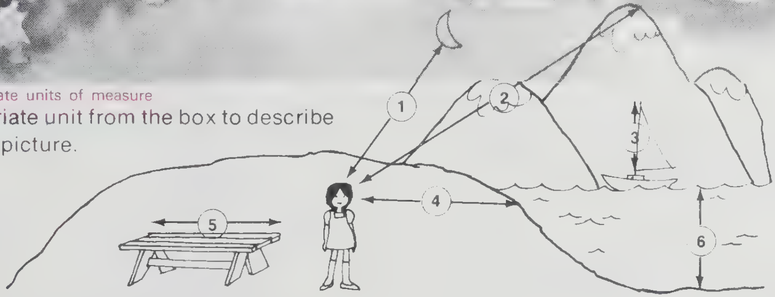
You may refer to the table on the last page of this book if you wish.

82

PROGRESS CHECK

Skill: Selecting appropriate units of measure

Select an appropriate unit from the box to describe the lengths in the picture.



kilometres, metres, centimetres, millimetres

1. distance from girl to moon kilometres
2. distance from girl to mountain kilometres
3. height of sail metres
4. distance from girl to lakeshore metres
5. length of picnic table metres or centimetres
6. depth of lake metres
7. height of girl centimetres
8. width of boat centimetres
9. height of mountain metres or kilometres
10. length of girl's shoes centimetres
11. length of boat metres
12. length of girl's hair centimetres

lesson Page 83

goal **Progress Check**—selecting appropriate units of measure

page 83 All students should do this page independently.

STOP Have the pupil list each unit of measure named on the page; then find an illustration or make a drawing of an object that can be appropriately measured by each unit listed. These can be used to make posters for display in the room.

things metresticks; centimetre rulers

With the appropriate measuring instruments, send your detectives out on a hunt for things to measure. How many objects can each person find that can be measured with each of these units of measure: metre, centimetre, millimetre.

lesson Pages 84, 85, 86, 87

goal Introduction to adding and subtracting measurements

memo Discussion is needed to get this new skill launched.

page 84 There is no renaming needed on this page. Emphasize that only like units of measure can be added or subtracted. Also watch for common addition and subtraction errors, as well as for errors in the use of symbols.

This is a good time to start some research projects. Have each pupil involved in at least one independent activity.

Project 1—Collect the names, descriptions (sketch or picture), and purposes of as many measuring devices as can be found.

Project 2—Start a scrapbook or a bulletin-board display of newspaper or magazine clippings that need measurement to complete the description of an object or an event. You might include clippings that range from supermarket ads to sports stories.

Project 3—Collect labels from packages showing the way the contents are measured. Measures will include length and mass as well as capacity. (There will be lots of metric units.)

Add your ideas and any the class might have.

You can add, subtract, multiply, or divide measurements with no trouble IF you remember one thing. Only like measurements can be computed. Look at some examples:

~~3 cm~~
~~+ 3 m~~
~~6~~

3 cm + 3 m is of course 3 m 3 cm .

One length of rope was 6 m . Another was 9 m . How much rope in all?

6 m
+ 9 m
15 m

One length of rope was 1 m 40 cm and another was 1 m . How much rope in all?

1 m 40 cm	
+ 1 m	
2 m 40 cm	total number of cm
	total number of m

The same thing holds for subtraction. For example: One rope was 2 m 60 cm long. The other was 1 m 10 cm long. How much longer was the first piece of rope?

2 m 60 cm	
− 1 m 10 cm	
1 m 50 cm	the difference in cm
	the difference in m

The computation you will be working with is not tricky or hard. Just remember to add or subtract like measurements. Try these. Watch the operation signs.

1.	2.	3.	4.
1 m 10 cm	3 m 55 cm	4 m 10 cm	3 m 45 cm
+ 1 m 8 cm	+ 1 m 40 cm	− 1 m 8 cm	− 1 m 20 cm
2 m 18 cm	4 m 95 cm	3 m 2 cm	2 m 25 cm

84

ARE YOU READY FOR SOMETHING NEW?

You know about money.
We write sums of money like this — $1 . 45

This is the dollar sign.
This is the number of dollars.
This is called the decimal point.
This is the number of cents.

1. How many dollars? How many cents?

a	$2.75	b	$16.50	c	$1.05	d	$3.10	e	$0.06
	$2.00 75¢		$16.00 50¢		$1.00 5¢		$3.00 10¢		$0.00 6¢

We use the decimal point in writing measurements too. 1 . 45 m

This is the number of metres.
This is the decimal point.
This is the number of centimetres.

2. How many metres? How many centimetres?

a	2.75 m	b	16.50 m	c	1.05 m	d	3.10 m	e	0.06 m
	2 m 75 cm		16 m 50 cm		1 m 5 cm		3 m 10 cm		0 m 6 cm

We can write 5 m 27 cm in metres, using the decimal point and symbol m .
5 m 27 cm = 5.27 m

3. Write these with a decimal point and the symbol m .

a	3 m 55 cm	b	1 m 20 cm	c	5 m 75 cm	d	1 m 18 cm
	3.55 m		1.20 m		5.75 m		1.18 m

85

lesson Page 85

goal Introduction of the decimal point in dollar and cents notation and in writing measurements

page 85 Children are already familiar with the use of the decimal point in money notation. By comparison they here learn how it is used to indicate part of a unit of measure. The simplicity and ease of working with the International System of Units here become apparent.

Mastery of the use of the decimal point is not expected at this point. That will come later. This is just a first acquaintance with it.

Not all children will be ready for Exercise 3. The concept that the centimetres can be expressed as a decimal fraction of a metre, and that the resulting figure is expressed in metres, may be too hard for some at this time. It can be learned later when decimals are dealt with more fully.

goal Practice in solving word problems involving measurements; **Progress Check**—adding and subtracting measurements requiring no renaming

page 86 Use the word problems with only your most capable students as an independent activity. You may want the others to have the experience also, but plan to work with these youngsters as a group. The computation isn't so hard, but the subject of the problems is quite sophisticated.

The Progress Check is independent work for everyone; however, not everyone should do problem 6. Most of the pupils will not have had sufficient experience for this problem. Watch for addition and subtraction errors in the remaining problems.

Sports records are being broken every year.

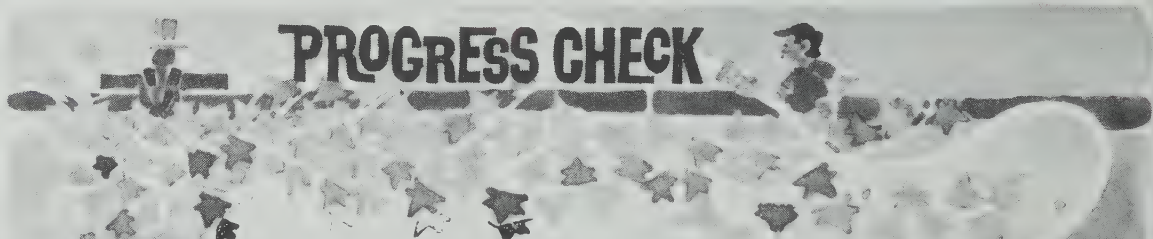
1. A really good athlete can throw a discus 57 m or more. But the world's record for the discus throw is about 67 m. What is the difference between the distances? 10 m

2. A recent record for the javelin throw was 79 m. The world's record is over 92 m. What is the difference between the two distances? 13 m

3. Fishing is a sport too.

   a Suppose you caught a fish 69 cm long. The longest ones grow to 75 cm. What's the difference? 6 cm

   b You would probably have your picture taken if you caught a brook trout 50 cm long. The record is 80 cm long. What's the difference? 30 cm



Compute. Watch the operation signs.

1. 1 km 300 m + 2 km 650 m 3 km 950 m	2. 4 km 100 m +3 km 625 m 7 km 725 m	3. 6 m 15 cm − 2 m 4 cm 4 m 11 cm
4. 6 m 11 cm − 2 m 6 cm 4 m 5 cm	5. 4 m +3 m 50 cm 7 m 50 cm	*6. 10 m − 9 m 50 cm 50 cm

Just for fun

All the underlined words have something to do with time. You will know some of these words. Others may be new.

What period of time does each phrase refer to?

1. fourscore and seven years ago 80 years
2. fortnightly dances 2 weeks
3. a centennial celebration 100 years
4. a bicentennial celebration 200 years
5. a silver anniversary 25 years
6. a golden anniversary 50 years
7. a quarterly magazine 3 months
8. an annual affair 1 year
9. a semiannual affair ½ year
10. biennial election 2 years
11. a perennial favorite 1 year
12. * just a second
13. * wait a minute
14. * in a moment
15. an hour ago 60 minutes

*Usually an indeterminate amount of time.

Make up a story. See how many of these time words you can use.

87

lesson Page 87

goal Investigating various measures of time

memo This page makes an excellent independent research project, to be followed by a class discussion. The dictionaries will get a real workout. You may not want to use the lesson with everyone.

things for each pupil: dictionary

page 87 Many of the words on this page pertaining to time will be new for the youngsters. This is definitely a time for dictionary practice. Have the pupils write definitions for each word as it is used in the phrases in the book. Discuss the completed assignment. Watch the definitions for **second**, **minute**, and **hour.** Discuss how they are used in the book. These are exact measurements of time, yet people usually use them as estimations. "Wait a minute" is a good example.

As the book suggests, let them try to write a story using as many of the words as possible. Each of the stories should be wonderfully original. You'll want volunteers to share theirs with the class.

lesson Pages 88, 89, 90

goal Establishing the need for being able to tell time

memo Discussion is needed for this new idea.

page 88 Everyone will enjoy reacting to the thought-provoking questions. Let the pupils air their ideas freely. This is one of the few times that they will get a chance to compare their thoughts about time. Many will be surprised to learn that others have similar thoughts on this subject.

The page focuses on the **feeling** of time, not on the accurate measurement of time. Think about examples from nature to illustrate this feeling. Snakes hibernate, birds fly south, flowers open and close with sunlight, some trees lose their leaves, and so on. How many examples can the group think of? Even though we have a feeling for time, we must have clocks for the telling of time. Man lives in a community. He interacts with many other people every day. Life is too complex to tell time by **feeling.**

There was a sign in a window nearby that provided this interesting perception of time: "Time is an invention to keep everything from happening at once."

When you want to go somewhere and you have to wait for someone to take you, does time seem to go slowly? Yes

When you're really having a good time and you have to be home at 9:00, does time seem to go fast? Yes

Does time seem to go slowly when you have to do something you don't like to do? Yes

Does summer seem to go faster than winter? Yes day faster than night? Yes

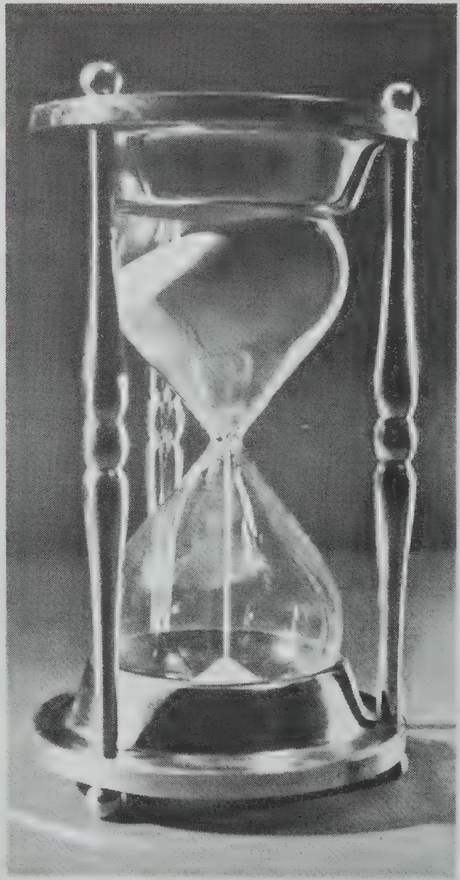
Will there be a 9:00 tomorrow? the next day? the day after that? and the day after that? Hopefully

What makes 10:00 in the morning different from 10:00 at night? Answers will vary.
Examples: the amount of daylight, the activities taking place

1. Must we have clocks for telling time?
No. There are less accurate instruments.
   - a A farmer can tell about what time it is by the length of the shadow he casts. (BUT that's only if the sun shines.)
   - b A waitress can tell about what time it is according to how many people there are to be served. (BUT that's only on days that she works.)
   - c A child can tell about what time it is by the TV programs that are on. (BUT that's only if the TV is working.)

2. Do all clocks look the same? No

3. Do all clocks measure the same thing?
They all measure time, but some measure it more precisely than others.



The earth turns. The turning earth also moves around the sun. We have night and day and we have a year because of the earth's turning.

It takes 12 months for the earth to go round the sun,

1 2 3 4 5 6 7 8 9 10 11 12 and here we start round the sun again.

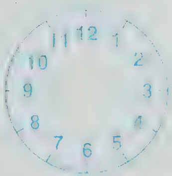
It takes 24 hours for the earth to make one complete turn on its axis,

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 and here we start round again.

We have on the average 12 hours of day and 12 hours of night.

1 2 3 4 5 6 7 8 9 10 11 12 and here $\frac{1}{2}$ of the day is over and we start into the next $\frac{1}{2}$. When that's over we start again into the next day.

Since hours go on and on, one into the other, we bend the number line.



From 12:00 midnight to 12:00 noon, the hour hand of the clock completes one turn.

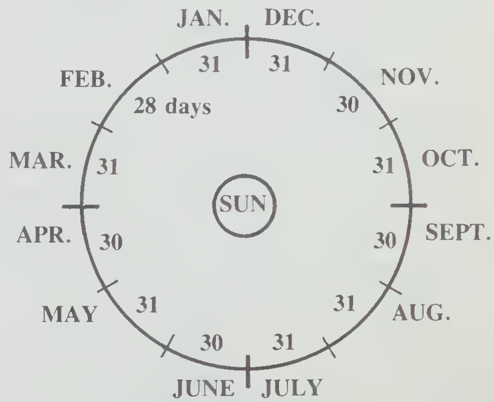
From 12:00 noon to 12:00 midnight, the hour hand completes another turn.

89

goal Investigating our system for measuring time

things globe, if available
flashlight

page 89 A diagram similar to the following should help the discussion on measuring time.



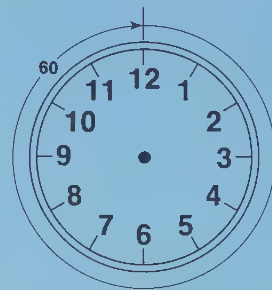
You may also want to use a globe and a flashlight (the sun) to show how rotation makes our days and nights. The purpose of this discussion is to show pupils that different measures of time are related and have meaning. This lesson can easily be integrated into a science unit on seasons, earth's rotation, revolution, axis tilt, and so on.

Keep going. The discussion continues to page 90.

goal Investigating our system for measuring time [*continued*]

page 90 When the discussion is completed, decide which pupils can handle the questions independently and which need to continue in a discussion group with you. Note that questions 8 and 9 are marked with an *. These are hard questions. Not everyone should tackle them.

It's probably not a surprise
that the number of minutes in one hour
is some number times 12.



When the minute hand makes one complete turn, 60 minutes, or 1 hour, have passed.

Now guess how many seconds in one minute? 60
The clock is neat and tidy. Go back to the picture. Imagine a second hand sweeping around that same curved number line. When it makes one complete turn, 60 seconds, or 1 minute, have passed and the next minute begins.

1. How many hours are there in 1 day? 24
   Is it always the same number? Yes
2. How many days are there in 1 week? 7
   Is it always the same number? Yes
3. How many days are there in 1 month? Between 28 and
   Is it always the same number? No
4. How many weeks are there in 1 month? About 4
   Is this number exact? No
5. How many days are there in 1 year? 365
   in 1 leap year? 366
6. How many weeks are there in 1 year? 52
   How many days are left over? 1, except for 2 in a leap ye
7. How many months are there in 1 year? 12
   Is it always the same number? Yes

*8. How many years are there in 1 decade? 10
   Is it always the same number? Yes

*9. How many years are there in 1 century? 100
   Is it always the same number? Yes

py and complete these tables.

1.

hours	minutes	
1	60	
2	?	120
5	?	300
12	?	720
24	?	1440

2.

minutes	seconds	
1	60	
2	?	120
5	?	300
10	?	600
60	?	3600

3.

days	hours	
1	24	
2	?	48
3	?	72
7	?	168
10	?	240

4.

	days	weeks	
	7	1	
	14	?	2
63	?	9	
252	?	36	

5.

months	weeks	
1	?	4
3	?	12
6	?	24
12	?	48

6.

	months	years	
	12	?	1
24	?	2	
	120	?	10
144	?	12	

these
tside
class
time
vith a
uddy

7. Get a clock or a watch that has a second hand. Watch the second hand as it moves around. When it gets to the 12 mark, start tapping with your foot. Tap your foot each time the second hand passes one of the marks on the rim. The time it takes for the second hand to get back to the 12 mark is *one minute* (min.).

8. How many seconds does it take you to do each of these things? Get a friend to time you.
   - **a** Count from 1 to 50.
   - **b** Compute 1344 + 973.
   - **c** Walk the length of a city block.
   - **d** Run the length of a city block.

Answers will vary.

9. How many seconds does it take your friend to do these things?

10. Can you tell how long 10 seconds is without a clock or a watch? Get a friend to time you. How close did you come?

91

lesson Page 91

goal Introduction to equivalent measures of time and to estimating time

page 91 The tables at the top of the page are independent work.

The outside-project section is optional. You'll want to take some time to discuss it, but let the pupils experiment on their own. Parts **a** and **b** of exercise 8 involve timing assigned tasks and could be done in pairs during class time. Have pupils write an estimate before they perform the tasks to be timed.

You may want to designate a distance for the city block (**8c** and **8d**). A city block has never been a standard unit of measure. With the current subdivisions of city property, the length of an actual block is less and less predictable.

Provide time on a later day to discuss the findings.

lesson Pages 92, 93

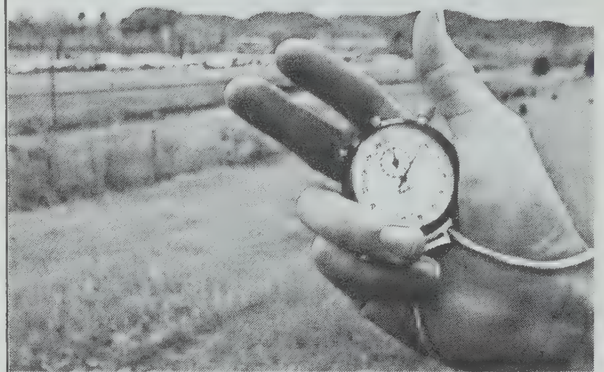
goal Practice in solving word problems involving addition and subtraction of time measurements

page 92 This page is not easy. Only you can judge how it should be used with your group. Independent learners should, however, be able to go on their own.

Watch for computational errors as well as selection of the wrong operation. If a pupil has trouble selecting the correct operation, he will need more practice with word problems. You may want him only to select the operation and not to compute the answer.

Don't expect all youngsters to rename the answers for problems 3 and 4. This has not been a point of emphasis yet.

WARNING —problem 7 is tricky!



We use symbols for units of time, too.

hour	h
minute	min
second	s

1. Mary took 6 min 52 s to do this page of mathemati
   John took 6 min 21 s .
   Who took longer? Mary
   How much longer? 31 s
2. It is now 1:20 in the afternoon. School is out at 3:2
   How long is it until school is out? 2 h
3. A chess clock shows how much time each player h spent making his moves. Rod has used 1 h 34 min Lynn has used 1 h 49 min . How long has the game lasted? How much more time has Lynn used than Rod? 15 min
4. In a relay race Hilda took 53 s . Virginia took 46 s . Anne took 48 s . How many seconds did the three girls take in all? The other team took 15 s more. Ho much time did the other team take? 162 s
5. Tad ran 500 m in 2 min 3 s . Bill ran it in 3 min 7 s How much longer did Bill take? 1 min 4 s
6. How many hours is it from breakfast to lunch? 3 to 5 h from lunch to dinner? 5 to from dinner to the next day's breakfast? 12 to 14 h
7. A sign on the barber's door reads "Back in 60 minutes." What is the least amount of time you 1 mi would expect to wait? What is the longest time you would expect to wait? 60 min

92

BUS SCHEDULE

WESTBOUND					EASTBOUND				
OLD TOWN	6:05	11:35	*3:50*	*8:20*	RIVERPORT	6:45	11:30	*4:00*	*8:15*
EAST BUTTON	6:55	*12:25*	*4:40*	*9:10*	CHANCE	7:10	11:55	*4:25*	*8:40*
DIXIEVILLE	7:30	*1:00*	*5:15*	*9:45*	DIXIEVILLE	7:30	*12:15*	*4:45*	*9:00*
CHANCE	7:50	*1:20*	*5:35*	*10:05*	EAST BUTTON	8:05	*12:50*	*5:20*	*9:35*
RIVERPORT	8:15	*1:45*	*6:00*	*10:30*	OLD TOWN	8:55	*1:40*	*6:10*	*10:25*

1. What time does the first bus leave Old Town going west? When does it reach Riverport? 6:05 8:15 How long does it take? 2 h 10 min
2. What time does the last bus leave Riverport going east? When does it reach Old Town? How long 8:15 10:25 does it take? Does it take the same length of time 2 h 10 min as the westbound bus? Yes
3. Do all the westbound buses take the same amount of time? Do all the eastbound buses take the same Yes amount of time? Yes
4. Why are some times in the schedule printed in different type? For A.M. and P.M.
5. How long does it take to go from Old Town to East Button? from East Button to Dixieville? from 50 min 35 min Dixieville to Chance? from Chance to Riverport? 20 min 25 min
6. How long does it take to go from Chance to East Button? from Chance to Old Town? from 55 min 1 h 45 min Riverport to Dixieville? 45 min

goal Using a timetable (schedule) to compute time problems

memo This page is a challenge and features a very real situation. Everyone must learn to read a **schedule** sooner or later. Why not start now? Schedules are usually a nightmare, so be prepared for anything.

page 93 Make sure that the pupils understand the bus schedule. Talk about it. Point out that the first time notations shown in each column for **westbound** buses are departure times from Old Town. The other notations below these are the arrival times at each of the respective cities. The **eastbound** schedules are, of course, read in a similar manner.

If you live in an area with a metropolitan transportation system – buses, commuter trains, subway – encourage pupils to get a schedule. Each schedule will look a bit different and will be a new challenge. (Ugh!) Maybe one of your students can discover a better way to make a schedule.

lesson Pages 94, 95

goal Exploring relationship between time, speed, and distance

page 94 Mastery of the concept of speed as a function of distance and time is of course not expected at this time. During the discussion arising from the exercises, the children should become familiar with the expression *kilometres per hour*. *Metres per second* may also be introduced in discussing running times if interest is high. The children should also be able to recognize and read the symbol km/h when they meet it.

The last exercise on the page is a fun exercise which requires out-of-class activity. Assign the children partners for this exercise.

1. If you were driving at a speed of 100 kilometres per hour, how far would you get in 1 hour? 100 km
2. At the same speed, how far would you get in:
   a 2 h? **b** 5 h? **c** 8 h? 200 km 500 km 800 km
3. We use the symbol **km/h** to stand for kilometres per hour. What part of the symbol stands for the word **per**? What part stands for the word **hour**? h The slanted line/
4. At a speed of 75 km/h, how far would you get in:
   a 1 h? **b** 2 h? **c** 4 h?
   75 km 150 km 300 km
5. What is the usual speed limit, in km/h, in the city? outside the city?
   Answers will vary.
6. Speed can be measured in other units as well. The speed of light is about 300 000 km/s. What do you think the symbol **km/s** stands for?
   Kilometres per second
7. If you were able to jump on a beam of light, how far would it carry you in 10 s? 3 000 000 km
8. The fastest land animal in the world is the cheetah. Look at the chart. How fast can it go? Do you think it can run at that speed all day? About how long do you think it could run at that speed? 125 km/h No
   Answers will vary
9. How many animals on the chart are faster than a man? 5
10. What is the difference between your top speed and the speed you can keep up for a full hour?

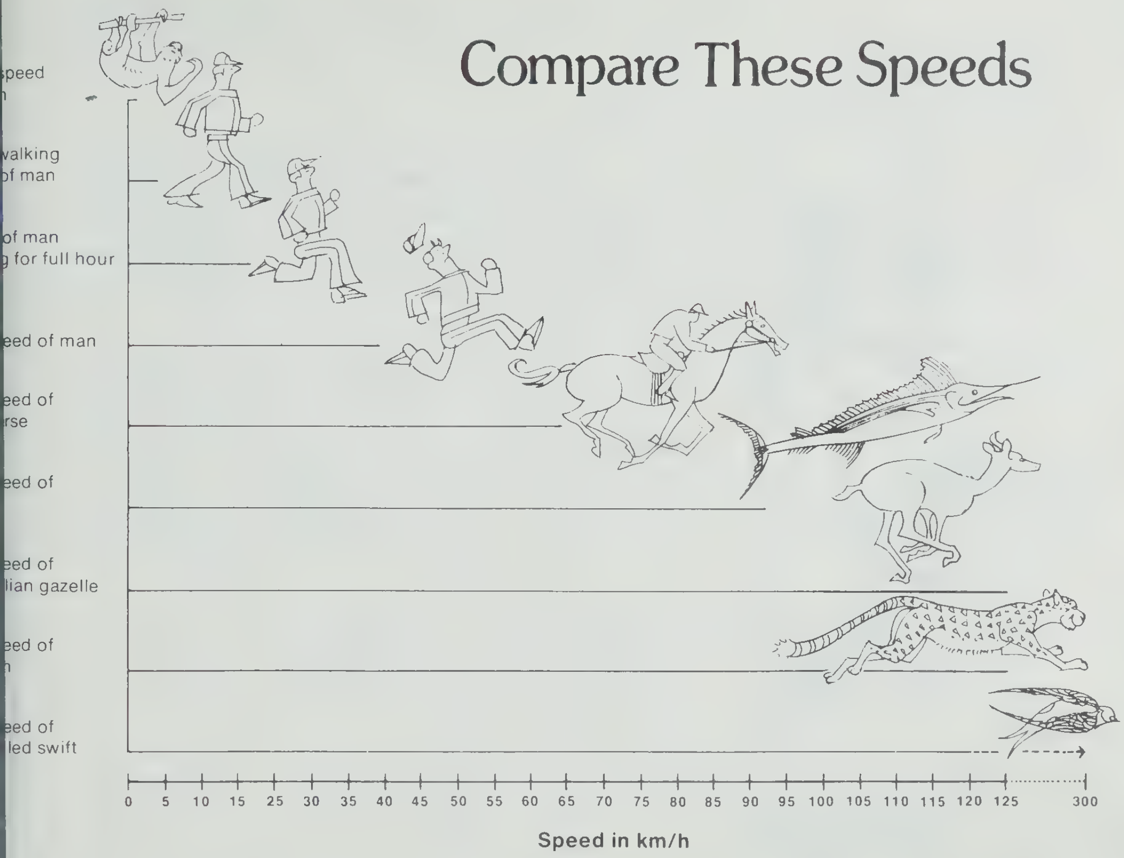
Work out your own top speed when you get a chance. Get a friend to help you. Measure off 50 m on the playground. Have someone time you, in seconds, while you run 50 m. Use the chart below to find out what your speed is in km/h.

Number of seconds to run 50 m	*5	6	7	8	9	10	11	12	13	14	15	16
Speed in km/h	36	30	26	23	20	18	16	15	14	13	12	11

*This is about the same speed as a world record would be.

94

Compare These Speeds



95

goal Reading speeds from a chart

page 95 The children can play a game in which each animal is represented by one child. Another child can call the animals in pairs. The children called must then stand with the one representing the faster animal ahead of the slower animal in each pair.

For variation, each child called may announce the speed in km/h of the animal he represents.

lesson Page 96

goal Checkout – length and time measurements

page 96 Everyone on his own! Skills are identified on the answer key to help you diagnose errors and prescribe additional help.

goal Think about and explore ideas through a picture clue

CHECKOUT

1. Which of the measurements fits
   - a the width of an ant's head? 1 mm — 28 cm
   - b the length of a cat's tail? 28 cm — 4 m
   - c the height of a giraffe? 4 m — 1 mm

2. List in order from shortest to longest unit of time.

   minute, day, second, year, week, hour

3. Compute. Watch the operation signs.

a	b	c
3 m 24 cm	8 m 48 cm	8 km 100 m
+ 1 m 59 cm	− 3 m 35 cm	− 7 km
4 m 83 cm	5 m 13 cm	1 km 100 m

4. How many minutes in 2 h? 120

5. How far can you travel in 3 h if you are going at 50 km/h? 150 km

6. How many centimetres in 5 m? 500

7. How many days in 9 weeks? 63

8. Write the symbols for
   a kilometre **b** gram **c** millimetre
   km g mm

96

See activity 4, page 96b.

STOP

things almanac

Look up world swimming records in an almanac. What does the time notation mean? (minutes: seconds, parts of a second) Compare the women's and the men's time difference for each event. Challenge very capable pupils to compute the actual time differences.

RESOURCES

another form of evaluation

for Progress Check — page 83

Select an appropriate unit from the box to describe the lengths in the picture.

kilometres	centimetres
metres	millimetres

1. distance from boy to father
   centimetres
2. distance from boy to sun
   kilometres
3. length of one leaf from the tree
   millimetres or centimetres
4. height of tree
   centimetres or metres
5. father's height
   centimetres
6. length of bench
   centimetres or metres

Which is longer?

7. 35 mm or 3 cm
8. 1 m or 1200 mm
9. 3 m or 275 cm
10. 1 km or 945 m
11. 3 km or 3005 m
12. 35 km or 35 075 m

for Progress Check — page 86

Compute. Watch the operation signs.

1. 15 m 75 cm
   − 8 m 20 cm
   7 m 55 cm

2. 5 km 350 m
   +4 km 200 m
   9 km 550 m

3. 35 m 45 cm
   +25 m 30 cm
   60 m 75 cm

4. 7 km 900 m
   −3 km 600 m
   4 km 300 m

5. 3 km
   +5 km 240 m
   8 km 240 m

*6 9 m
   −7 m 50 cm
   1 m 50 cm

1. 5 km 500 m
   −2 km 200 m
   3 km 300 m

2. 25 m 30 cm
   +12 m 60 cm
   37 m 90 cm

3. 4 m 90 cm
   −2 m 35 cm
   2 m 55 cm

4. 6 km 120 m
   +2 km 200 m
   8 km 320 m

5. 5 km
   +1 km 500 m
   6 km 500 m

*6. 7 m
   −4 m 50 cm
   2 m 50 cm

for Checkout — page 96

1. Name the largest unit of measurement.
   a) day, week, second, minute
   b) kilometre, metre, centimetre
   c) year, decade, century

2. Compute. Watch the operation signs.

a) 2 m 35 cm
   +5 m 55 cm
   7 m 90 cm

b) 7 m 98 cm
   −3 m 46 cm
   4 m 52 cm

c) 2 km 275 m
   +3 km 550 m
   5 km 825 m

d) 1 m 46 cm
   +3 m 35 cm
   4 m 81 cm

e) 9 km 250 m
   −4 km
   5 km 250 m

f) 12 km 180 m
   −9 km 70 m
   3 km 110 m

3. How many days in 7 weeks? 49
4. How many centimetres in 4 metres? 400
5. How many minutes in 3 hours? 180
6. How many millimetres in 20 centimetres? 200
7. How many metres in 5 kilometres? 5000

activities

1. Follow up exercises 1 and 2 by having the children actually measure from the floor to the top of their desks. Use any available measuring device. The length of a book, the length of a pencil, or the length of a chalkboard eraser makes a satisfactory unit of measure.

Compare the measurements that result. Are they all the same? Why are they different?

Make an agreement that one step equals one unit of measure. Pupils who walk can then **measure** the distance in steps from their homes to school. Let time be the unit of measure for those who ride the bus. Who lives the farthest from school in each group? the nearest to school?

2. things boxes; string; inch and centimetre rulers; metrestick; yardstick

Individual project. Estimate the least amount of string necessary to wrap and tie the box. You can measure the sides of the box to help you make your estimate. Use either yards and inches, feet and inches, or metres and centimetres. Record your estimate.

Now actually wrap and tie the box with string. Measure the amount of string you really used. Use the same units of measure as those in your estimate. Compare the two measurements. Did you overestimate? underestimate? How much?

3. first day Provide each group with several duplicated sheets to use in recording the results of the exploration. Taking one article of clothing at a time, the pupils are to look at the tagged size and try to find a measurement that corresponds to that size. Before hypothesizing, each group should look at two or three similar articles that vary in size. Once the pupils have decided what measurement determines the size of the article, they are to note it in the proper column. They should also note the unit of measure by which the sizes differ.

Example:

Type of clothing	Size	What measurement determines size?	Unit of measure
shirt	16-33	collar	$\frac{1}{2}$ inch
		sleeve	1 inch

There will be some wild answers with this type of exploration. Continue until each group has examined several types of articles and has noted its conclusions.

second day Each group should prove its hypotheses with catalog-size charts and make revisions on another list. Encourage the pupils to research further with parents, friends, and neighbors. Discussion of their initial thoughts and what their research uncovered should be very revealing.

The pupils should now be ready to apply what they have learned to the questions on page 94.

4. Have the youngsters choose a day at random and become clock watchers. They are to record the length of each class period—art, music, reading, math, and so on. When this information is complete, have them decide in which class they spend the greatest amount of time and in which they spend the least amount of time. Then have them compute the difference in time spent in these two classes. Might be interesting to ask them what their favorite class is after this study in time.

additional learning aids

concept—chapter objectives 1, 3

SRA products
Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: M 1, P 4
Mathematics Involvement Program, SRA (1971)
Cards: 174, 194

other learning aids (described on page 144e)
Good Time Mathematics, Learning About Measurement,

operation—chapter objectives 4, 5, 6

SRA products
Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: M2, 3
Skill through Patterns, level 4, SRA (1974)
Spirit master: 54

other learning aids—Spin-A-Yard

5 MULTIPLICATION

before this chapter the learner has—

1. Mastered the multiplication facts
2. Rounded numbers to the nearest ten or hundred

in chapter 5 the learner is—

1. Multiplying a 2- or 3-digit factor by a 1-digit factor
2. Multiplying two 2-digit factors
3. Estimating products to check the reasonableness of a computed product

in later chapters the learner will—

1. Master multiplying any 2- or 3-digit factor by any 1-digit factor
2. Master multiplying any two 2-digit factors to find their product

Notes & Things

The development of the multiplication computational skills is based on the same knowledge that was used for addition and subtraction.

1. Mastery of the basic facts must come first. Every child must be able to confidently name the product of any multiplication combination.
2. The knowledge of place value is the key to making any pupil independent in computation. If the child knows the facts and place value, he can multiply any of these problems with no trouble:

$$\begin{array}{r} 27 \\ \times\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 285 \\ \times\ \ 9 \\ \hline \end{array} \qquad \begin{array}{r} 3261 \\ \times\ \ \ 8 \\ \hline \end{array} \qquad \begin{array}{r} 465{,}120 \\ \times\ \ \ \ \ \ \ 6 \\ \hline \end{array}$$

   It will take only a minimal amount of instruction to have the pupil confidently operating with 2- or 3-digit multipliers **IF** he knows the basic facts and understands place value.

3. Knowing how to estimate answers will help eliminate careless mistakes and serve as a guide to the computation itself. If a pupil can look at 28 × 529 and think "30 × 500 is 15,000, so the product will be close to that," he will have mastered the skill of multiplication in no time at all.

Rounding numbers is of course a prerequisite to the development of estimation skills. Since estimation is a mental operation, most numbers will be rounded in such a way that the pupil will be operating with a number fact while also keeping in mind the place value (the number of zeros).

In general, a 2-digit number is rounded to the nearest ten, a 3-digit number to the nearest hundred, a 4-digit number to the nearest thousand, and so on.

You will find a heavy emphasis on rounding and estimation throughout the chapter. And you will see the computational form for multiplication build from the multiplication skills that the learner has already developed.

The major computational emphasis will be on 1-digit multipliers so that an understanding of the operation itself, the related place-value implications, and the computational form can be thoroughly developed. Multiplying two 2-digit factors is introduced at the end of the chapter, but minimal practice is provided at this point. Let's get knowledge of 1-digit multipliers mastered first. That's the goal.

things

large map of North America
spirit master of blank multiplication table
newspaper grocery ads

page 97 This photo of abandoned buildings should stimulate your history buffs without a spoken word, but in order to get the notion of research ideas started please consider some direct questions.

How old do you think these buildings might be? Does it look as if the buildings are in a city? Does it look as if people are living in them now? Could buildings like these be part of a ghost town? What is a ghost town? What part of the country do we think about when we think of a ghost town? Do you think the world was different at the time these buildings were built? How? Did the people then have the same food? the same forms of transportation? the same jobs? Did things cost the same? Why were these towns abandoned?

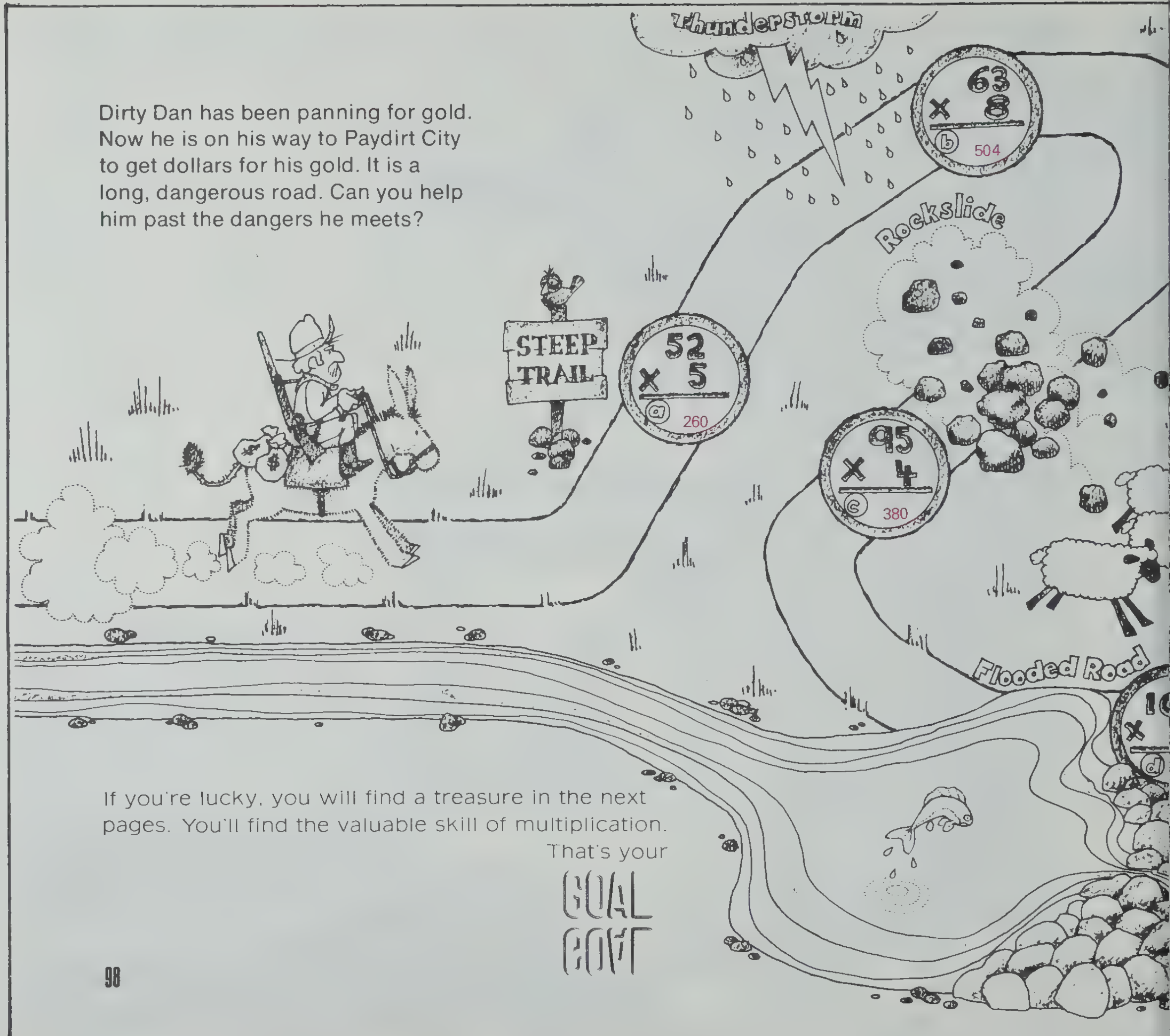
Now you have the stage set for the chapter itself and lots of independent research. Your school library will have many appropriate history reference books, and the reprints of turn-of-the-century mail-order catalogs will open a new dimension of history for your pupils. Hopefully a reprint of either an early Sears Roebuck catalog or an early Montgomery Ward catalog can be found in your public library.

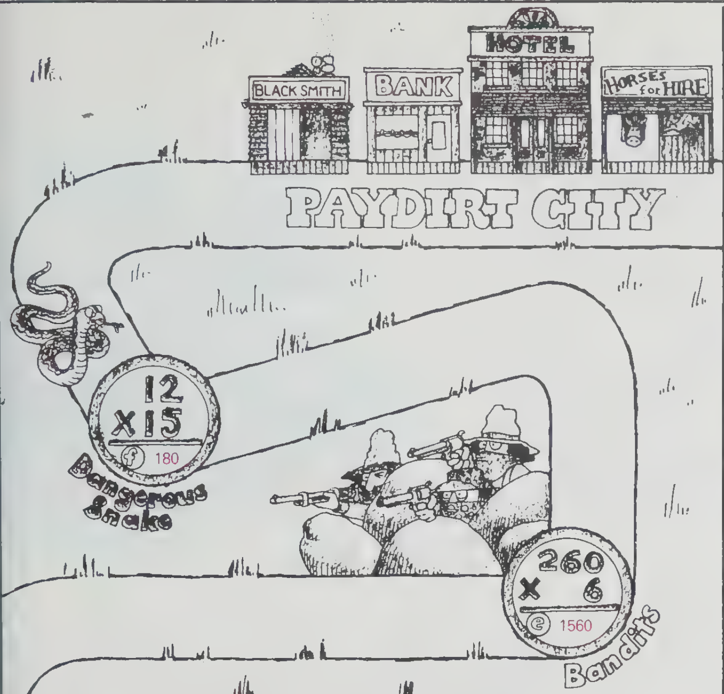
And chances are that you have some golden-agers in your community who would be very happy to tell individuals or groups about the different economic conditions that existed when they were children of the same age as your investigators. Won't it be grand to have youngsters know that arithmetic was important way back then!

goal Survey—multiplication with 1-, 2-, and 3-digit factors

page 98 Dirty Dan's activities are appropriate for the time setting with which the chapter opens. They provide the background for the problems on this page and on several pages that follow.

Any pupil who follows Dan's trail quickly and successfully will have little trouble getting through the chapter independently. For those who find the problems difficult, emphasize the learning goal.





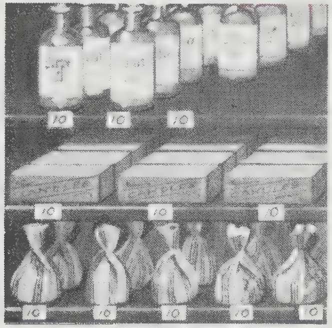
This is part of an assayer's chart.
Copy and complete it.

Grams of gold

Dollar value of gold	1	2	3	4	5	6	7	8	9	10
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20								

Can you guess what it was used for?
A quick way of telling a miner how much he should be paid for his gold

1. Dan visited the General Store in Paydirt City. There were many kinds of goods on the shelves. The picture below shows an easy way to count each type of item. Can you figure out the system?
Counting by 10s



2. Two bottles of castor oil are gone. How many are still on the shelf? 28 There are no boxes of candles gone from the middle shelf. How many candles are on the shelf? 30

99

lesson Pages 99, 100

goal Readiness for multiplying a 2-digit factor by a 1-digit factor

page 99 Explain the meaning of the word *assayer*, and then use the chart as independent work. It serves as a review of multiplication facts and will help you to identify the pupils who will need some extra practice each day until they have mastered the facts.

Exercises 1 and 2 focus on place value and multiples of 10. This development is continued on page 100. Independent learners should be on their own so that you will have time to work with those who have a reading problem.

See activity 1, page 120a.

Go on a hunt for some famous places to visit. An added challenge would be finding some less-known historical landmarks like Virginia City, Nevada. Locate the landmarks on a map. Now plan an interesting family vacation. Some youngsters may even wish to plan the route.

goal Finding multiples of 10

page 100 Check whether the words **multiple** and **factor** are understood.

factor × factor = product

The product is also a multiple of one of the factors.

You may want to handle this page quickly as oral practice or as independent work. Either way the computation should be done without paper and pencil. Give more drill if necessary to get youngsters to multiply mentally by some number of tens or hundreds.

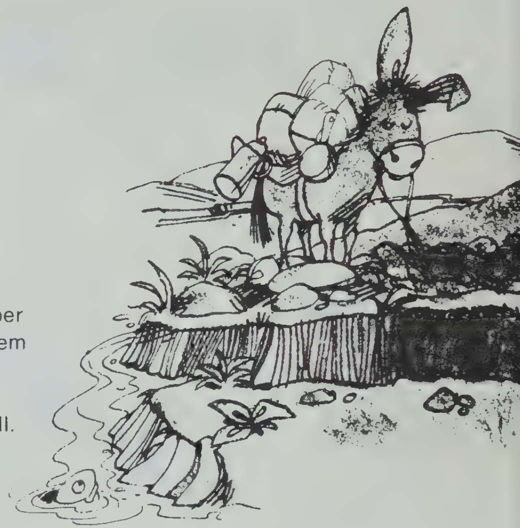
1. How many in all if 1 row had 10 objects? 10

2. How many in all if 2 rows had 10 objects in each row? 20

3. How many in all if 4 rows had 10 objects in each row? 40

4. How many in all if 6 rows had 10 objects in each row? 60

5. How many in all if 7 rows had 10 objects in each row? 70



Each of your answers is a multiple of 10. The number ten was used as a factor in each problem. The system for filling the shelves in the old grocery store used this idea. Each row was to have ten items. The number of rows × 10 would tell you how many in all.

Here are some problems involving multiples of 10. Find the answers.

6. 10 boxes of salt in a row.
3 complete rows.
30 ? boxes of salt in all.

7. 10 sacks of beans in a row.
5 complete rows.
50 ? sacks of beans in all.

8. 10 bags of sugar in a row.
8 complete rows.
80 ? bags of sugar in all.

9. 10 cans of lye in a row.
4 complete rows.
40 ? cans of lye in all.

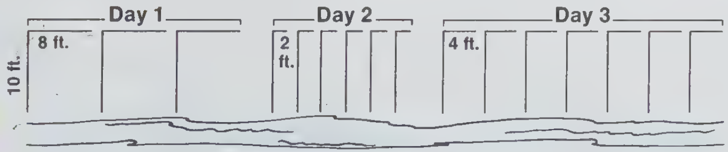
10. 100 boxes of matches in a row.
5 complete rows.
500 ? boxes of matches in all.

11. 100 packages of needles in a row.
9 complete rows.
900 ? packages of needles in all.

100

There once lived a prospector called Dirty Digger. He was the pickiest prospector there ever was. He always dug trenches in straight lines away from the riverbank. When he got ten feet from the river, he would turn right and dig a few more feet. Then he would stop.

Here are some of the patterns he made. Each day he made a new pattern and repeated it.



At night Dirty would add up the number of feet he had dug. Here is his addition.

DAY 1		DAY 2		DAY 3	
10	8	10	2	10	4
10	8	10	2	10	4
+10	+8	10	2	10	4
30	24	10	2	10	4
		10	2	10	4
		+10	+2	10	4
		60	12	+10	+4
				70	28
30		60		70	
+24		+12		+28	
54 feet in all		72 feet in all		98 feet in all	

irty could have saved some time.
e could have multiplied.

101

lesson Pages 101, 102, 103

goal Introduction to multiplying a 2-digit factor by a 1-digit factor

memo Pages 101 and 102 introduce a new concept and a new algorithm (computational form). These therefore are discussion pages.

things for each pupil:
blank multiplication table

warm-up It is very important that there is a time-out for review of the multiplication facts. Get another copy of the blank multiplication table. (See page 53.) Have each pupil complete the table. They should compare the results to make sure there are no errors.

page 101 Dirty Digger certainly did everything the hard way—even his computation. Go right on to page 102.

goal Introduction to the algorithm for multiplying a 2-digit factor by a 1-digit factor

page 102 Develop the page with the group. Stress the importance of place value in the partial products. Aligning ones and tens in straight columns is necessary in order to add the final product. (You might suggest that the pupils turn lined paper sideways to reinforce the ideas of columns.)

Don't require the youngsters to write the thinking step, shown to the right, when writing the partial products. They need write only this:

$$\begin{array}{r} 12 \\ \times\ 6 \\ \hline 12 \\ 60 \\ \hline 72 \end{array}$$

Problems 1 through 7 should be completed independently. Watch for those pupils who are already in trouble. Check on mastery of multiplication facts. Addition errors are another possibility.

Ask your pupils to hunt for newspaper grocery ads and bring them to class tomorrow. And then plan on at least one day of computation practice outside the text. Use activity 2, part a described on page 120a of the Resource Section.

If Dirty had known how to multiply, this is how his work would have looked.

DAY 1

$$\begin{array}{r} 10 \\ \times\ 3 \\ \hline 30 \end{array} \qquad \begin{array}{r} 8 \\ \times 3 \\ \hline 24 \end{array}$$

$$\begin{array}{r} 30 \\ +24 \\ \hline 54 \end{array} \text{ feet in all}$$

DAY 2

$$\begin{array}{r} 10 \\ \times\ 6 \\ \hline 60 \end{array} \qquad \begin{array}{r} 2 \\ \times 6 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 60 \\ +12 \\ \hline 72 \end{array} \text{ feet in all}$$

DAY 3

$$\begin{array}{r} 10 \\ \times\ 7 \\ \hline 70 \end{array} \qquad \begin{array}{r} 4 \\ \times 7 \\ \hline 28 \end{array}$$

$$\begin{array}{r} 70 \\ +28 \\ \hline 98 \end{array} \text{ feet in all}$$

Here is an even shorter method.

DAY 1
Each trench is 10 + 8,
or 18, feet long.
He dug 3.

$$\begin{array}{r} 10 + 8 \\ \times\ \ \ 3 \\ \hline 30 + 24 \end{array} \qquad \begin{array}{rl} 18 & \\ \times\ 3 & \\ \hline 24 & 3 \times 8 \\ 30 & 3 \times 10 \\ \hline 54 & \text{in all} \end{array}$$

DAY 2
Each trench is
12 feet long.
He dug 6.

$$\begin{array}{rl} 12 & \\ \times\ 6 & \\ \hline 12 & 6 \times 2 \\ 60 & 6 \times 10 \\ \hline 72 & \end{array}$$

DAY 3
Each trench is
14 feet long.
He dug 7.

$$\begin{array}{rl} 14 & \\ \times\ 7 & \\ \hline 28 & 7 \times 4 \\ 70 & 7 \times 10 \\ \hline 98 & \end{array}$$

Your turn to multiply.

1. $\begin{array}{r} 12 \\ \times\ 8 \\ \hline 16 \\ 80 \\ \hline 96 \end{array}$
2. $\begin{array}{r} 13 \\ \times\ 7 \\ \hline 21 \\ 70 \\ \hline 91 \end{array}$
3. $\begin{array}{r} 17 \\ \times\ 6 \\ \hline 42 \\ 60 \\ \hline 102 \end{array}$
4. $\begin{array}{r} 15 \\ \times\ 4 \\ \hline 20 \\ 40 \\ \hline 60 \end{array}$
5. $\begin{array}{r} 19 \\ \times\ 9 \\ \hline 81 \\ 90 \\ \hline 171 \end{array}$
6. $\begin{array}{r} 16 \\ \times\ 5 \\ \hline 30 \\ 50 \\ \hline 80 \end{array}$
7. $\begin{array}{r} 18 \\ \times\ 8 \\ \hline 64 \\ 80 \\ \hline 144 \end{array}$

nce a month, Dirty Dan, the
rospector, made the long trip into
own for supplies.
ook at Dan's shopping list. Then
ook at the lists of prices.
igure out how much Dan had to
ay for each item on his list.

1. 5 packages cheese 60¢
2. 8 cans corned beef 96¢
3. 2 cans sardines 26¢
4. 2 packages tea 28¢
5. 5 packages coffee 85¢
6. 5 packages apricots 75¢
7. 4 cans peaches 64¢
8. 8 cans pears $1.52
9. 6 cans cherries $1.14
10. 7 cans pineapple 77¢
11. 8 cans peas $1.36
12. 8 cans beans $1.12

ind out how much his supplies cost in all. $10.25

CHEESE . . . . 12¢ a package
SAUSAGES . . . . 13¢ a dozen
CORNED BEEF . . . . . 12¢ a can
SARDINES . . . . 13¢ a can
COFFEE . . . . 17¢ a package
TEA . . . . 14¢ a package
DRIED APRICOTS . . . 15¢ a package
RAISINS . . . . 13¢ a package

CANNED FRUITS & VEGETABLES
PEACHES . . . . 16¢ a can
PEARS . . . . 19¢ a can
CHERRIES . . . . 19¢ a can
PINEAPPLE . . . . . . 11¢ a can
PEAS . . . . 17¢ a can
BEANS . . . . 14¢ a can
SWEET POTATOES . . 13¢ a can

103

Pretend you are a grocery store manager. Design an ad for next week's paper that attracts customers into your store.

Some youngsters may want to run a different kind of store. That's fine! Or they may prefer to write a radio or TV commercial.

goal Practice in multiplying a 2-digit factor by a 1-digit factor

memo IMPORTANT! Use activity 2 on page 120a of the Resource Section before doing this page.

things newspaper grocery ads

page 103 Put things into perspective. Have the pupils use those newspaper ads and compare the prices in the late 1800s with today's prices. You can work in some good subtraction practice here too.

Some guidance in setting up problems 1 through 12 may be necessary. They should be completed independently. Determining the total cost of the groceries could turn into a group project. Make sure that the item costs are correct before the pupils begin to compute the total.

lesson Pages 104, 105

goal Development of estimation in multiplication

memo Rounding and estimating are used extensively throughout this program in all computation. These two skills are especially important in multiplication. Estimation **before** computation is one of the most effective ways to reinforce concepts of place value. It allows some sense to be made from the "magic" of partial products.

page 104 Independent learners are on their own. You'll have to decide about your other groups. The strugglers will need you. Use number-line diagrams again if the pupils have forgotten about rounding.

1. Dirty Dan said, "A year's supply of soap will cost me about 50¢."
   What does that statement really say? (Notice the word "about.")
   - a Does it tell exactly how much was paid?
   - b Does it tell if it cost more than 50¢? No
   - c Does it tell if it cost less than 50¢? No
   - d Would he say that if he really paid 25¢? 75¢? No No
   - e Would he say that if he paid 40¢? 60¢? No No
   - f Would he say that if he paid 45¢? 55¢? Probably Probably
   - g Would he say that if he paid 48¢? 52¢? Probably Probably
   - h What was probably the lowest price he would have paid to make that statement? the highest price?
     45¢ 55¢

2. Socks cost 36¢ a pair.
   About how much do 2 pairs cost?
   $2 \times 36 = ?$ Round 36 up to 40. $2 \times 40 = ?$ 80
   Will the 2 pairs cost less or more than 80¢?

3. Pick handles cost 18¢ each. Dan picks up six.
   About how much did he pay for the pick handles?
   $6 \times 18 = ?$ Round 18 up to 20. $6 \times 20 = ?$ 120
   Did they cost more or less than \$1.20?

4. Dan's mule plods at a speed of 8 km/h.
   About how far would they travel if Dan can push the mule for 21 h non-stop?
   8×21 is about 8×20.
   $8 \times 20 = 160$
   Is 8×21 more or less than 160?

104

Round the 2-digit number to the nearest ten. Then estimate the answer. Is the exact answer greater than or less than the estimate? Use > or < when you write the estimate on your paper.

Remember > means "is greater than" and < means "is less than."

1. 4 × 23 is about 4 × 20.
   4 × 20 = 80
   4 × 23 ■ 80 (> or <) >

2. 6 × 19 is about 6 × 20.
   6 × 20 = ? 120
   6 × 19 ■ 120 <

3. 7 × 42 is about 7 × 40.
   7 × 40 = ? 280
   7 × 42 ■ 280 >

4. 3 × 38 is about 3 × 40.
   3 × ? = ? 40 120
   3 × 38 ■ 120 <

5. 8 × 27 is about 8 × ?. 30
   8 × ? = ? 30 240
   8 × 27 ■ 240 <

6. 2 × 53 is about 2 × ?. 50
   2 × ? = ? 50 100
   2 × 53 ■ 100 >

Estimates help you avoid silly mistakes. Estimate before you complete each problem. Write it down. Then multiply. Compare your answer with your estimate. Find out if your answer is reasonable. (Estimated answer in parentheses)

	a	b	c	d	e	f	g
7.	37 × 5 (200) 185	42 × 7 (280) 294	57 × 6 (360) 342	32 × 8 (240) 256	28 × 6 (180) 168	67 × 4 (280) 268	63 × 7 (420) 441
8.	69 × 2 (140) 138	95 × 7 (700) 665	65 × 4 (280) 260	74 × 9 (630) 666	83 × 8 (640) 664	58 × 3 (180) 174	49 × 6 (300) 294

105

goal Using estimation to check the reasonableness of a product

page 105 Problems 1 through 6 are independent work. Be careful of those **less-than** and **greater-than** signs. The focus here is on estimation, not on exact products. Check the pupil's progress before he completes the page.

How can an estimate check the reasonableness of an answer? You'll want to make sure that the pupil knows why he is to estimate and find the exact answer for rows 7 and 8.

Plan to go outside the text for the next day's work. Please use the computation practice suggested in activity 2, part **b** on page 120b of the Resource Section.

lesson Pages 106, 107, 108

goal Progress Check – multiplying a 2-digit factor by a 1-digit factor; extension of estimation skills

memo Make sure that you have used the day-long activity 2 on page 120a of the Resource Section **before** you use this page.

page 106 The Progress Check is short, but it will signal the trouble spots.

$$\begin{array}{r} 69 \\ \times\ \ 2 \\ \hline 16 \\ 180 \\ \hline 136 \end{array} \quad \begin{array}{l} \\ \\ \left.\begin{array}{l} \\ \\ \end{array}\right\} \text{Fact errors} \\ \\ \text{Addition error} \end{array} \qquad \begin{array}{r} 58 \\ \times\ \ 5 \\ \hline 40 \\ 250 \\ \hline 2540 \end{array} \quad \begin{array}{l} \\ \\ \text{Columns not} \\ \text{aligned} \\ \\ \end{array}$$

Errors in multiplication facts mean more practice. Lined paper turned sideways will help with place-value column alignment. Estimation will help catch gross errors.

If much additional work is needed, stop here. Complete mastery is not necessary, but a feeling of confidence is. Put your pupils to work making patterned practice sheets as suggested in activity 2 on page 120a. Ask your capable pupils to help you with peer-tutor work. Get involved with any supplemental materials you have available.

Now on with the page! A review of rounding to hundreds and to thousands may be necessary – it's been a long time.

Skill: Multiplying 2-digit by 1-digit number

Multiply.

1.	2.	3.	4.	5.	6.	7.
69	58	17	74	86	45	92
× 2	× 5	× 9	× 8	× 7	× 6	× 3
138	290	153	592	602	270	276

Let an estimate help you with multiplication of larger numbers.

1. The gym at Washington School can hold 312 people. What is the greatest number of people who can go to 7 home basketball games? Estimate the answer.

   Round 312 to the nearest hundred.

   7 × 312 is about 7 × 300.
   7 × 300 = 2100

   Is 312 less than or greater than 300?
   Is 7 × 312 less than or greater than 2100?

2. The football stadium at Washington School can hold 1942 people. What is the greatest number of people who can go to 6 home football games?

   Round 1942 to the nearest thousand.

   6 × 1942 is about 6 × 2000.
   6 × 2000 = 12,000

   Is 1942 less than or greater than 2000?
   Is 6 × 1942 less than or greater than 12,000?

When estimating, round –
2-digit numbers to the nearest ten,
3-digit numbers to the nearest hundred, and
4-digit numbers to the nearest thousand.

106

See activity 3, page 120b.

See activity 4, page 120b.

Round so that you can estimate. Is the exact answer greater than or less than the estimate? Use $>$ or $<$.

1. 4×572 is about 4×600.
$4 \times 600 = 2400$
4×572 ■ 2400 ($>$ or $<$)
$<$

2. 7×2134 is about 7×2000.
$7 \times 2000 = 14{,}000$
7×2134 ■ $14{,}000$
$>$

3. 6×389 is about 6×400.
$6 \times 400 = 2400$
6×389 ■ 2400
$<$

4. 7×212 is about 7×200.
$7 \times 200 =$? 1400
7×212 ■ 1400
$>$

5. 3×4962 is about 3×5000.
$3 \times 5000 =$? 15,000
3×4962 ■ $15{,}000$
$<$

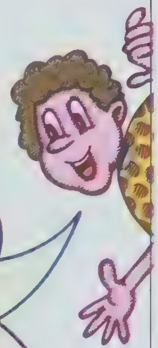
6. 8×6035 is about 8×6000.
$8 \times 6000 =$? 48,000
8×6035 ■ $48{,}000$
$>$

7. 5×94 is about 5×90.
$5 \times$? $=$? 90 450
5×94 ■ ? 450
$>$

8. 7×812 is about 7×800.
$7 \times$? $=$? 800 5600
7×812 ■ ? 5600
$>$

9. 9×634 is about $9 \times$?. 600
$9 \times$? $=$? 600 5400
9×634 ■ ? 5400
$>$

10. 5×3341 is about $5 \times$?. 3000
$5 \times$? $=$? 3000 15,000
5×3341 ■ ? 15,000
$>$



goal Practice in estimating products when one factor is a 3- or 4-digit number, the other factor a 1-digit number

page 107 Everyone on his own. Watch for those who still reverse the greater-than and less-than signs.

Eventually pupils should generalize that where they round down, the exact answer will be greater than the estimate; when they round up, the exact answer will be less than the estimate. This is an idea to be discovered. Telling won't help. Praise those who do see this relationship.

It is very important that you do not ask that the exact products be computed now. Save this work. You can come back to this page after page 114.

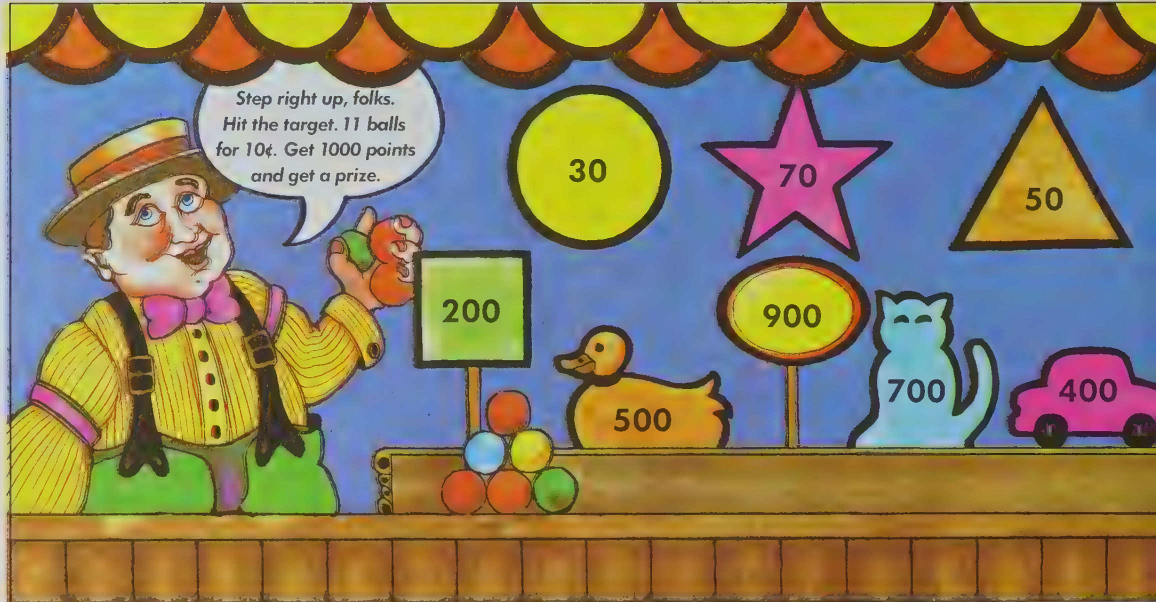
goal Practice in multiplying a multiple of 10 or of 100 by a 1-digit factor

page 108 Everyone on his own. Be alert for multiplication-fact errors and carelessness with zeros.

Place value and straight columns will be extremely important when adding the total number of points. Remember—lined paper turned sideways will help keep place-value columns straight.

You may want to extend the page by simulating the activity. A target can be hit by a student as he closes his eyes and points with a finger. A new rule will be needed: Points will be allowed only if the number is covered by the finger—no points allowed if the finger lands on the edge of a target or between two targets.

The hits should be tallied in a chart. To compete with Bill, 33 hits are needed. *O.K. Let's figure out how many points we made. How does our score compare with Bill's? How many prizes do we get?*



Bill had 30¢. He spent it all right here. He hit something with every ball.

1. How many points did Bill get? Complete the table.

2. How many prizes did he get? 16

Target		Number of hits	Total number of points
a	duck	3	? 1500
b	star	5	? 350
c	car	8	? 3200
d	square	4	? 800
e	cat	6	? 4200
f	oval	7	? 6300

108

machine operator can punch out 315 pieces per our. At this rate, how many pieces could 3 operators unch out in an hour?

$$\begin{array}{r} 315 \\ \times \quad 3 \\ \hline 945 \end{array}$$

(Don't expect everyone to answer here.)

ack figures it this way: The 3 operators first punch ut 5 pieces each. Second, they punch out 10 pieces ach. Third, they punch out 300 pieces each. Then you dd to find out how many pieces they punch out in all.

first $\begin{array}{r} 5 \\ \times 3 \\ \hline 15 \end{array}$ second $\begin{array}{r} 10 \\ \times \quad 3 \\ \hline 30 \end{array}$ third $\begin{array}{r} 300 \\ \times \quad 3 \\ \hline 900 \end{array}$ $\begin{array}{r} 15 \\ 30 \\ +900 \\ \hline 945 \end{array}$ in all

low many pieces could the 3 operators punch out in n 8-hour day?

$$\begin{array}{r} 945 \\ \times \quad 8 \\ \hline 7560 \end{array}$$

(Don't expect everyone to answer here.)

etty figures it out like this: In each of the 8 hours e operators first punch out 5 pieces. Second, they unch out 40 pieces. Third, they punch out 900 ieces. She adds to find how many pieces they punch ut during the whole day.

$$\begin{array}{rl} 945 & \\ \times \quad 8 & \\ \hline 40 & \text{first } (8 \times 5) \\ 320 & \text{second } (8 \times 40) \\ 7200 & \text{third } (8 \times 900) \\ \hline 7560 & \text{in all} \end{array}$$

109

lesson Pages 109, 110

goal Extension of multiplication to a 3-digit factor multiplied by a 1-digit factor

memo A new idea – discussion please. The algorithm must be introduced carefully. Understanding of it now will prevent problems later. Pages 109 and 110 are considered introductory.

page 109 Straight columns are imperative. **Thinking steps** are shown at the right on the pupil page. Don't ask the pupil to write them – they'll get in the way.

goal Practice in multiplying a 3-digit factor by a 1-digit factor

page 110 Same rules as for the preceding page. Problems 1 through 3 should provide added confidence for the less capable students. Provide all the help necessary for the remaining problems.

A truck is carrying 473 crates. There are 5 boxes in each crate. How many boxes are there in the truck?

4 7 **3**	4 **7** 3	**4** 7 3
How many in 3 crates?	How many in 70 crates?	How many in 400 crates?
$3 \times 5 = 15$ Right?	$70 \times 5 = 350$ O.K.?	$400 \times 5 = 2000$ Agree?

Now how many boxes in all 473 crates?

$$\begin{array}{r l}
473 & \\
\times \quad 5 & \\
\hline
15 & 5 \times 3 \longrightarrow \text{This is part of the product.} \\
350 & 5 \times 70 \longrightarrow \text{This is part of the product.} \\
2000 & 5 \times 400 \longrightarrow \text{This is part of the product.} \\
\hline
2365 & \text{in all} \longrightarrow \text{This IS the product.}
\end{array}$$

Copy and complete these.

1. 124×6
- 24 ■■ 6×4
- 120 ■■■ 6×20
- 600 ■■■ 6×100
- 744 ■■■ in all

2. 247×2
- 14 ■■ 2×7
- 80 ■■ 2×40
- 400 ■■■ 2×200
- 494 ■■■ in all

3. 489×3
- 27 ■■ 3×9
- 240 ■■■ 3×80
- 1200 ■■■■ 3×400
- 1467 ■■■■ in all

4. 132×2 264

5. 379×5 1895

6. 423×3 1269

7. 251×7 1757

8. 431×8 3448

Many radio stations have contests. WBBQ has one. Once every hour they give a new clue to the answer to their puzzle. Each clue gives a better idea of what the answer is. The first person to answer correctly wins a prize.

Let's use this contest to learn more about multiplication.

In the following puzzles, select the multiplication problem that is described by the clues.

1

Clues: **1.** The first part of the product is 12.
2. The second part of the product is 40.

A $\begin{array}{r} 13 \\ \times\ 4 \\ \hline \end{array}$	*B* $\begin{array}{r} 18 \\ \times\ 4 \\ \hline \end{array}$	*C* $\begin{array}{r} 14 \\ \times\ 3 \\ \hline \end{array}$	*D* $\begin{array}{r} 15 \\ \times\ 5 \\ \hline \end{array}$

After which clue do you know the answer? 2

The first clue describes part of problems *A* and *C*. The second clue is 40. So it's not *C*. The answer is *A*. The answer is known after the second clue.

2

Clues: **1.** The first part of the product is 24.
2. The second part of the product is 180.
3. The third part of the product is 1200.

A $\begin{array}{r} 452 \\ \times\ 5 \\ \hline \end{array}$	*B* $\begin{array}{r} 234 \\ \times\ 6 \\ \hline \end{array}$	*C* $\begin{array}{r} 268 \\ \times\ 3 \\ \hline \end{array}$	*D* $\begin{array}{r} 782 \\ \times\ 2 \\ \hline \end{array}$

After which clue do you know the answer? 3

lesson Pages 111, 112

goal Reinforcing the development of the multiplication model

memo You may decide to skip the contests on pages 111 and 112 with your struggling students and instead use the problems as straight practice. If you decide to use the pages as intended, these young people will need your guidance all the way.

page 111 Arithmetic doesn't always have to be computed with pencil and paper. In the real world, we must often **think** our answers. Some of your students will be capable of completing the page with no written computation. Wonderful! Others may find it necessary to multiply **at least** ones in written form—just to narrow down the number of possibilities.

Continue on to page 112.

goal Reinforcing the development of the multiplication model

page 112 Same rules as for the preceding page. Notice the last line on the page. This is a lot of work! You may want to modify this assignment to meet individual needs.

Any individuals who really had to struggle with the clues would benefit from a group discussion and a sharing of strategy.

PUZZLE 3

Clues: **1.** The first part of the product is 21.
2. The second part of the product is 140.
3. The third part of the product is 700.

A	*B*	*C*	*D*
467×3	123×7	19×9	74×4
1401	861	171	296

After which clue do you know the answer? 2

PUZZLE 4

Clues: **1.** The first part of the product is 64.
2. The second part of the product is 640.
3. The third part of the product is 6400.

A	*B*	*C*	*D*
934×6	533×5	277×7	888×8
5604	2665	1939	7104

After which clue do you know the answer? 1

PUZZLE 5

Clues: **1.** The first part of the product is 8.
2. The second part of the product is 160.
3. The third part of the product is 2400.

A	*B*	*C*	*D*
421×8	142×4	284×2	642×4
3368	568	568	2568

After which clue do you know the answer? 3

Did you know the answer to the multiplication problem or the answer to the puzzle? Answer to the puzzle is probable answer.

Compute to find the answers to each of the problems on this page.

504 × 8 32 400 432	504 × 8 32 4000 4032	504 × 8 32 0 4000 4032

Same factors

How are these problems alike? How are they different? Parts of products different; answers different

Which problems are worked correctly? 2d and 3d

Which are worked wrong? 1st

It is helpful to use estimation to check the answer when a multiplication problem has zeros in it. Remember, you will want to round—

2-digit numbers to the nearest ten,
3-digit numbers to the nearest hundred, and
4-digit numbers to the nearest thousand.

Now look at the problems at the top of the page. 504 rounded to the nearest hundred is 500.

$$\begin{array}{r} 500 \\ \times \quad 8 \\ \hline 4000 \end{array}$$

Does this give you a clue as to which answer is correct?

Estimate. Then multiply. (Estimated answer in parentheses)

1. $\begin{array}{r} 150 \\ \times \quad 9 \\ \hline \end{array}$ (1800) 1350

2. $\begin{array}{r} 605 \\ \times \quad 4 \\ \hline \end{array}$ (2400) 2420

3. $\begin{array}{r} 3004 \\ \times \quad 6 \\ \hline \end{array}$ (18,000) 18,024

4. $\begin{array}{r} 3079 \\ \times \quad 8 \\ \hline \end{array}$ (24,000) 24,632

5. $\begin{array}{r} 9070 \\ \times \quad 3 \\ \hline \end{array}$ (27,000) 27,210

113

STOP See activity 5, page 120c.

lesson Page 113

goal Practice in checking products by estimation; extension of multiplication to a 4-digit factor multiplied by a 1-digit factor

memo Skip this page with learners who are struggling. They'll meet these skills again later.

page 113 Those who estimate should realize immediately that there is something wrong with the products shown in the illustration.

Discuss the page with pupils who need guidance. Where does the 0 under the 32 come from in the problem on the right? Why isn't it in the middle problem? Do we have to multiply the 0? Is it wrong? When we multiply the 500, does that automatically take care of the 0? What mistake was made in the problem on the left?

Try rounding the products for problems 1 through 5. Use your good judgment in assigning any written problems.

lesson Page 114

goal Introduction to the short algorithm for multiplication

memo No one knows the abilities of your students better than you do. Are only some of the students ready to jump to the short algorithm? Then introduce it just to them. Let the others continue with the long form. If all pupils are ready, then go full steam ahead. If no one is operating with confidence, skip the page.

warm-up Brush up on a skill that is absolutely necessary for short-form multiplication – multiplication **plus** simple addition.

$(3 \times 5) + 4 = \underline{?}$ $(4 \times 4) + 2 = \underline{?}$
$(6 \times 7) + 3 = \underline{?}$ $(4 \times 5) + 1 = \underline{?}$
$(8 \times 4) + 5 = \underline{?}$ $(7 \times 9) + 3 = \underline{?}$

page 114 The questions on the page force the youngsters to analyze the shortened form. Correlate the identical steps in Bruce's problem to his dad's problem. Repeat for the second problem.

Bruce	**Dad**
47	5 47
× 8	× 8
56	376
320	
376	

For extra practice, you can go back to page 107 and ask that the exact products be computed. These are the answers:

1. 2288	2. 14,938	3. 2334
4. 1484	5. 14,886	6. 48,280
7. 470	8. 5684	9. 5706
10. 16,705		

One night Bruce was having trouble working this problem:

$$\begin{array}{r} 47 \\ \times\ 8 \\ \hline \end{array}$$

So he asked his dad for help. His dad said, "I'll write down the answer for you, but I don't have time to tell you how I got it. You'll have to figure that for yourself."

Here is what Bruce's dad wrote:

$$\begin{array}{r} {}^{5}47 \\ \times\ 8 \\ \hline 376 \end{array}$$

Bruce was really confused. He had never seen multiplication like this before. "What is dad doing now?" he thought.

Finally Bruce remembered how to do the problem.

$$\begin{array}{r} 47 \\ \times\ 8 \\ \hline 56 \\ 320 \\ \hline 376 \end{array}$$

Bruce got the same answer! He wondered how his dad could work the problem doing all his work on only one line.

Can you help Bruce? Look at what his dad wrote.
Why are there 6 ones in the answer?
What happens to the 5 tens? Added to the 32 tens
Why are there 37 tens in the answer?

$$\begin{array}{r} {}^{5}47 \\ \times\ 8 \\ \hline 376 \end{array}$$

8 × 7 = 56 (5 tens, 6 ones)

8 × 40 = 320 and then 5 tens added

1. Try to explain this computation:

$$\begin{array}{r} {}^{1\,2}546 \\ \times\ 4 \\ \hline 2184 \end{array}$$

a Why do we write 4 ones in the answer? 4 × 6 = 24 (2 tens, 4 ones)
b Why do we write 2 in the tens column above the 4? 4 × 6 = 24 (2 tens, 4 ones)
c Why do we write 8 tens in the answer? 4 × 40 + 20 = 180 (1 hundred, 8 tens)
d Why do we write 1 in the hundreds column above the 5? 4 × 40 + 20 = 180 (1 h
e Why do we write 1 hundred in the answer? 4 × 500 + 100 = 2100 (2 thousands, 1 hun
f Why do we write 2 thousands in the answer? 4 × 500 + 100 = 2100 (2 thousands, 1 h

114

STOP Write a problem on the board. Call on one pupil at a time to write only one digit in the process of finding the product.
Example:

$$\begin{array}{r} 54 \\ \times\ 7 \\ \hline \end{array} \quad \begin{array}{r} 54 \\ \times\ 7 \\ \hline 8 \end{array} \quad \begin{array}{r} {}^{2}54 \\ \times\ 7 \\ \hline 8 \end{array} \quad \begin{array}{r} {}^{2}54 \\ \times\ 7 \\ \hline 78 \end{array} \quad \begin{array}{r} {}^{2}54 \\ \times\ 7 \\ \hline 378 \end{array}$$

This activity forces the learner to look at one step at a time.

A store gets a shipment of 48 coats. They will sell for \$30 apiece. What is the total value of the coats?

3 × 48 is 144. So 30 × 48 must be 1440.

$$\begin{array}{r} 48 \\ \times\ 3 \\ \hline 144 \end{array} \qquad \begin{array}{r} 48 \\ \times 30 \\ \hline 1440 \end{array}$$

Find the answers.

1. $\begin{array}{r} 52 \\ \times 30 \\ \hline \end{array}$ 1560	2. $\begin{array}{r} 47 \\ \times 60 \\ \hline \end{array}$ 2820	3. $\begin{array}{r} 61 \\ \times 50 \\ \hline \end{array}$ 3050
4. $\begin{array}{r} 83 \\ \times 30 \\ \hline \end{array}$ 2490	5. $\begin{array}{r} 28 \\ \times 70 \\ \hline \end{array}$ 1960	6. $\begin{array}{r} 19 \\ \times 90 \\ \hline \end{array}$ 1710
7. $\begin{array}{r} 55 \\ \times 40 \\ \hline \end{array}$ 2200	8. $\begin{array}{r} 76 \\ \times 80 \\ \hline \end{array}$ 6080	9. $\begin{array}{r} 94 \\ \times 40 \\ \hline \end{array}$ 3760
10. $\begin{array}{r} 87 \\ \times 80 \\ \hline \end{array}$ 6960	11. $\begin{array}{r} 63 \\ \times 50 \\ \hline \end{array}$ 3150	12. $\begin{array}{r} 39 \\ \times 70 \\ \hline \end{array}$ 2730

13. Big Wad chewing gum is shipped in boxes. There are 40 packages in each box. How many packages are there in a shipment of 37 boxes? 1480

14. Happy Harry's Hickory Nuts come in 50-pound crates. The store orders 25 crates. How many pounds did the total order weigh? 1250

15. There are 30 seats in each row. There are 30 rows. How many seats are there in all? 900

lesson Pages 115, 116, 117

goal Introduction to multiplying a 2-digit factor by a multiple of 10

memo A new concept is introduced on this page. Get everyone together for discussion.

page 115 Work through the example together. You might remind the pupils that they already know 3 × 4 is related to 3 × 40 and 3 × 400. This new idea isn't really so different.

As pupils become confident, let them work independently. You decide how best to handle the word problems.

goal Practice in multiplying two 2-digit factors

page 116 Problems 1 through 10 are for everyone. You will want to work closely with those who are struggling.

Independent learners can complete problems 11 through 15. These are a challenge and require a detective to figure them out.

Suppose the 48 coats are to sell for $32 apiece. What is their total value now?

$$\begin{array}{r} 48 \\ \times\ \ 2 \\ \hline 96 \end{array} \qquad \begin{array}{r} 48 \\ \times 30 \\ \hline 1440 \end{array} \qquad \begin{array}{rl} 48 & \\ \times 32 & \\ \hline 96 & 2 \times 48 \\ 1440 & 30 \times 48 \\ \hline 1536 & \end{array}$$

Multiply. (Partial products shown at bottom of page.)

1. 54 × 23 = 1242	**2.** 18 × 61 = 1098	**3.** 57 × 17 = 969	**4.** 46 × 92 = 4232	**5.** 73 × 55 = 4015
6. 35 × 87 = 3045	**7.** 69 × 26 = 1794	**8.** 76 × 95 = 7220	**9.** 80 × 89 = 7120	**10.** 19 × 77 = 1463

Just for fun. Be a detective. Somebody did these problems the long way. Find the missing parts of each of the problems. Match the number of the problem with the letter that shows parts of the product and the answer.

11. 15 × 12 (E)	**12.** 21 × 32 (D)	**13.** 41 × 21 (A)	**14.** 78 × 56 (C)	**15.** 83 × 74 (B)

A	**B**	**C**	**D**	**E**
1	12	48	2	10
40	320	420	40	20
20	210	400	30	50
800	5600	3500	600	100
861	6142	4368	672	180

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
162	18	399	92	365	245	414	380	720	133
1080	1080	570	4140	3650	2800	1380	6840	6400	1330
1242	1098	969	4232	4015	3045	1794	7220	7120	1463

116

STOP

Go back a few steps to help.

3 × 4
3 × 40
30 × 4
30 × 40

Rewrite these problems in the form in which the youngsters will use the skill.

$$\begin{array}{r} 4 \\ \times 3 \\ \hline \end{array} \quad \begin{array}{r} 40 \\ \times\ \ 3 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ \times 30 \\ \hline \end{array} \quad \begin{array}{r} 40 \\ \times 30 \\ \hline \end{array}$$

Review these steps orally. Then change the zero in 40 to a nonzero digit.

$$\begin{array}{r} 46 \\ \times\ \ 3 \\ \hline \end{array}$$

Have them find the product.

You found that 3 times 46 is 138. This should help you find the product for 30 × 46.

PROGRESS CHECK

This is a checkup on what you have learned. Use any type of multiplication you can.

Skill: Multiplication facts

1. 9×7 = 63
2. 8×6 = 48
3. 5×3 = 15
4. 7×4 = 28

Skill: Multiplying 2- or 3-digit by 1-digit number

5. 22×3 = 66
6. 34×2 = 68
7. 58×6 = 348
8. 279×9 = 2511

Skill: Multiplying 2-digit by 2-digit number

9. 575×7 = 4025
10. 42×13 = 546
11. 35×78 = 2730
12. 83×93 = 7719

Skill: Solving problems by multiplying

They decided to have a carnival to raise money for their project.

13. 83 adults bought tickets.
An adult's ticket cost 75¢.
How much money here? $62.25

14. 67 children bought tickets.
A child's ticket cost 35¢.
How much money here? $23.45

15. 92 fishpond tickets were sold.
Each ticket cost 15¢.
How much money here? $13.80

16. 85 fun-house tickets were sold.
Each ticket cost 10¢.
How much money here? $8.50

17. 62 ringtoss tickets were sold.
Each ticket cost 20¢.
How much money here? $12.40

18. 99 cakewalk tickets were sold.
Each ticket cost 25¢.
How much money here? $24.75

*How much money did they collect in all? $145.15

goal **Progress Check** – multiplication skills

memo Multiplying two 2-digit factors is not a mastery objective at this time. We are striving for a level of confidence. More experience will come later.

Don't panic when you see the decimals in problems 13 through 19. The pupils aren't expected to multiply with decimals. If they end up with an answer of 6225¢ for problem 13, it's O.K. This generation is so sophisticated in using money, chances are good that you will get the correct answer if you ask how many dollars and cents there are in 6225¢. If not, tell them. The product 6225 is the most important thing. Some kind of label such as ¢ is of secondary importance. The notation of $62.25 is the bonus.

page 117 Skills being checked are indicated on the answer key. All computations are independent work. You may need to provide help in reading the word problems. Put your ambitious ones to work on the How much in all? question. That question is a monster!

Again look for errors in multiplication facts, renaming, addition, and messy place-value columns.

STOP See activity 6, page 120c.

See activity 7, page 120c.

goal Extension of multiplication skills to multiplying a 3-digit factor by a 2-digit factor

memo Use this page with accelerated students only. This work is not right for students who are having difficulty.

page 118 The pupils who complete this page not only will be applying the knowledge they have gained so far. They will be going even one small step further. Anyone who gets all these problems correct is probably completely independent in all multiplication of whole numbers now. Give praise—oodles of it! You have a mighty fine operator in your class.

An automobile dealer figures he makes about $379 in profit on each car he sells. He has sold 26 cars so far this year. How much profit did he make on the first 6 cars? on the next 20? on all 26?

$$\begin{array}{r} 379 \\ \times\ \ 6 \\ \hline 2274 \end{array} \qquad \begin{array}{r} 379 \\ \times\ 20 \\ \hline 7580 \end{array} \qquad \begin{array}{rl} 379 & \\ \times\ 26 & \\ \hline 2274 & 6 \times 379 \\ 7580 & 20 \times 379 \\ \hline 9854 & \end{array}$$

In the third problem, why did we multiply 379 by 6?*
Why did we multiply 379 by 20?*
How did we get 9854? Added two parts of the product together.

$$^{*}\ \begin{array}{r} 379 \\ \times\ 26 \\ \hline \end{array} \text{ is the same as } \begin{array}{r} 379 \\ \times\ \ 6 \\ \hline \end{array} + \begin{array}{r} 379 \\ \times\ 20 \\ \hline \end{array}$$

Multiply

1. 247 × 25 = 6175	**2.** 384 × 48 = 18,432	**3.** 717 × 85 = 60,945
4. 684 × 63 = 43.092	**5.** 909 × 98 = 89,082	**6.** 474 × 32 = 15,168
7. 575 × 68 = 39,100	**8.** 875 × 59 = 51,625	**9.** 625 × 48 = 30,000

10. There are 24 hours in a day. There are 365 days in a year. How many hours are there in one year? 8760

11. You attend school about 185 days each year. If you go to school for 13 years, about how many days of school will you attend? 2405

12. There are 60 minutes in one hour. How many minutes in one day? 1440

13. Jan's mother works in a store 5 hours a day, 6 days a week. Jan's dad works in a factory 8 hours a day, 5 days a week.

a Who works more hours each week? Dad

b How many more hours each week? 10 hours

c Both work 50 weeks each year. How many hours does each work in one year?
Mother—1500 hours, dad—2000 hours

See activity 8, page 120c.

Sometimes prices of things increase. It doesn't seem like much to pay a penny or two more for something. But extra pennies can count up!

Your family buys certain food every week. Find out how much more you will pay for an item over a period of time. Compute the amount you would pay with the old price and with the new price. Figure out the amount paid in one month (4 weeks), in 6 months (26 weeks), and in 1 year's time (52 weeks).

YOU BUY	NUMBER IN 1 WEEK	OLD PRICE FOR 1	NEW PRICE FOR 1	TOTAL AMOUNT PAID IN					
				4 WEEKS		26 WEEKS		52 WEEKS	
				old	new	old	new	old	new
1. BREAD	3 LOAVES	25¢ (75¢)	26¢ (78¢)	a $3.00	b $3.12	c $19.50	d $20.28	e $39.00	f $40.56
2. MILK	4 HALF-GALLONS	59¢ ($2.36)	61¢ ($2.44)	a $9.44	b $9.76	c $61.36	d $63.44	e $122.72	f $126.88
3. ICE CREAM	1 HALF-GALLON	85¢	87¢	a $3.40	b $3.48	c $22.10	d $22.62	e $44.20	f $45.24
4. COFFEE	1 CAN	93¢	97¢	a $3.72	b $3.88	c $24.18	d $25.22	e $48.36	f $50.44
5. HOT DOGS	1 POUND	82¢	84¢	a $3.28	b $3.36	c $21.32	d $21.84	e $42.64	f $43.68
*6. EGGS	1 DOZEN	49¢	50¢	a $1.96	b $2.00	c $12.74	d $13.00	e $25.48	f $26.00

Talk about these questions.

Are prices for any one thing the same all over the country? No Have you heard about other price changes? How do these price changes affect your family?

Discuss, please. Eating habits may have to be changed and different budgets made.

119

goal Application of multiplication skills to problem solving

memo This page is a real challenge! It is included for accelerated students only. They may want to tackle it as a group rather than individually.

page 119 Pages such as this one give you a worthwhile activity for your able pupils while you spend time with individuals or small groups that continue to need your help and guidance. This page can launch some good research projects after a brief discussion of the questions at the bottom of the page.

lesson Page 120

goal **Checkout**—multiplication skills

memo Just a reminder—rows 5 and 6 are not included in the mastery objectives for this chapter. The learner should, however, feel confident, not frustrated and confused by these problems.

page 120 Too many problems! Be selective. Rows 1, 3, and 5 will indicate the learner's ability. Use the remaining rows to recheck the progress of those youngsters who will be given additional help.

CHECKOUT

Skill: Multiplying 2-digit by 1-digit number

	a	b	c	d	e
1.	14 × 4 = 56	42 × 6 = 252	27 × 8 = 216	58 × 9 = 522	91 × 7 = 637
2.	39 × 2 = 78	85 × 8 = 680	76 × 3 = 228	74 × 7 = 518	89 × 5 = 445

Skill: Multiplying 3-digit by 1-digit number

	a	b	c	d	e
3.	342 × 6 = 2052	765 × 8 = 6120	809 × 9 = 7281	762 × 5 = 3810	801 × 4 = 3204
4.	539 × 7 = 3773	642 × 6 = 3852	810 × 2 = 1620	375 × 8 = 3000	692 × 3 = 2076

Skill: Multiplying 2-digit by 2-digit number

	a	b	c	d	e
5.	26 × 53 = 1378	54 × 45 = 2430	13 × 78 = 1014	82 × 34 = 2788	69 × 67 = 4623
6.	77 × 19 = 1463	48 × 87 = 4176	31 × 46 = 1426	37 × 62 = 2294	37 × 38 = 1406

120

STOP A little art makes practice more interesting and encourages mental computation.

×	60	+	5	
4	(240)		(20)	= (260)

×	30	+	5	
40	(1200)		(200)	= (1610)
+				
6	(180)		(30)	

See activity 9, page 120d.

RESOURCES

another form of evaluation

for Progress Check – page 106

Multiply.

1.	2.	3.	4.	5.
48 × 3 144	56 × 9 504	29 × 7 203	85 × 8 680	76 × 4 304

for Progress Check – page 117

This is a checkup on what you have learned. Use any type of multiplication you can.

1. 8 ×7 56	2. 9 ×8 72	3. 7 ×6 42	4. 4 ×5 20
5. 32 × 3 96	6. 23 × 2 46	7. 64 × 7 448	8. 268 × 4 1072
9. 463 × 8 3704	10. 54 ×26 1404	11. 93 ×47 4371	12. 27 ×78 2106

Two classes are going to the zoo.

13. 64 are going on the bus. Bus fare is 45¢. How much money here? $28.80
14. 59 buy children's tickets. A child's ticket costs 65¢. How much money here? $38.35
15. 26 buy bags of peanuts. Each bag costs 29¢. How much money here? $7.54
16. 34 buy picture postcards. Each postcard costs 10¢. How much money here? $3.40
17. 47 ride the zoo train. Each ticket costs 55¢. How much money here? $25.85
18. They all buy milk for lunch. Each carton costs 15¢. How much money here? $9.60

*How much money did the classes spend in all? $113.54

for Checkout – page 120

	(a)	(b)	(c)	(d)	(e)
1.	24 × 2 48	35 × 3 105	53 × 7 371	81 × 9 729	46 × 4 184
2.	78 × 6 468	92 × 5 460	37 × 8 296	64 × 3 192	57 × 9 513
3.	423 × 4 1692	635 × 7 4445	320 × 6 1920	594 × 5 2970	708 × 9 6372
4.	563 × 8 4504	876 × 3 2628	614 × 7 4298	957 × 9 8613	482 × 6 2892
5.	72 ×46 3312	93 ×58 5394	54 ×82 4428	27 ×59 1593	38 ×64 2432
6.	87 ×89 7743	41 ×93 3813	76 ×27 2052	49 ×35 1715	65 ×72 4680

activities

1. things 4 sets of numeral cards 0 through 9

Group game: 2, 3, or 4 players. The cards are shuffled and completely dealt facedown. The cards remain facedown in a stack before each player. Each player turns over the 2 top cards in his stack and gives the product, going once around the group. The player with the greatest product wins and takes all the cards used in the round. Should there be a tie for the greatest product, the cards remain and only those who tied play another round. The winner for this round collects all cards that are faceup.

Products may be challenged. If the challenged player did give an incorrect product, his 2 cards automatically go to the challenger. If the product is correct, however, the challenger gives his cards to the player he challenged.

2. part a Involvement is one of the most important ingredients of that thing called **motivation.** Yet what can turn off involvement more quickly than a bunch of dreary drill problems? Many youngsters think, "O.K., so I have to get the answers down. So what if I get some of them wrong?"

The all-time challenge is to create computational practice that will be appropriate for many levels of ability and that will demand at least some intellectual involvement.

One way of doing such drill is to get the pupils into the act by having them make up their own practice sheets. Below you will find two complete getting-started ideas. Use the first one after page 102 is completed. Use the second after page 105. They will provide comprehensive practice in every multiplication fact. They are fun, but there is a lot of computation involved too. Be careful that the pupils don't get overtired.

Instructions to pupils: *Be a number detective. Plan space on your paper so that you can write 9 problems across a row. Write the numbers 11 through 19 in one row. . . . Multiply each number by 2.*

Again write the numbers 11 through 19 directly under the numbers in your first row of problems. . . . If you multiply each number by 4 this time, do you think these products will have a pattern similar to the pattern in the first row of problems? . . . How do you think the products will be related? (Each product will be twice as large as that of the corresponding problem above. Each product in the first row increases by 2, each product in the second row increases by 4.)

Write the numbers 11 through 19 in a row again. Try to keep the problems lined up with those in the first row. . . . Multiply each number by 6. . . . There will be another pattern in your products, and these products will be related in some way to your first row of problems. Do you know how they will be related? This is a challenge to your brightest pupils, although it is hard to predict which children will come up with the right answer. *If you think you know the relationship, prove it. If you don't know, find the products. Then maybe you will be able to untangle the mystery.* (The products will be 3 times the corresponding problem in the first row.)

Now look at the factors you multiplied by — 2, 4, 6. What factor do you think you should use next to multiply the numbers 11 through 19? (8) *Will there be another pattern in your products? Will these products be related in some way to the products in the first row of problems? If you think you know, prove it. If you don't know, complete the problems and then maybe you will know.*

Look at all the products of all the problems. Are the products odd or even numbers?

If interest is still high, repeat the process for the odd-number factors 3, 5, 7, and 9 on the same day. If the pupils are tired, please wait for another day. Interest is much more important than computation right now. Without it, this whole series of drill problems will fall flat and be as dreary as any old set of problems.

part b Second set of clues for the number detectives: *You will have to plan your paper so that you can write 9 numbers in a row again.* Consider having the youngsters turn their paper so that they can use the longer side. *Write the numbers 11, 21, 31, 41. What do you think will be the last 5 numbers that you are to write?* (51, 61, 71, 81, 91)

What is the smallest number we can use as a factor and get an odd-number product? (1) *If we use 1 as a factor, what will the products be?* (The same as the first factor) *Since you already know all the answers, what is the next-largest factor you can use to get an odd-number product?* (3) *O.K. Use 3 as the second factor. Find the products. . . . Is the digit in the ones place the same in all your products? . . . Aha! Another number clue.*

The next row of 9 numbers will also follow a pattern. I'll give you the first 4 numbers; you write the remaining 5. Here they are: 13, 23, 33, 43. You finish. . . . This time you want the second factor to be 2 times as great as the factor you used in the first row. What will you use as the second factor? (6) *Will these products be odd or even numbers? Will the digit in the ones place be the same for all the products? Prove it by finding the products.*

One more row of problems. I'll give you the first 4 numbers to be used; you complete the last 5. Here they are: 16, 26, 36, 46. What are the last 5? This time the second factor is to be the greatest 1-digit number there is. What is that number? (9) *Is that number 3 times as great as the second factor you used in the first row of problems?* (Yes) *Will your products be 3 times as great as the products in the first row of problems?* You will catch a lot of people on this question. No; the products will not be 3 times as great, because the first factors were 11, 21, 31 and now they have changed to 16, 26, 36. It's a good mistake, however, because the question was a real curve ball. These questions are great fun if they are not handled too seriously. *Will there be a pattern in the products? Write down what you think the pattern will be and then prove that your guess was right or wrong.*

You should be graduating some detectives from basic training after these exercises. Put those people to work devising sets of problems for the others to use. Transfer the problems to duplicating paper. Then everyone will qualify for the training session to come later.

3. Some youngsters will still be having problems with multiplication in the estimation step. Have them focus on the basic fact involved and the number of zeros in the multiples 10, 100, and 1000.

$3 \times 5000 = \underline{?}$ Think: $3 \times 5 = \underline{?}$
$3 \times 50 = \underline{?}$
$3 \times 500 = \underline{?}$
$3 \times 5000 = \underline{?}$

Some will see the pattern immediately. If not, try one or two more examples. Do not tell them the pattern. If they don't "see" it, telling won't do much good.

4. things small cards

Prepare a deck of playing cards as follows:
- 3 cards each for 1 through 10
- 2 cards each for 11 through 17
- 1 card each for 18 through 25

Rules:
- Each player is dealt 5 cards facedown.
- The top card of the remaining deck is turned faceup.
- Using addition, subtraction, multiplication, or division, each player tries to combine his 5 cards to obtain the number shown on the faceup card.
- Each card may be used only once.
- Any player who reaches the objective is expected to verify his solution with the other players.
- Players predetermine how to score.

5. things 3 sets of numeral cards 0 through 9

Two to four persons can play the Multifact Game. Shuffle the cards. Give one card to each person. It is placed faceup. Place the rest of the cards facedown in a pile. Each player in turn draws a card. After he draws, he is to tell the product of his card and the one drawn. If the product is correct, he puts one of the cards in a discard pile. If an incorrect product is given, he must keep both cards. The play continues.

The next time the play comes to a person with more than one card faceup, he must tell the product of the card drawn and each of his 2 faceup cards. If both products are correct, he can discard 2 cards. If either of the products is wrong, he keeps all 3 cards.

When the facedown deck has been used, switch partners. Pair the winners for the next game. The number facts that need practice will become painfully obvious. This information will be a source for a new set of flash cards to be used as often as possible.

6. things game board; 25 small cards

Prepare a game board as shown. The factors can be varied to provide the type of practice needed. You may want to write the factors in order to simplify locating the numbers.

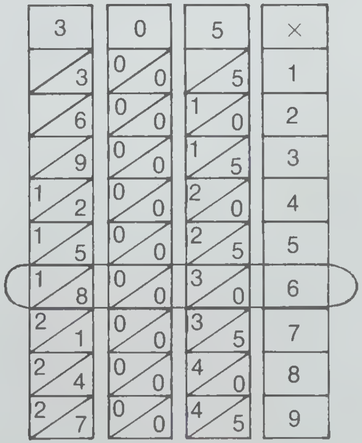
×	40	300	80	600	60
2					
5					
7					
8					
9					

Write one of the products on each small card. These cards (tiles) should be the same size as the empty boxes on the game board.

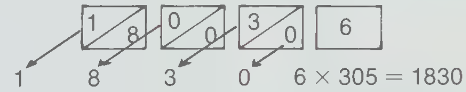
Turn the answer tiles facedown in random order. Each player selects 4 tiles. The first player places an answer tile on any box for which the numeral on the tile is the product. Each succeeding player then attempts to place a tile on the board so that it is a correct product and touches (vertically or horizontally) a tile played previously by another player. If a player does not have a tile he can play, he draws from the facedown tiles until he finds one he can use. The winner is the first player to use all his tiles.

7. things for each pupil: 11 tongue depressors or strips of cardboard

Napier's bones are fun to make and to use. To form the bones, write a numeral for 0 through 9 at the top of ten depressors or strips. Write the multiples of the number written at the top in the boxes below as shown—tens above the diagonal, ones below the diagonal. The last strip is the multiplier bone.

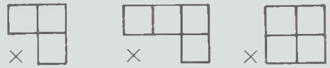


To multiply 305 by 6, line up the multiplier with the bones for 3, 0, and 5 in the appropriate place-value positions. Locate the 6 row on the multiplier bone. Move across the row from right to left, adding along the diagonal lines to find the product.



8. things 2 sets of numeral cards 0 through 9

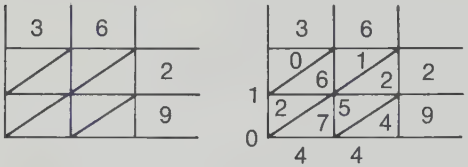
Pupils make their own grids for the appropriate type of practice.



The cards are shuffled and placed in a stack facedown. The dealer draws the number of digits needed to form a problem in the grid and displays them to the group. Each person may arrange the digits as he wishes, then multiply to find the product.

Added challenge: Arrange the cards to find the greatest product; the least product.

9. The lattice method of multiplication is an extension of Napier's bones. Renaming in multiplication occurs automatically. To multiply 29 × 36 make a lattice as shown.



Only the facts are needed to fill in the boxes: 2 × 6, 2 × 3, 9 × 6, and 9 × 3. Tens are recorded above the diagonal, ones below the diagonal. To find the product, add from right to left along the diagonal lines. 29 × 36 = 1044 Renaming in addition may be necessary—as indicated in the circles in the example.

additional learning aids

operation—chapter objectives 1, 2, 3

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: W 4, 7, 8, 9
P 3
Computapes, SRA (1972)
Module 3, Lessons: MD 10, 11, 12
Module 4, Lessons: MD 21, 22
Computational Skills Development Kit, SRA (1969)
Multiplication cards: 1, 2, 3, 4
Cross-Number Puzzles (Whole Numbers), SRA (1966)
Multiplication cards: 1, 2, 3, 4
Diagnosis: an instructional aid—Mathematics Level A, SRA (1973)
Probe: L-3
Skill Modes in Mathematics, SRA (1974)
Level I, Molecule: C
Skill through Patterns, level 4, SRA (1974)
Spirit masters: 21, 22, 23, 24, 28, 29, 30, 38, 40, 46, 49, 50, 51, 52, 53, 59, 64, 67, 71, 72
Visual Approach to Mathematics, level 3, SRA (1967)
Visuals: 15, 16, 17, 18, 20

other learning aids (described on page 144e)
Napier's Rods, Veri-Tech Senior (multiplication books)

6 FRACTIONS

before this chapter the learner has—

1. Named the fractions associated with the marked parts of a region
2. Made a model to illustrate a fraction
3. Compared two fractions with like denominators
4. Identified the numerator and the denominator of a fraction

in chapter 6 the learner is—

1. Mastering naming a fraction associated with a fraction model—number line, region, or set
2. Mastering making a model to illustrate a fraction
3. Mastering ordering a set of fractions with like denominators
4. Ordering a set of unit fractions (numerator of 1)
5. Mastering the addition and subtraction of two fractions with like denominators
6. Mastering the comparison of two fractions with like denominators
7. Mastering the identification of fractions equal to 0 or 1
8. Identifying fractions that are greater than 1

in later chapters the learner will—

1. Rename fractions
2. Rename appropriate fractions as whole or mixed numbers
3. Add or subtract fractions with like denominators and rename answers in simplest form when possible

Notes & Things

The definition of a fraction and the related vocabulary are very difficult to handle if you include all the different meanings for a simple idea such as $\frac{1}{2}$. You have seen the symbol $\frac{1}{2}$ called a numeral (or fractional numeral, common fraction, and so on) that represents a fractional number (or rational number, quotient, and so on). Rather than make life difficult, we're simply going to call numbers such as $\frac{1}{2}$ *fractions*. And we won't make a big fuss about the difference between numbers and numerals.

At this level the numerator of a fraction is a number that tells how many same-size parts of a given region, set, or number line are being considered; the denominator is a number that tells how many same-size parts are in the whole thing; and the fraction (numerator and denominator together) tells *how much* of a region, set, or number line is being considered.
If you prefer to define a fraction or its parts in another way, it's O.K. The chapters have been written in a way that lets you do this. However, you should know that this series does not thoroughly investigate the fraction as a quotient until level 7.

After the concept and notation of fractions are reviewed, the emphasis is placed on getting each pupil to see that fractions have order, just like whole numbers. If the denominators are the same, then the numerator tells the comparative size:

$$\frac{1}{7}\ \frac{2}{7}\ \frac{3}{7}\ \frac{4}{7}\ \frac{5}{7}\ \frac{6}{7}\ \frac{7}{7}\ \ldots$$

If the numerators are the same, then the denominators tell the story:

$$\ldots\ \frac{1}{8}\ \frac{1}{7}\ \frac{1}{6}\ \frac{1}{5}\ \frac{1}{4}\ \frac{1}{3}\ \frac{1}{2}\ \frac{1}{1}$$

Knowledge of the order of fractions is as important as knowledge of the order of whole numbers. It is the basis for the "common sense" that is so frequently lacking in work with fractions.
Estimation also is as important for work with fractions as it is for work with whole numbers.

The adoption of the metric system of measure will eventually bring quite a different emphasis to the study of fractions. Decimal fractions will be much more important and will be studied earlier. But that time is not here yet. The present expectation that pupils will perform all operations with fractions is reinforced by standardized tests and by tradition. Yet everyday use of operations with fractions is limited. This series therefore deliberately controls the set of fractions used in computational-skill development and practice. Don't look for exotic fractions such as $\frac{4}{33}$. You won't find them.

There is very little emphasis on renaming fractions in this chapter. That skill is deferred until the basic concept of fractions is studied.

You may be surprised to see so few real-world pictures used in this chapter. There is a reason. The notion of same-size parts underlies the concept. Unfortunately, youngsters argue about who got the bigger half of the candy bar long before they get to the study of fractions. The region models therefore are drawn with precision to reinforce the notion of same-size parts.

All these things have been done to make your job easier. You'll like this chapter; the youngsters might like it also.

things

spirit master of number lines (optional)

For the extra activities you will want to have these things available:

- spirit master of 3 types of region models
- 3-by-3 array game boards
- 2 rubber jar rings
- sets of colored plastic sticks

goal Think about and explore ideas through a picture clue

page 121 What better way to get started thinking about fractions than with the real thing? Circular regions may be meaningful to some math people but they rarely make much of an impression on children. Every youngster knows that if you have 4 whole things you want to share among 4 people, everyone gets 1 thing, and if you have 1 whole thing to share among 4 people, you can get that job done too. Surely, at least some youngsters wonder why books make such a big deal about fractions.

The most important idea to gain from this photo is the notion that 2 people could share the gooey dessert, or 3 or 4 or 5 and so on. But the more people that share, the smaller the piece each person will get.

The activity that can be extrapolated from this photo is a personal diary. During the next week, how many things can you find that are divided into fractional parts? You might even consider turning it into a competitive situation. And be prepared for some incredible findings. The peer group will have to be the judges as to whether each entry in the diary is or is not valid.

goal Survey—naming a FRACTIONAL PART of a region; adding and subtracting two fractions with like denominators

memo So that the humor of the situation presented can be appreciated, this page should be discussed together and any answers to the questions should be recorded.

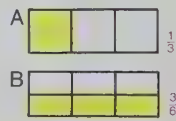
page 122 Pupils who knew the answers to problems 1 through 4 have prior knowledge of the material presented in the chapter. You may want to further check these pupils with the Progress Checks in order to determine if parts of the chapter can be skipped.

The youngsters who have difficulty answering the questions need not worry. Their goal in this chapter will be to develop the necessary skills.

Ziggy and Zap started a club. They called themselves "The Zany Zees." There were only two people in the club: Ziggy and Zap.

They decided to have a party for all club members. They got a cake. They made popcorn balls. They bought a big jug of apple cider.

1. Ziggy was a thinker. He drew two pictures to show how they could cut the cake. He shaded the part he wanted in each picture. Write the fractions that tell how much cake he wanted in A and how much he wanted in B.



2. Zap said no to Ziggy's plans. He wanted $\frac{1}{4}$ of the cake. Draw a picture to show how much Zap wanted for himself. 

3. The friends decided to cut the cake in 8 pieces of the same size.

   a What fraction would represent 1 piece? $\frac{1}{8}$

   b They kept a record of how much was gone as each took a piece. In what order do you think they wrote these fractions?
   $\frac{5}{8}$ $\frac{3}{8}$ $\frac{1}{8}$ $\frac{2}{8}$ $\frac{4}{8}$ $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$

4. Ziggy ate $\frac{5}{9}$ of the popcorn balls.
   Then Zap ate $\frac{2}{9}$ of them.
   How much of the popcorn did they eat? $\frac{7}{9}$

   a Who ate more? Ziggy b How much more? $\frac{3}{9}$

They didn't drink much cider. Neither one of them felt very good.

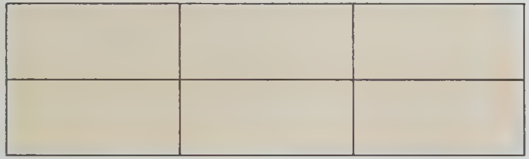
122

But you will feel good if you learn about fractions. Try it, anyway. Make it your goal.

Mr. Smith owns an apartment house. He knows that people sometimes have a hard time sharing things equally. He doesn't like arguments. He works hard to avoid them. Here are some of the things he did to help people get along.

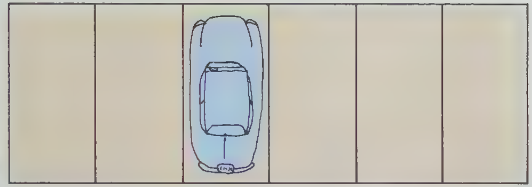
Six families live in the building.
The clothesline is marked into six parts of equal size.

1. The storage space in the basement is divided into six equal spaces.

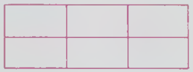


Could the space be divided into six equal parts in another way?
Yes

2. The parking lot is divided into six equal spaces.



Could the space be divided into six equal parts in another way?
Maybe. It depends on the size of the lot. Two cars could park parallel.



3. There is only one washing machine. How could Mr. Smith divide this into six equal parts?
Six equal time periods could be established.

4. One family has two boys. They always argue. Mr. Smith can't help that. One day he heard one boy say to his brother, "You got the biggest half of the candy bar." Is it possible to get a half that is bigger than another half? No

123

STOP Extension activity to follow problem 4.

things 1 large apple, 1 small apple

Cut each apple in half. Have the pupil name each of the four pieces with a fraction.
Which half do you want? Why?

lesson Pages 123, 124, 125

goal Examining fractions as they are used in everyday situations

memo Fractions seem abstract, yet they are a very real part of our everyday lives. You'll want to discuss this page. Focus on how fractions help us to share things equally.

page 123 Experiment with problem 1. How many different ways could the space be divided into six equal parts? Which way yields the most convenient storage spaces?

A twist for problem 2. Assume the lot and car are drawn to scale. Can the lot be divided in another way and contain room for more than 6 cars? What about compact cars?

Take a critical look at problem 4. Think about a cantaloupe and a watermelon, each cut in half. Is $\frac{1}{2}$ of the cantaloupe equal to $\frac{1}{2}$ of the watermelon? Pupils should generalize that if they are working with the fraction as a number, then $\frac{1}{2}$ does equal $\frac{1}{2}$. But when a fraction describes part of a real thing, then they must compare parts of the same thing before they can say $\frac{1}{2}$ does equal $\frac{1}{2}$.

goal Naming fractional parts of a region

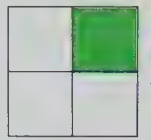
page 124 Problems 1 through 4 can be completed independently, although you may want to share ideas. The functional meaning of a numerator and of a denominator will be examined on several pages. It is up to you how much emphasis you want to place on this extremely informal definition at this point. Please be sure to reinforce the idea that all parts must be the same size to be named by a fraction.

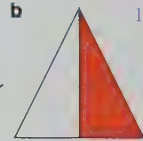
Sue baked an apple pie for dessert. She cut it into 5 equal pieces. Sue really likes apple pie.

She ate 2 pieces. What fractional part of the pie did she eat? $\frac{2}{5}$

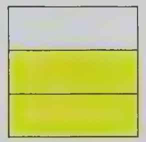
$\frac{2}{5}$ 2 ← This number is called the *numerator*. The number of pieces Sue ate.
5 ← This number is the *denominator*. The number of pieces in the whole pie.

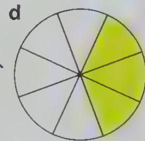
1. Answer the questions for each region.

a  1, 4, $\frac{1}{4}$

b  1, 2, $\frac{1}{2}$

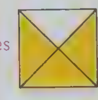
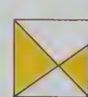
How many parts are shaded?
How many parts in all?
Write what fractional part is shaded.

c  2, 3, $\frac{2}{3}$

d  3, 8, $\frac{3}{8}$

2. Draw and shade regions to show each fraction. Accept other appropriate models.

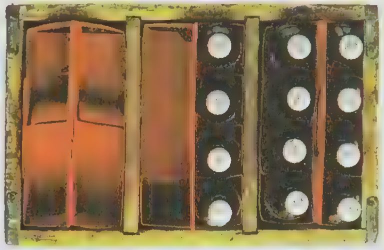
a $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{1}{8}$ **d** $\frac{2}{3}$

3. **a** Does this show $\frac{3}{4}$? Yes  **b** Does this show $\frac{3}{4}$? No 

4. Sue's kid brother ate none of the 5 pieces. He ate $\frac{0}{5}$ of the pie. Does this make sense? Yes

124

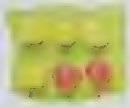
1. A case has 24 slots for bottles.
Bottles are in 12 of the slots.
What fraction of the slots has bottles? $\frac{12}{24}$

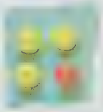


If the case were full, it would represent 1 whole case—the total. The fraction $\frac{12}{24}$ tells what part of the case is filled.

12 ← The numerator tells how many slots are filled.
24 ← The denominator tells the total number of slots.

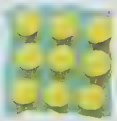
2. Answer the questions for each set of candles.

a  2, 6, $\frac{2}{6}$

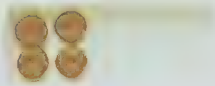
b  1, 4, $\frac{1}{4}$

How many red?
How many in all?
What fraction of the set is red?

c  3, 8, $\frac{3}{8}$

d  0, 9, $\frac{0}{9}$

3. A box can hold 12 doughnuts. Some of the doughnuts have been eaten. 4 doughnuts are left in the box. What fractional part remains? $\frac{4}{12}$



4. Draw doughnuts in boxes to show each fraction.

a $\frac{1}{12}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** $\frac{1}{3}$

goal Naming fractional parts of a set

memo Problem 1 should be a group discussion. Praise students who give the correct equivalent fraction, $\frac{1}{2}$. But please don't stop to teach this concept now.

page 125 The question box in the center of problem 2 is designed to help pupils write the correct fraction by first identifying the numerator and then the denominator before the fraction itself is written.

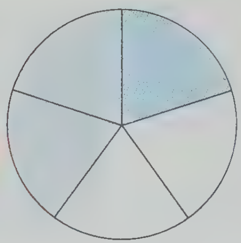
lesson Page 126

goal Comparing two fractions with like denominators

page 126 Be careful. Your less capable students may need additional guidance. Shower them with praise if they are managing independently.

Encourage everyone to use his imagination with problem 2. The region drawn need not be square or circular or rectangular, but each of the parts of the region must be the same size.

TIME TO PRACTICE



1. a How many same-size parts? 5
   b Each part is what fraction of the whole region? $\frac{1}{5}$
   c What fractional part has stripes? $\frac{1}{5}$
   d What fractional part is dotted? $\frac{2}{5}$
   e Which is more, $\frac{2}{5}$ or $\frac{1}{5}$ of the region?

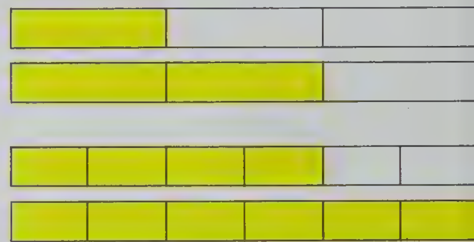
2. Draw a region that shows fourths. Shade $\frac{1}{4}$.

   a What fraction of the region is not shaded? $\frac{3}{4}$
   b Which is less, $\frac{1}{4}$ or $\frac{3}{4}$ of the region?

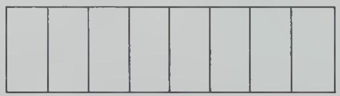
3. a What fraction of region A is shaded? $\frac{1}{3}$
   b What fraction of region B is shaded? $\frac{2}{3}$
   c Which is more, $\frac{1}{3}$ or $\frac{2}{3}$ of a region?

4. a What fraction of region C is shaded? $\frac{4}{6}$
   b What fraction of region D is shaded? $\frac{6}{6}$
   c Which is less, $\frac{4}{6}$ or $\frac{6}{6}$ of a region?

A
B
C
D



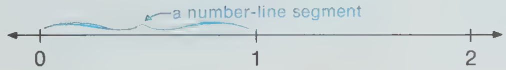
5. Which fraction of a region is more?
   a $\frac{5}{8}$ or $\frac{3}{8}$
   b $\frac{0}{8}$ or $\frac{6}{8}$
   c $\frac{8}{8}$ or $\frac{2}{8}$



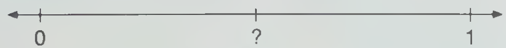
6. Which fraction of a region is less?
   a $\frac{6}{10}$ or $\frac{4}{10}$
   b $\frac{1}{10}$ or $\frac{0}{10}$
   c $\frac{9}{10}$ or $\frac{10}{10}$

126

A piece of the number line is called a number-line segment.



1. Use the number-line segment from 0 to 1. The segment on the right is divided into 2 equal parts.

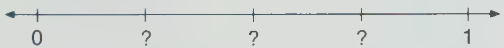


   a What fraction of the segment is each part? $\frac{1}{2}$
   b What fraction can name this point? $\frac{1}{2}$

2. Now the segment is divided into 3 equal parts.

   a What fraction is each part? $\frac{1}{3}$
   b What fraction can name this point? $\frac{1}{3}$
   c What fraction can name this point? $\frac{2}{3}$

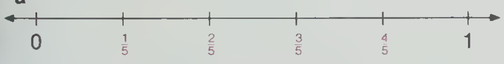
3. Now the segment is divided into 4 equal parts.



   a What fraction is each part? $\frac{1}{4}$
   b What fraction names this point? $\frac{1}{4}$
   c What fraction names this point? $\frac{2}{4}$
   d What fraction names this point? $\frac{3}{4}$

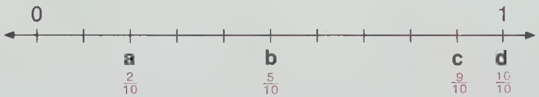
4. Copy each of these number lines and name each point.

a



b

5. The length of the unit from 0 to 1 will be longer on the next number-line segment. This length is divided into 10 equal parts. Write the fraction that names each of the points marked with a letter.



127

lesson Page 127

goal Examining fractions on a number line

memo This is an important page for future success.

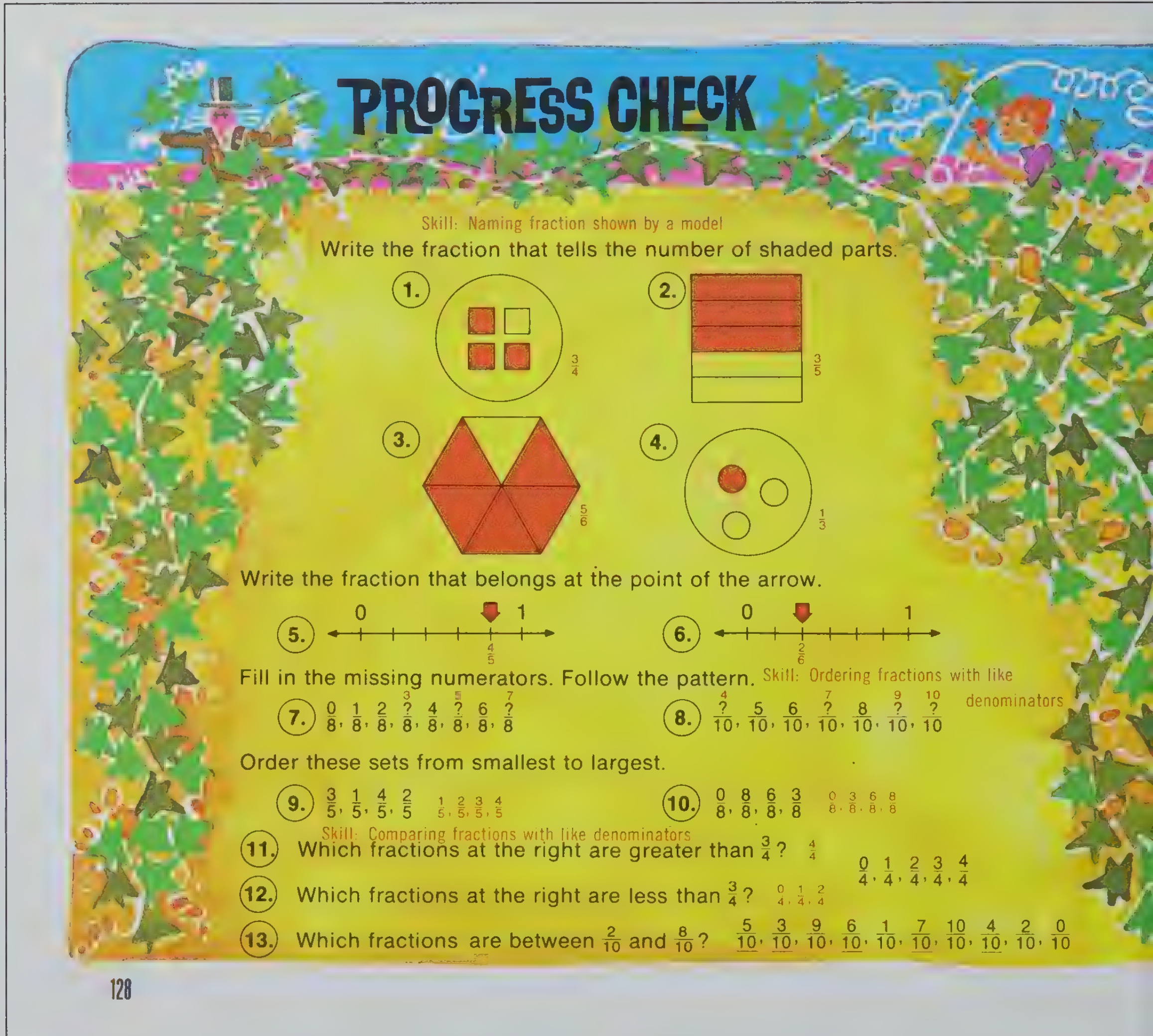
page 127 Fractions on a number line is a difficult concept for many youngsters. Nice and easy. Notice that the questions make the pupil first think what fraction of the segment each part is and then name the point that indicates the end of each part.

lesson Page 128

goal **Progress Check**—naming fractional parts of a region, a set, a number line; ordering and comparing fractions with like denominators

page 128 Strictly independent work. Skills are identified on the answer key to help you pinpoint the learner's strengths and weaknesses.

Examine carefully the errors on problems 1 through 6. Three different fraction models are used—region, set, number line. Is the learner having problems with only one model?



PROGRESS CHECK

Skill: Naming fraction shown by a model

Write the fraction that tells the number of shaded parts.

1. $\frac{3}{4}$
2. $\frac{3}{5}$
3. $\frac{5}{6}$
4. $\frac{1}{3}$

Write the fraction that belongs at the point of the arrow.

5. 0 1 $\frac{4}{5}$
6. 0 1 $\frac{2}{6}$

Fill in the missing numerators. Follow the pattern. Skill: Ordering fractions with like denominators

7. $\frac{0}{8}, \frac{1}{8}, \frac{2}{8}, \frac{?}{8}, \frac{4}{8}, \frac{?}{8}, \frac{6}{8}, \frac{?}{8}$ — 3, 5, 7
8. $\frac{?}{10}, \frac{5}{10}, \frac{6}{10}, \frac{?}{10}, \frac{8}{10}, \frac{?}{10}, \frac{?}{10}$ — 4, 7, 9, 10

Order these sets from smallest to largest.

9. $\frac{3}{5}, \frac{1}{5}, \frac{4}{5}, \frac{2}{5}$ — $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$
10. $\frac{0}{8}, \frac{8}{8}, \frac{6}{8}, \frac{3}{8}$ — $\frac{0}{8}, \frac{3}{8}, \frac{6}{8}, \frac{8}{8}$

Skill: Comparing fractions with like denominators

11. Which fractions at the right are greater than $\frac{3}{4}$? $\frac{4}{4}$
12. Which fractions at the right are less than $\frac{3}{4}$? $\frac{0}{4}, \frac{1}{4}, \frac{2}{4}$

$\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$

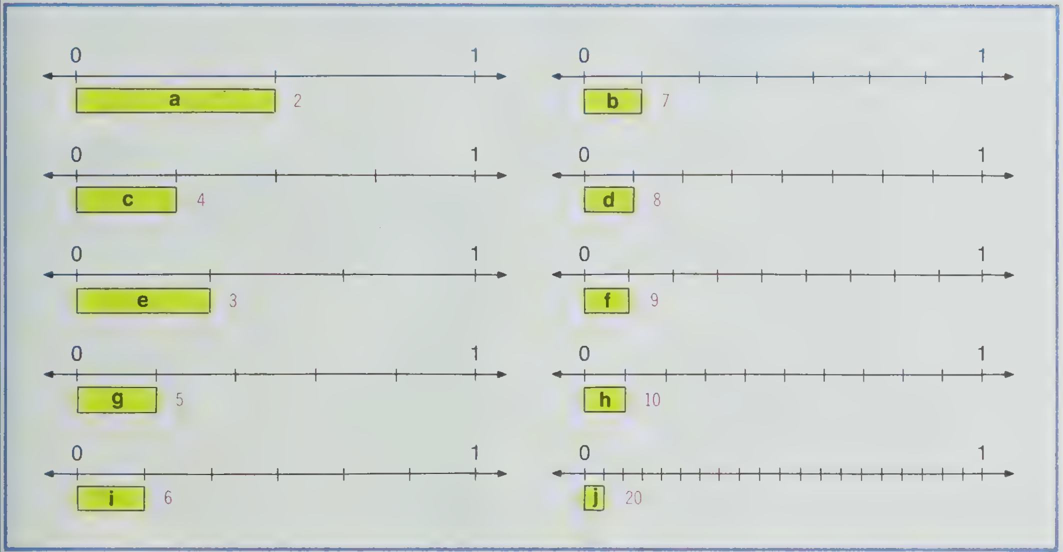
13. Which fractions are between $\frac{2}{10}$ and $\frac{8}{10}$? $\frac{5}{10}, \frac{3}{10}, \frac{9}{10}, \frac{6}{10}, \frac{1}{10}, \frac{7}{10}, \frac{10}{10}, \frac{4}{10}, \frac{2}{10}, \frac{0}{10}$

128

See activity 1, page 144a.

See activity 2, page 144a.

1. One unit of length can be equally divided in many ways. Look at the strip below each number-line segment. Write how many strips it would take to have 1 whole unit of length.



2. Which is more?

a $\frac{1}{2}$ unit or $\frac{2}{2}$ unit   b $\frac{4}{7}$ unit or $\frac{3}{7}$ unit   c $\frac{1}{4}$ unit or $\frac{3}{4}$ unit

3. Can 1 be renamed as a fraction? Yes
Use the number lines above for help.
Name at least 6 fraction names for 1. $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \ldots$

4. Can 0 be renamed as a fraction? Yes **Think!**
How many $\frac{1}{2}$s from 0 to 0? 0

lesson Pages 129, 130

goal Renaming 1 and 0 as fractions

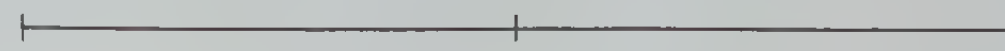
page 129 The development on the page should lead most students to the answers for problems 3 and 4. You may need to help less capable students by asking questions. *What fraction names the length of strip* **a**? ($\frac{1}{2}$) *How many strips would it take to have 1 whole unit of length?* (Two strips of length $\frac{1}{2}$) Continue until they realize that both numerator and denominator are the same number whenever 1 is renamed as a fraction.

goal Comparing two fractions with like numerators or like denominators

page 130 Problems 1 through 5 reinforce the development on page 129. With pupils who are struggling, you might try real licorice sticks. Watch out for tummy aches! The models are a ready reference for problem 6.

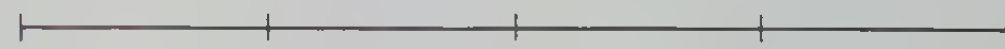
If there is real trouble, copy each of the number lines on the plastic you use for the overhead projector. Ask a child to label where $\frac{1}{2}$ ends and where $\frac{2}{2}$ ends. Label the next where $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$ end, and so on, until each of the five lines are marked. Then work only with the two lines needed to determine any of the comparisons listed for problem 6.

1. Pretend this is a licorice stick.
   - **a** How many parts? 2
   - **b** How much is one part? $\frac{1}{2}$



2. **a** Now how many parts? 3
   - **b** How much is one part? $\frac{1}{3}$

3. **a** Now how many parts? 4
   - **b** How much is one part? $\frac{1}{4}$



4. **a** Now how many parts? 6
   - **b** How much is one part? $\frac{1}{6}$

5. **a** Now how many parts? 8
   - **b** How much is one part? $\frac{1}{8}$

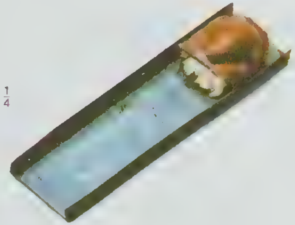
6. Pretend you really like licorice.
   There is 1 stick and no more.
   Name the amount you'd rather have.

a $\frac{1}{2}$ or $\frac{1}{3}$	**b** $\frac{1}{3}$ or $\frac{1}{4}$	**c** $\frac{1}{4}$ or $\frac{1}{6}$	**d** $\frac{1}{6}$ or $\frac{1}{8}$
e $\frac{1}{3}$ or $\frac{2}{3}$	**f** $\frac{1}{4}$ or $\frac{2}{4}$	**g** $\frac{1}{6}$ or $\frac{2}{6}$	**h** $\frac{1}{8}$ or $\frac{2}{8}$
i $\frac{2}{2}$ or $\frac{2}{3}$	**j** $\frac{2}{3}$ or $\frac{2}{4}$	**k** $\frac{2}{4}$ or $\frac{2}{6}$	**l** $\frac{2}{6}$ or $\frac{2}{8}$

LOOK OUT! Use the pictures above to help.

m $\frac{2}{3}$ or $\frac{3}{4}$	**n** $\frac{1}{2}$ or $\frac{2}{6}$	**o** $\frac{2}{4}$ or $\frac{3}{8}$	**p** $\frac{5}{6}$ or $\frac{6}{8}$

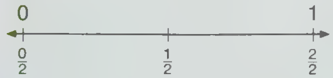
Dave broke a big candy bar into 4 equal pieces. On Tuesday he ate 1 piece. On Wednesday he ate 2 pieces.



- **a** What fraction of the candy bar did he eat on Tuesday? $\frac{1}{4}$
- **b** What fraction did he eat on Wednesday? $\frac{2}{4}$
- **c** What fraction did he eat over the two days? $\frac{3}{4}$
- **d** The math sentence $\frac{1}{4} + \frac{2}{4} = \underline{?}$ fits this story. $\frac{3}{4}$

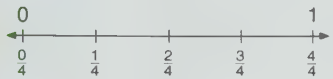
Number lines can help picture the next problems. You find the answers.

2. He used $\frac{1}{2}$ the string for one box. He used another $\frac{1}{2}$ for the other box. How much string did he use in all?



$\frac{1}{2} + \frac{1}{2} = \underline{?}$ $\frac{2}{2}$ (or 1 whole piece)

3. She used $\frac{1}{4}$ of the ribbon for her sister. She used $\frac{2}{4}$ of the ribbon for herself. How much ribbon was used?



$\frac{1}{4} + \frac{2}{4} = \underline{?}$ $\frac{3}{4}$

4. They ran $\frac{1}{3}$ of the distance. They walked $\frac{1}{3}$ of the distance. And then they rested. How far had they gone before they rested?

$\frac{1}{3} + \frac{1}{3} = \underline{?}$ [illegible]/3

131

lesson Pages 131, 132, 133, 134, 135

goal Adding two fractions with like denominators, using a number-line model

page 131 Independent learners are on their own. The concepts and questions are not difficult. You will have to help your poor readers. Focus on how fractions are used in the real world.

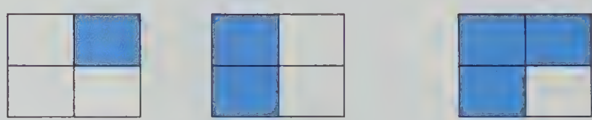
It would be great if each pupil could have his own sheet of number lines to mark to find the answers.

goal Adding two fractions with like denominators, using a region model

memo There is always a problem in showing any kind of diagram or model to be used with fractions. We adults can determine that  is $\frac{1}{3}$ of a circular region. We have had lots of experience. Children have not had our experience. They need to see the model of the fraction as it relates to the whole region.  is 1 shaded part out of 3 same-size parts in all. That model does show $\frac{1}{3}$. If you want to appreciate the child's problem, guess what fractional part of a rectangular region the next diagram shows.  Is it $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{10}$, $\frac{1}{100}$? You can't know unless it is related to the whole region.

page 132 This is an important idea. Each model can be tricky. The first region shows the first addend, the second region shows the second addend, and the third region shows the sum. This is **not** obvious to children. The first two regions are not being put together to make one. Focus on the shaded parts of these two regions. It is the shaded parts that are put together and shown in the third region.

Consider doing the entire page in a discussion group and then continuing right on to page 133.



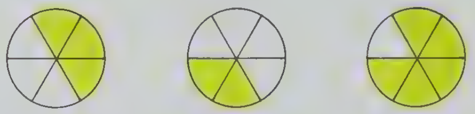
$\frac{1}{4} \quad + \quad \frac{2}{4} \quad = \quad \frac{3}{4}$

1. In the diagram above, look at the denominator of each fraction. What does it tell you? Look at the numerator in the sum. Why is it 3? The number of parts in all. Three parts are shaded in all.

2. Write the addition sentence for each of these diagrams.

a  $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$

b  $\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$

c  $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$

3. Find the sums.

a $\frac{3}{10}+\frac{4}{10}=$? $\frac{7}{10}$	**b** $\frac{1}{8}+\frac{5}{8}=$? [illegible]	**c** $\frac{2}{5}+\frac{2}{5}=$? $\frac{4}{5}$	**d** $\frac{5}{8}+\frac{2}{8}=$? $\frac{7}{8}$
e $\frac{2}{7}+\frac{3}{7}=$? $\frac{5}{7}$	**f** $\frac{3}{8}+\frac{4}{8}=$? $\frac{7}{8}$	**g** $\frac{6}{11}+\frac{3}{11}=$? $\frac{9}{11}$	**h** $\frac{1}{5}+\frac{3}{5}=$? $\frac{4}{5}$

ou have been doing a good job

as anyone found a very fast way to add fractions?

his region has been separated into 10 parts.

$\frac{1}{10}$ $\frac{2}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ $\frac{5}{10}$ $\frac{6}{10}$ $\frac{7}{10}$ $\frac{8}{10}$ $\frac{9}{10}$ $\frac{10}{10}$

$$\frac{4}{10} + \frac{3}{10} = \frac{?}{10} \quad 7$$

me out to review

/hat do the parts of a fraction stand for?

$\frac{4}{10}$ ← The numerator tells the number of parts we are looking at.
← The denominator tells the number of parts in all.

o back to the addition problem above.

$\frac{4}{10} + \frac{3}{10}$ ← We are adding 4 parts and 3 parts.
← There are 10 parts in all.

ould we think about addition like this: $\frac{4}{10} + \frac{3}{10} = \frac{4+3}{10} = \frac{7}{10}$? Yes

the denominators of both fractions show the same number
f equal parts, then add the numerators.

. $\frac{2}{3} + \frac{1}{3} = \frac{2+1}{3} = \frac{?}{3}$ 3
2. $\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{?}{4}$ 3
3. $\frac{6}{12} + \frac{2}{12} = \frac{6+2}{12} = \frac{?}{12}$ 8
4. $\frac{5}{10} + \frac{2}{10} = \frac{5+2}{10} = \frac{?}{10}$ 7

. $\frac{1}{5} + \frac{3}{5} = \frac{?+?}{5} = \frac{?}{5}$ 1 3 4
6. $\frac{3}{7} + \frac{2}{7} = \frac{?+?}{7} = \frac{?}{7}$ 3 2 5
7. $\frac{5}{9} + \frac{2}{9} = \frac{?+?}{9} = \frac{?}{9}$ 5 2 7
8. $\frac{1}{8} + \frac{7}{8} = \frac{?+?}{8} = \frac{?}{8}$ 1 7 8

133

goal Development of the algorithm for adding fractional numbers

page 133 Although there is a risk of establishing a rote operational skill, it is necessary to focus the learner's attention on this operation and on the role of the numerator and denominator. Emphasize that since the size of each part does not change when joined, the denominator remains the same. It's the number of parts (numerator) that we're interested in. These can be added.

The development on the page will be sufficient for most students. You'll want to work with those who are struggling.

goal Practice in adding two fractions with like denominators; more on understanding fraction names for 1 and 0

page 134 Zero as a numerator causes little or no difficulty—zero parts shaded. Zero as a denominator is a little more involved. We examine only one case for the present: $\frac{0}{0}$. 0 parts of 0 pies is nonsense! And that's enough for the learner to handle right now. If some bright one asks about $\frac{4}{0}$ as a fraction, there seems to be only one appropriate answer for now: $\frac{4}{0}$ says that we have 0 parts in all, so we can't very well talk about having 4 of 0 parts. Inverse relationships have not as yet been sufficiently developed for the pupils to understand that $\frac{4}{0}$ means $\underline{?} \times 0 = 4$. Such a sentence is impossible to solve.

Practice addition.

1. $\frac{2}{7} + \frac{4}{7} = \frac{2+4}{7} = \underline{?}$ $\frac{6}{7}$
2. $\frac{4}{9} + \frac{4}{9} = \frac{4+4}{9} = \underline{?}$
3. $\frac{1}{6} + \frac{4}{6} = \frac{1+4}{6} = \underline{?}$ $\frac{5}{6}$
4. $\frac{3}{8} + \frac{2}{8} = \frac{3+2}{8} = \underline{?}$
5. $\frac{1}{4} + \frac{3}{4} = \underline{?}$ $\frac{4}{4}$
6. $\frac{2}{10} + \frac{7}{10} = \underline{?}$ $\frac{9}{10}$
7. $\frac{1}{8} + \frac{0}{8} = \underline{?}$ $\frac{1}{8}$
8. $\frac{0}{4} + \frac{0}{4} = \underline{?}$ $\frac{0}{4}$

If you were really hungry, would you rather have—

$\frac{2}{2}$ of a hamburger or $\frac{3}{3}$ of a hamburger? Either on

$\frac{5}{5}$ of a bag of candy or $\frac{6}{6}$ of a bag of candy? Eith

$\frac{4}{4}$ of a pie or $\frac{0}{0}$ of a pie?

If you have 0 pies cut into 0 parts, could I have one of these parts?

THINK $\cancel{\frac{0}{0}}$ NO!

THAT'S NONSENSE!

If you have a pie cut into 4 parts, would you give me 0 parts? Sure you would.

$\frac{0}{4}$ is O.K.

134

Add.

a $\frac{1}{5} + \frac{1}{5}$ $\frac{2}{5}$ b $\frac{2}{4} + \frac{1}{4}$ $\frac{3}{4}$ c $\frac{3}{8} + \frac{3}{8}$ $\frac{6}{8}$ d $\frac{5}{10} + \frac{4}{10}$ $\frac{9}{10}$ e $\frac{3}{6} + \frac{3}{6}$ $\frac{6}{6}$ f $\frac{1}{3} + \frac{2}{3}$ $\frac{3}{3}$

et ready to shift gears. You will still be thinking
bout fractions. But watch out! Draw pictures to help
u find answers.

There were 5 out of 6 pieces of pie left in the pan. That's $\frac{5}{6}$ of the pie. Three more pieces were eaten. That's $\frac{3}{6}$ of the pie. How much of the pie was left in the pan? $\frac{2}{6}$

3. Alan has $\frac{3}{4}$-inch nails. But he needed some $\frac{1}{4}$-inch shorter. What length of nail did he need? $\frac{2}{4}$ (or $\frac{1}{2}$)

The race announcer said, "The horses are entering the first-quarter turn." If the horses had run $\frac{1}{4}$ of the race, how much of the race was left to run? $\frac{3}{4}$

5. Dave had a half dollar. He spent a quarter dollar. How much of a dollar did he have left? $\frac{1}{4}$

There is a sale. The sign reads: If you bought something at this sale, how much of the original price would you pay? $\frac{2}{3}$

The recipe for a cake called for $\frac{2}{4}$ cup of sugar. $\frac{1}{4}$ cup was to be saved for the topping. How much was to be used in the cake? $\frac{1}{4}$

goal Practice in adding two fractions with like denominators; exploration in subtracting fractions, using real-world problem situations

page 135 Everyone should complete exercise 1 independently. The word problems involve very real uses of fractions and require the learner to think subtraction to find the answers. You decide which of your pupils can handle these independently.

lesson Pages 136, 137

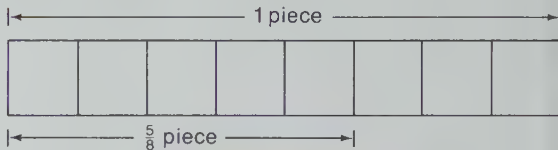
goal Development of subtraction of fractional numbers

memo You'll want to make sure that everyone gets off to a good start. Use this as a discussion page.

page 136 The examples get things started. If more visualization is needed, use felt pieces. You may want to make up some word problems to help establish a thinking pattern for problems 1 through 8: Sammy had $\frac{3}{3}$ of a candy bar. I took $\frac{2}{3}$. How much does Sammy have left? You had $\frac{3}{5}$ of a pie. You gave me $\frac{1}{5}$. How much do you have left? Elizabeth had $\frac{2}{2}$ of a pack of gum. She gave you $\frac{1}{2}$. How much does she have left? And so on. Fractions can be made very real. Sharing good food is something nearly everyone responds to.

The operation of subtraction with fractions really isn't hard. This is the kind of problem that got you started thinking subtraction.

Janice entered Max in the dog show. Max isn't very handsome. Janice put a bow around his neck. She put another bow on his tail. She had a piece of polka dot ribbon. She used $\frac{5}{8}$ of it for the bow around his neck. How much ribbon did she have left for the tail?



You can show subtraction with other models too.

There were $\frac{5}{5}$, but $\frac{1}{5}$ is gone. Now there are $\frac{4}{5}$.
Take away $\frac{1}{5}$ from $\frac{4}{5}$. How much remains?

$\frac{4}{5} - \frac{1}{5} = \underline{?}$ $\frac{3}{5}$

Your turn. Subtract. Draw models if you need to.

1. $\frac{3}{3} - \frac{2}{3}$ $\frac{1}{3}$ **2.** $\frac{3}{5} - \frac{1}{5}$ $\frac{2}{5}$ **3.** $\frac{2}{2} - \frac{1}{2}$ $\frac{1}{2}$ **4.** $\frac{3}{6} - \frac{2}{6}$ $\frac{1}{6}$

5. $\frac{4}{5} - \frac{2}{5}$ $\frac{2}{5}$ **6.** $\frac{6}{8} - \frac{3}{8}$ $\frac{3}{8}$ **7.** $\frac{1}{4} - \frac{1}{4}$ $\frac{0}{4}$ **8.** $\frac{3}{4} - \frac{1}{4}$ $\frac{2}{4}$

136

There were $\frac{6}{6}$, but $\frac{1}{6}$ is gone.
Now there are $\frac{5}{6}$.
Take $\frac{3}{6}$ away from $\frac{5}{6}$.
How much remains? $\frac{5}{6} - \frac{3}{6} = \frac{?}{6}$

Look at that problem again.
Do the denominators of both fractions show the same number of equal parts?

Can you think about subtraction like this: $\frac{5}{6} - \frac{3}{6} = \frac{5-3}{6} = \frac{2}{6}$?
Sure you can. Go to it. Show how good you are with subtraction.

1. $\frac{2}{3} - \frac{1}{3} = \frac{2-1}{3} = \frac{?}{3}$ 1
2. $\frac{7}{10} - \frac{3}{10} = \frac{7-3}{10} = \frac{?}{10}$ 4
3. $\frac{1}{2} - \frac{0}{2} = \frac{1-0}{2} = \frac{?}{2}$ 1
4. $\frac{5}{8} - \frac{5}{8} = \frac{5-5}{8} = \frac{?}{8}$ 0

Even the hard ones aren't hard this way.

5. $\frac{10}{10} - \frac{1}{10} = \frac{10-1}{10} = \underline{?}$ $\frac{9}{10}$
6. $\frac{7}{7} - \frac{2}{7} = \frac{7-2}{7} = \underline{?}$ $\frac{5}{7}$
7. $\frac{8}{9} - \frac{4}{9} = \underline{?}$ $\frac{4}{9}$
8. $\frac{5}{6} - \frac{4}{6} = \underline{?}$ $\frac{1}{6}$

You are doing a good job.
Try your skill with some word problems.

9. I had $\frac{3}{4}$ of the candy. I gave you $\frac{1}{4}$. How much do I have left? $\frac{2}{4}$

10. He had $\frac{1}{2}$ of the job done. His brother did $\frac{1}{2}$ of the job. How much of the job was left to do? $\frac{0}{2}$

11. You had 1 whole candy bar. You gave me $\frac{1}{2}$ of it. How much do you have left? $\frac{1}{2}$

137

goal Development of the algorithm for subtracting fractional numbers

memo Here is that art problem again. Words can help tell about the fraction that is there to start with. The diagram must also help. The fraction $\frac{5}{6}$ is shown. That's the fraction from which $\frac{3}{6}$ must be subtracted. The $\frac{3}{6}$ is shown being taken away.

page 137 Parallel this development to that of the addition algorithm. The size of the pieces doesn't change—so leave the denominator alone. Concentrate on the numerators. That's where the action takes place.

You will want to work with the pupils who are struggling.

lesson Pages 138, 139, 140

goal Practice in adding and subtracting two fractions with like denominators

memo If each pupil can accept the generalization at the top of the page, you're home free. Establishing these skills now is important to future work.

page 138 As pupils progress through the set of problems, they are required to do more independent thinking. If necessary, give additional help where the guide steps no longer appear. Watch for those youngsters who understand the concepts but are careless with the operation signs. Be selective in making assignments. Every child does not have to do every problem.

If the denominators show the same number of equal— size parts, then we focus on the numerator whether you add or subtract.

$$\frac{5}{12}+\frac{2}{12}=\frac{5+2}{12}=\frac{7}{12}$$

$$\frac{7}{12}-\frac{2}{12}=\frac{7-2}{12}=\frac{5}{12}$$

Practice addition on this side.

1. $\frac{1}{6}+\frac{3}{6}=\frac{1+3}{6}=\frac{?}{6}$ 4
3. $\frac{2}{9}+\frac{7}{9}=\frac{2+7}{9}=\frac{?}{9}$ 9
5. $\frac{7}{10}+\frac{2}{10}=\frac{\overset{7}{?}+\overset{2}{?}}{10}=\frac{?}{10}$ 9
7. $\frac{3}{5}+\frac{1}{5}=\frac{\overset{3}{?}+\overset{1}{?}}{5}=\frac{?}{5}$ 4
9. $\frac{4}{7}+\frac{2}{7}=\frac{\overset{4}{?}+\overset{2}{?}}{7}=\underline{?}$ $\frac{6}{7}$
11. $\frac{1}{8}+\frac{6}{8}=\underline{?}$ $\frac{7}{8}$
13. $\frac{3}{10}+\frac{4}{10}=\underline{?}$ $\frac{7}{10}$
15. $\frac{4}{6}+\frac{1}{6}=\underline{?}$ $\frac{5}{6}$

Practice subtraction on this side.

2. $\frac{6}{7}-\frac{2}{7}=\frac{6-2}{7}=\frac{?}{7}$ 4
4. $\frac{7}{10}-\frac{4}{10}=\frac{7-4}{10}=\frac{?}{10}$ 3
6. $\frac{6}{8}-\frac{1}{8}=\frac{\overset{6}{?}-\overset{1}{?}}{8}=\frac{?}{8}$ 5
8. $\frac{7}{8}-\frac{6}{8}=\frac{\overset{7}{?}-\overset{6}{?}}{8}=\frac{?}{8}$ 1
10. $\frac{5}{6}-\frac{1}{6}=\frac{\overset{5}{?}-\overset{1}{?}}{6}=\underline{?}$ $\frac{4}{6}$
12. $\frac{9}{10}-\frac{2}{10}=\underline{?}$ $\frac{7}{10}$
14. $\frac{4}{6}-\frac{3}{6}=\underline{?}$ $\frac{1}{6}$
16. $\frac{4}{5}-\frac{1}{5}=\underline{?}$ $\frac{3}{5}$

138

PRACTICE

Compute. Be careful! There is practice in addition and subtraction.

1. $\frac{1}{4}+\frac{2}{4}$ $\frac{3}{4}$	2. $\frac{5}{6}-\frac{2}{6}$ $\frac{3}{6}$	3. $\frac{5}{6}-\frac{1}{6}$ $\frac{4}{6}$
4. $\frac{1}{5}+\frac{1}{5}$ $\frac{2}{5}$	5. $\frac{4}{6}+\frac{2}{6}$ $\frac{6}{6}$	6. $\frac{2}{8}+\frac{2}{8}$ $\frac{4}{8}$
7. $\frac{11}{12}-\frac{9}{12}$ $\frac{2}{12}$	8. $\frac{4}{9}+\frac{4}{9}$ $\frac{8}{9}$	9. $\frac{0}{5}+\frac{3}{5}$ $\frac{3}{5}$
10. $\frac{3}{8}-\frac{3}{8}$ $\frac{0}{8}$	11. $\frac{4}{5}-\frac{3}{5}$ $\frac{1}{5}$	12. $\frac{3}{4}+\frac{0}{4}$ $\frac{3}{4}$
13. $\frac{5}{8}-\frac{4}{8}$ $\frac{1}{8}$	14. $\frac{2}{10}+\frac{4}{10}$ $\frac{6}{10}$	15. $\frac{1}{12}+\frac{1}{12}$ $\frac{2}{12}$
16. $\frac{7}{9}-\frac{5}{9}$ $\frac{2}{9}$	*17. $\frac{58}{100}-\frac{31}{100}$ $\frac{27}{100}$	18. *$\frac{295}{314}-\frac{231}{314}$ $\frac{64}{314}$

19. Bill had sold only $\frac{1}{4}$ of his book of tickets. How much of the book did he have left to sell? $\frac{3}{4}$

20. Tim sold $\frac{3}{4}$ of his book of tickets. How much more of a book had Tim sold than Bill? $\frac{2}{4}$
How much of a book did Tim have left to sell? $\frac{1}{4}$

139

goal Practice in adding and subtracting two fractions with like denominators

page 139 Be careful! Mixed practice—**watch those signs.** Recopying problems 1 through 16 is tedious. Answers only are sufficient. Problems 17 and 18 are marked for roadrunners or group attack. You decide how to handle the word problems.

goal **Progress Check** – adding and subtracting two fractions with like denominators

page 140 Assign problems 1 through 6 to check the learner's ability. Directions are stated on the pupil page for set 1 and set 2. Check whether the learner was just careless with addition or subtraction or whether he needs individual attention. This may be the time for a peer tutor.

PROGRESS CHECK

Skill: Adding fractions

Add.

1. $\frac{3}{5}+\frac{1}{5}$ $\frac{4}{5}$ 2. $\frac{1}{4}+\frac{3}{4}$ $\frac{4}{4}$ 3. $\frac{3}{9}+\frac{6}{9}$ $\frac{9}{9}$

Skill: Subtracting fractions

Subtract.

4. $\frac{3}{4}-\frac{1}{4}$ $\frac{2}{4}$ 5. $\frac{4}{5}-\frac{3}{5}$ $\frac{1}{5}$ 6. $\frac{8}{8}-\frac{6}{8}$ $\frac{2}{8}$

If you missed one addition problem, do set 1 below.
If you missed one subtraction problem, do set 2.

SET 1

1. $\frac{1}{3}+\frac{1}{3}$ $\frac{2}{3}$ 2. $\frac{2}{5}+\frac{2}{5}$ $\frac{4}{5}$ 3. $\frac{5}{9}+\frac{2}{9}$ $\frac{7}{9}$ 4. $\frac{4}{10}+\frac{5}{10}$ $\frac{9}{10}$ 5. $\frac{3}{6}+\frac{2}{6}$ $\frac{5}{6}$

6. $\frac{1}{2}+\frac{1}{2}$ $\frac{2}{2}$ 7. $\frac{1}{6}+\frac{5}{6}$ $\frac{6}{6}$ 8. $\frac{3}{9}+\frac{5}{9}$ $\frac{8}{9}$ 9. $\frac{2}{5}+\frac{3}{5}$ $\frac{5}{5}$ 10. $\frac{5}{8}+\frac{2}{8}$ 7/[illegible]

SET 2

1. $\frac{3}{5}-\frac{2}{5}$ $\frac{1}{5}$ 2. $\frac{6}{8}-\frac{4}{8}$ $\frac{2}{8}$ 3. $\frac{5}{6}-\frac{2}{6}$ $\frac{3}{6}$ 4. $\frac{4}{4}-\frac{2}{4}$ $\frac{2}{4}$ 5. $\frac{4}{6}-\frac{1}{6}$ $\frac{3}{6}$

6. $\frac{3}{6}-\frac{2}{6}$ $\frac{1}{6}$ 7. $\frac{7}{10}-\frac{5}{10}$ $\frac{2}{10}$ 8. $\frac{7}{8}-\frac{1}{8}$ $\frac{6}{8}$ 9. $\frac{6}{7}-\frac{5}{7}$ $\frac{1}{7}$ 10. $\frac{9}{9}-\frac{7}{9}$ $\frac{2}{9}$

140

STOP See activity 3, page 144b.

See activity 4, page 144b.

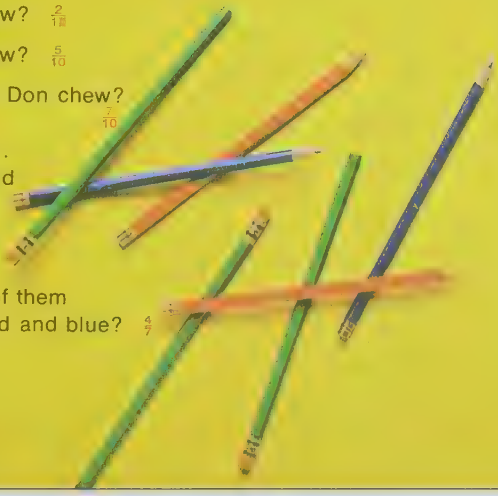
1. The pie was cut in 5 pieces.
   - **a** Mom didn't eat any of the pieces. What fraction can show the amount she ate? $\frac{0}{5}$
   - **b** Big Ben ate 5 of the 5 pieces. He ate it all! What fraction can show the amount he ate? $\frac{5}{5}$

2. There are 9 marbles. You have 4. I have 3.
   - **a** What part of the 9 marbles do you have? $\frac{4}{9}$
   - **b** What part of the 9 marbles do I have? $\frac{3}{9}$
   - **c** What part of the 9 marbles do we have together? $\frac{7}{9}$

There were 10 sticks of gum. Ron chewed 2. Don chewed 5.
- **a** What part of the 10 sticks did Ron chew? $\frac{2}{10}$
- **b** What part of the 10 sticks did Don chew? $\frac{5}{10}$
- **c** What part of the 10 sticks did Ron and Don chew? $\frac{7}{10}$

There were 12 pieces of candy. Ellen ate 1. Helen ate 8. What part of the 12 pieces did Ellen and Helen eat? $\frac{9}{12}$

There are 7 pencils. 2 of them are red. 2 of them are blue. What part of the 7 pencils are red and blue? $\frac{4}{7}$



lesson Page 141

goal Practice with word problems involving fractions

page 141 Independent learners are on their own. Don't let reading difficulties stand in the way of achievement in math. Give any necessary help with reading.

lesson Pages 142, 143

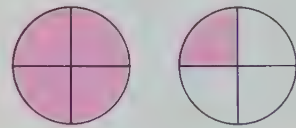
goal Introduction to MIXED NUMBERS

memo A big new idea for exploration only. It does signal a discussion page. Consider 142 and 143 as readiness pages for the next chapter on fractions. In fact, you may want to hold the pages for later—but don't forget to use them sometime. Don't expect any skill performance now.

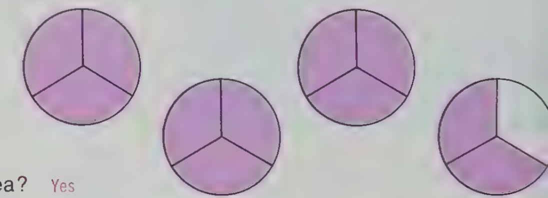
page 142 Nice and easy. One step at a time. The development on the page should get everyone off to a sure start. Challenge them to find a clue that will signal when a fraction can be renamed as a mixed number (numerator greater than denominator).

Is anyone confused by the term **rename**? Remind them they renamed 1 whole as $\frac{2}{2}$, $\frac{3}{3}$, $\frac{11}{11}$, and so on. Now they are renaming a fraction as a mixed number. *What does rename mean?* (To give another name for the same quantity)

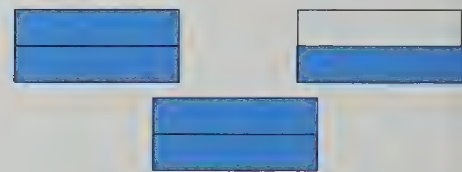
1. How many fourths are shaded? 5
How many wholes are shaded? 1
   - **a** How many more fourths than one whole are shaded? 1
   - **b** Is the sentence $\frac{5}{4} = 1\frac{1}{4}$ true? Yes
   - **c** Do $\frac{5}{4}$ and $1\frac{1}{4}$ name the same shaded area? Yes



2. How many thirds are shaded? 11
How many wholes are shaded? 3
   - **a** How many more thirds than three wholes are shaded? 2
   - **b** Is the sentence $\frac{11}{3} = 3\frac{2}{3}$ true? Yes
   - **c** Do $\frac{11}{3}$ and $3\frac{2}{3}$ name the same shaded area? Yes



3. How many halves are shaded? 5
How many wholes are shaded? 2
   - **a** How many more halves than two wholes are shaded? 1
   - **b** Is the sentence $\frac{5}{2} = 2\frac{1}{2}$ true? Yes
   - **c** Do $\frac{5}{2}$ and $2\frac{1}{2}$ name the same shaded area? Yes



Numbers like $1\frac{1}{4}$, $3\frac{2}{3}$, and $2\frac{1}{2}$ are called *mixed numbers*.
Can you think of a reason why they are called this? The numbers are mixtures of whole and fractional numbers.

4. Which fractions can be renamed as mixed numbers? Try to rename these fractions.
a $\frac{3}{2}$ $1\frac{1}{2}$ **b** $\frac{5}{4}$ $1\frac{1}{4}$ **c** $\frac{7}{4}$ $1\frac{3}{4}$ **d** $\frac{3}{4}$ **e** $\frac{12}{15}$ **f** $\frac{7}{3}$ 2
Draw diagrams if you need help.

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Add. Put a star by any answer that is greater than 1.

BOYS do these.

1. $\frac{3}{4}+\frac{3}{4}$ $\frac{6}{4}$★ 2. $\frac{4}{7}+\frac{6}{7}$ $\frac{10}{7}$★ 3. $\frac{4}{5}+\frac{2}{5}$ $\frac{6}{5}$★ 4. $\frac{9}{10}+\frac{9}{10}$ $\frac{18}{10}$★

5. $\frac{4}{6}+\frac{5}{6}$ $\frac{9}{6}$★ 6. $\frac{5}{8}+\frac{6}{8}$ $\frac{11}{8}$★ 7. $\frac{6}{8}+\frac{5}{8}$ $\frac{11}{8}$★ 8. $\frac{4}{8}+\frac{7}{8}$ $\frac{11}{8}$★

GIRLS do these.

1. $\frac{4}{5}+\frac{4}{5}$ $\frac{8}{5}$★ 2. $\frac{2}{4}+\frac{3}{4}$ $\frac{5}{4}$★ 3. $\frac{7}{8}+\frac{2}{8}$ $\frac{9}{8}$★ 4. $\frac{3}{7}+\frac{6}{7}$ $\frac{9}{7}$★

5. $\frac{4}{6}+\frac{4}{6}$ $\frac{8}{6}$★ 6. $\frac{6}{10}+\frac{9}{10}$ $\frac{15}{10}$★ 7. $\frac{5}{8}+\frac{4}{8}$ $\frac{9}{8}$★ 8. $\frac{7}{9}+\frac{3}{9}$ $\frac{10}{9}$★

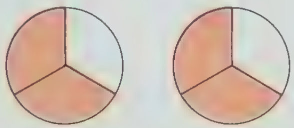
EVERYBODY do these.

Add. Put a star by any answer that is greater than 1 in these problems, too.

9. $\frac{2}{5}+\frac{4}{5}$ $\frac{6}{5}$★ 10. $\frac{5}{6}+\frac{5}{6}$ $\frac{10}{6}$★ 11. $\frac{2}{7}+\frac{6}{7}$ $\frac{8}{7}$★ 12. $\frac{7}{9}+\frac{5}{9}$ $\frac{12}{9}$★

13. $\frac{6}{9}+\frac{2}{9}$ $\frac{8}{9}$ 14. $\frac{1}{5}+\frac{3}{5}$ $\frac{4}{5}$ 15. $\frac{1}{8}+\frac{2}{8}$ $\frac{3}{8}$ 16. $\frac{4}{8}+\frac{5}{8}$ $\frac{9}{8}$★

Draw a picture like this one to show just one of the problems you starred.



$\frac{2}{3}+\frac{2}{3}=\frac{4}{3}$ or $1\frac{1}{3}$

goal Practice in adding fractions; identifying sums that are greater than 1

page 143 Read the directions carefully! There's no need to do more problems than indicated on the page. Note that pupils are directed only to star those answers that are greater than one. They are not directed to rename the answers. One thing at a time. The first step is to recognize when the answer can be renamed. They will be required to rename in later chapters.

lesson Page 144

goal **Checkout**—comparing, ordering, adding, and subtracting fractions with like denominators; comparing unit fractions

page 144 Independent work for everyone. Consider requiring just answers for problems 3 and 4. One row each in problems 1, 3, and 4 is sufficient to check the skill. The second row in each of these problems can then be used for additional practice. Skills are identified on the answer key to help you diagnose trouble areas.

CHECKOUT

Skill: Comparing fractions with like denominators

1. Which is more?

a $\frac{1}{4}$ or $\frac{3}{4}$ **b** $\frac{2}{3}$ or $\frac{1}{3}$ **c** $\frac{1}{6}$ or $\frac{3}{6}$ **d** $\frac{3}{5}$ or $\frac{2}{5}$

e $\frac{1}{9}$ or $\frac{7}{9}$ **f** $\frac{3}{10}$ or $\frac{2}{10}$ **g** $\frac{7}{8}$ or $\frac{5}{8}$ **h** $\frac{1}{2}$ or $\frac{2}{2}$

Skill: Ordering fractions

2. Order these fractions from smallest to largest.

$\frac{4}{8}$ $\frac{1}{8}$ $\frac{5}{8}$ $\frac{3}{8}$ $\frac{7}{8}$ $\frac{2}{8}$ $\frac{8}{8}$ $\frac{6}{8}$ $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$

Skill: Adding fractions

3. Add these fractions.

a $\frac{3}{5}+\frac{1}{5}$ $\frac{4}{5}$ **b** $\frac{1}{4}+\frac{3}{4}$ $\frac{4}{4}$ **c** $\frac{3}{9}+\frac{6}{9}$ $\frac{9}{9}$ **d** $\frac{1}{3}+\frac{1}{3}$ $\frac{2}{3}$

e $\frac{2}{5}+\frac{2}{5}$ $\frac{4}{5}$ **f** $\frac{5}{9}+\frac{2}{9}$ $\frac{7}{9}$ **g** $\frac{4}{10}+\frac{5}{10}$ $\frac{9}{10}$ **h** $\frac{1}{2}+\frac{1}{2}$ $\frac{2}{2}$

Skill: Subtracting fractions

4. Subtract these fractions.

a $\frac{3}{4}-\frac{1}{4}$ $\frac{2}{4}$ **b** $\frac{4}{5}-\frac{3}{5}$ $\frac{1}{5}$ **c** $\frac{8}{8}-\frac{5}{8}$ $\frac{3}{8}$ **d** $\frac{3}{5}-\frac{2}{5}$ $\frac{1}{5}$

e $\frac{7}{8}-\frac{0}{8}$ $\frac{7}{8}$ **f** $\frac{4}{4}-\frac{1}{4}$ $\frac{3}{4}$ **g** $\frac{3}{4}-\frac{3}{4}$ $\frac{0}{4}$ **h** $1-\frac{1}{3}$ $\frac{2}{3}$

Skill: Comparing fractions with unlike denominators

5. Which is more?

a $\frac{1}{2}$ or $\frac{1}{4}$ **b** $\frac{1}{8}$ or $\frac{1}{4}$ **c** $\frac{1}{3}$ or $\frac{1}{6}$ **d** $\frac{1}{2}$ or $\frac{1}{8}$

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STOP See activity 5, page 144b.

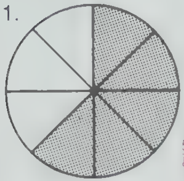
See activity 6, page 144b.

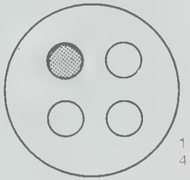
RESOURCES

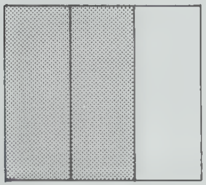
another form of evaluation

for Progress Check—page 128

Write the fraction that tells the number of shaded parts.

1.

$\frac{5}{8}$

2.

$\frac{1}{4}$

3.

$\frac{2}{3}$

4.

$\frac{3}{5}$

Write the fraction that belongs at the point of each arrow.

5.


$\frac{3}{8}$

6.


$\frac{2}{5}$

Fill in the missing numerators. Follow the pattern.

7. $\frac{0}{6}, \frac{1}{6}, \frac{?}{6}, \frac{?}{6}, \frac{?}{6}, \frac{5}{6}, \frac{?}{6}$ 2 3 4 6

8. $\frac{?}{9}, \frac{5}{9}, \frac{?}{9}, \frac{7}{9}, \frac{?}{9}, \frac{?}{9}$ 4 6 8 9

Order these sets from smallest to largest.

9. $\frac{5}{6}, \frac{3}{6}, \frac{2}{6}, \frac{4}{6}$ $\frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}$

10. $\frac{5}{10}, \frac{2}{10}, \frac{9}{10}, \frac{8}{10}$ $\frac{2}{10}, \frac{5}{10}, \frac{8}{10}$, [illegible]$/10$

11. Which fractions below are greater than $\frac{2}{5}$?
$\frac{0}{5}, \frac{1}{5}, \frac{2}{5}, \underline{\frac{3}{5}}, \underline{\frac{4}{5}}, \underline{\frac{5}{5}}$

12. Which fractions in 11 are less than $\frac{2}{5}$? $\frac{0}{5}, \frac{1}{5}$

13. Which fractions are between $\frac{3}{9}$ and $\frac{7}{9}$?
$\frac{3}{9}, \underline{\frac{6}{9}}, \frac{1}{9}, \underline{\frac{4}{9}}, \frac{2}{9}, \underline{\frac{5}{9}}, \frac{7}{9}, \frac{0}{9}, \frac{8}{9}, \frac{9}{9}$

for Progress Check—page 140

Add.

1. $\frac{1}{6} + \frac{4}{6}$ 5/[illegible]   2. $\frac{2}{8} + \frac{4}{8}$ $\frac{6}{8}$ 3. $\frac{2}{3} + \frac{1}{3}$ $\frac{3}{3}$

Subtract.

4. $\frac{8}{9} - \frac{3}{9}$ $\frac{5}{9}$ 5. $\frac{5}{6} - \frac{2}{6}$ $\frac{3}{6}$ 6. $\frac{10}{10} - \frac{7}{10}$ $\frac{3}{10}$

for Checkout—page 144

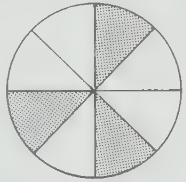
1. Which is more?
   a) $\underline{\frac{4}{5}}$ or $\frac{2}{5}$ b) $\frac{3}{9}$ or $\underline{\frac{5}{9}}$ c) $\frac{2}{4}$ or $\underline{\frac{3}{4}}$
   d) $\underline{\frac{6}{8}}$ or $\frac{4}{8}$ e) $\frac{1}{6}$ or $\underline{\frac{5}{6}}$ f) $\underline{\frac{1}{2}}$ or $\frac{0}{2}$
   g) $\frac{5}{10}$ or $\underline{\frac{7}{10}}$ h) $\frac{1}{3}$ or $\frac{3}{3}$
2. Order these fractions from smallest to largest.
   $\frac{3}{6}, \frac{1}{6}, \frac{4}{6}, \frac{6}{6}, \frac{5}{6}, \frac{2}{6}$ $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}$
3. Add these fractions.
   a) $\frac{2}{6} + \frac{3}{6}$ $\frac{5}{6}$ b) $\frac{2}{4} + \frac{1}{4}$ $\frac{3}{4}$ c) $\frac{4}{8} + \frac{2}{8}$ $\frac{6}{8}$ d) $\frac{1}{5} + \frac{3}{5}$ $\frac{4}{5}$
   e) $\frac{4}{9} + \frac{4}{9}$ $\frac{8}{9}$ f) $\frac{0}{2} + \frac{1}{2}$ $\frac{1}{2}$ g) $\frac{1}{10} + \frac{6}{10}$ $\frac{7}{10}$ h) $\frac{1}{3} + \frac{2}{3}$ $\frac{3}{3}$
4. Subtract these fractions.
   a) $\frac{2}{4} - \frac{1}{4}$ $\frac{1}{4}$ b) $\frac{5}{6} - \frac{2}{6}$ $\frac{3}{6}$ c) $\frac{7}{8} - \frac{1}{8}$ $\frac{6}{8}$ d) $\frac{9}{10} - \frac{3}{10}$ $\frac{6}{10}$
   e) $\frac{3}{5} - \frac{3}{5}$ $\frac{0}{5}$ f) $\frac{3}{3} - \frac{2}{3}$ $\frac{1}{3}$ g) $\frac{4}{9} - \frac{0}{9}$ $\frac{4}{9}$ h) $1 - \frac{1}{2}$ $\frac{1}{2}$
5. Which is less?
   a) $\frac{1}{4}$ or $\underline{\frac{1}{8}}$ b) $\underline{\frac{1}{6}}$ or $\frac{1}{3}$
   c) $\frac{1}{2}$ or $\underline{\frac{1}{8}}$ d) $\underline{\frac{1}{4}}$ or $\frac{1}{2}$

activities

1. things spirit master

Prepare a spirit master including the 3 types of models shown on the page.

How much is shaded? $\left(\frac{3}{8}\right)$



How much is not shaded? $\left(\frac{5}{8}\right)$

How many shaded? How many in all? $\left(\frac{3}{8}\right)$

How many not shaded? How many in all? $\left(\frac{5}{8}\right)$

2. things game boards; markers

Prepare several different game boards similar to the one shown. Each player will need some markers. Pupils play in pairs. In turn, the player picks a box. Before he can place his marker, he must indicate whether the first fraction is less than or greater than the second fraction. First player to place 3 markers in a row, column, or diagonal wins. Provide an answer key for each game board to settle arguments.

$\frac{7}{8}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{0}{4}$	$\frac{1}{9}$	$\frac{3}{9}$
$\frac{2}{7}$	$\frac{7}{7}$	$\frac{5}{6}$	$\frac{1}{6}$	$\frac{0}{8}$	$\frac{8}{8}$
$\frac{2}{10}$	$\frac{8}{10}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{2}$

3. things game board; 2 rubber jar rings; masking tape

Have the pupils prepare a 3-by-3 array game board as shown. Make sure that all the denominators are alike.

$\frac{0}{12}$	$\frac{5}{12}$	$\frac{3}{12}$
$\frac{1}{12}$	$\frac{2}{12}$	$\frac{6}{12}$
$\frac{7}{12}$	$\frac{12}{12}$	$\frac{4}{12}$

Tape the game board to the floor. Mark a line several feet in front of the game board. In turn each player stands behind this line and tosses 1 ring at a time. The sum of the 2 numbers hit is the player's score. The player with the highest score wins the round. Let the players determine a rule for when a ring touches 2 or 4 squares.

Variation: Subtract the two numbers hit by the rings. The difference is the player's score. The player with the lowest score wins the round.

4. things sets of colored plastic sticks

Assign a fractional value to the sticks of each color. For each different set assign different values. **But** make sure that for each set the denominators are all alike. For example:

	Set A	**Set B**
orange	$\frac{1}{5}$	$\frac{3}{4}$
red	$\frac{7}{5}$	$\frac{2}{4}$
green	$\frac{2}{5}$	$\frac{1}{4}$
blue	$\frac{3}{5}$	$\frac{0}{4}$
yellow	$\frac{5}{5}$	$\frac{4}{4}$
black	$\frac{4}{5}$	$\frac{5}{4}$

Each player takes a turn picking up sticks with a "neutral" stick until he moves a stick he is not trying to pick up or has picked up all the sticks. The sum of the values of the sticks picked up is the player's score. Player with the highest score wins the round.

Variation: Subtract the value of the sticks moved during the player's turn.

5. things small cards; colored felt pen

Prepare sets of cards by writing a fraction on each card. Each set must consist of fractions with either like numerators or like denominators. Vary the number of cards in each set. Emphasize the like numerator or denominator by using a colored felt pen.

Set A: $\frac{2}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{9}{8}$

Set B: $\frac{2}{2}, \frac{2}{3}, \frac{2}{5}, \frac{2}{7}, \frac{2}{8}, \frac{2}{12}, \frac{2}{15}$

Mix up the cards in each set. The goal is to order a set of cards from least to greatest number. Make sure the youngster has experience with both types of sets.

6. things small cards

Prepare sets of 10 or 12 cards by writing a fraction on each card. Each set must consist of fractions with either like numerators or like denominators. These sets can be prepared by the youngsters.

Pupils work in pairs. The cards in a set are mixed and dealt facedown—each player receiving half the cards. These are placed in a stack facedown before the player. Each turns over his top car. The player with the fraction that has the greater value takes both cards. Play continues until all the cards are used. The player with more cards wins. Pairs of players can exchange sets of cards.

Variation: The holder of the card with the smaller value takes both cards.

additional learning aids

concept—chapter objectives 1, 2, 3, 4, 6

SRA products

Mathematics Involvement Program, SRA (1971)
  Cards: 282, 253, 323, 333, 224, 244, 264, 185, 156
Visual Approach to Mathematics, level 3, SRA (1967)
  Visuals: 23, 24, 26
Visual Approach to Mathematics, Rational Numbers, SRA (1967)
  Visuals: 1, 2, 3, 4, 5

other learning aids (described on page 144e)
Action Fraction Games, Fraction Bars Student Activity Book, Fraction Dominoes, Fraction Line Set, Fraction Wheel

notation—chapter objectives 7, 8

other learning aids—Student Fraction Sets (Circular, Square)

operation—chapter objectives 5

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
  Spirit masters: F 1, 2, 3
Computapes, SRA (1972)
  Module 5, Lessons: FR 1, 2, 3, 4, 6, 7
Computational Skills Development Kit, SRA (1965)
  Addition of fractions cards: 1, 2, 3, 4
  Subtraction of fractions cards: 1, 2
Diagnosis: an instructional aid—Mathematics Level A, SRA (1973)
  Probe: L-9

number-line fractions

game 1

This is a game for two people.

Here is what you play with:

(1) A playing board
(2) Three tokens
(3) A deck of thirty cards

All players sit on the same side of the playing board. Each player takes a token and puts it on the number line at 0.

Someone shuffles the cards. They go facedown in a pile next to the playing board. The first player draws the top card from the deck. Then he moves his token forward along the number line. The number on the card tells how far.

The second player draws a card and moves. The players always take turns.

Suppose your token stops on a point taken by another token. You get an extra turn. Draw another card. The next player does not get his turn until your token lands on a point that is not taken. This is so even if you get several extra turns.

Suppose you draw a card numbered 0, $\frac{0}{2}$, or $\frac{0}{4}$. Your move is zero.

Suppose you make a mistake moving your token. You must take back your move when caught. You lose a turn.

The first player to pass 4 wins.

game 2

Here is another way to play. You need forty cards if there is a third player.

Choose a dealer. He deals two cards to each player. He puts the other cards facedown in a pile. All tokens start at 0, the same as before.

The first player discards one of his cards. This card tells how far he moves his token. Then the player draws the top card from the deck. Now he holds two cards again. The next player plays.

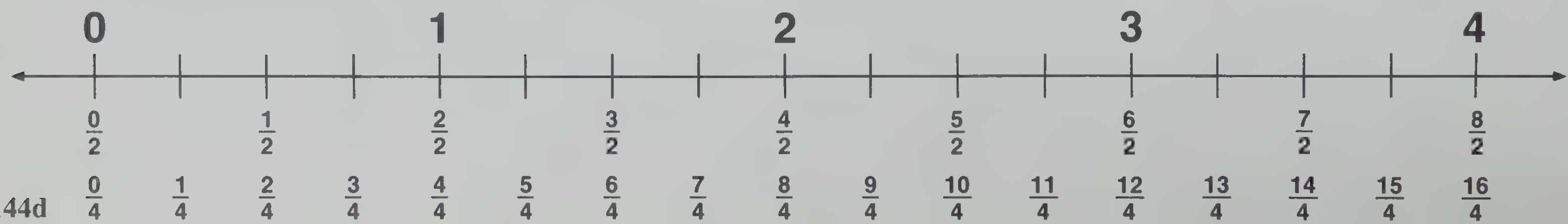
Suppose your token stops on a point taken by another token. First you draw another card. Then you take your extra turn. All other rules are the same.

playing cards for number-line fractions

A deck is three each of these cards. If three players play Game 2, a deck is four each of these cards.

0	1	$\frac{0}{2}$	$\frac{1}{2}$	$\frac{2}{2}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$

playing board for number-line fractions



7 GEOMETRY SHAPES

before this chapter the learner has—

1. Manipulated real-world models of geometric solids—boxes, cans, paper rolls, and so on
2. Identified flat surfaces and curved surfaces
3. Classified objects that are alike
4. Identified how two or more objects are alike or different

in chapter 7 the learner is—

1. Identifying geometric shapes in his environment
2. Mastering the identification of rectangular prisms, triangular prisms, cylinders, cubes, and spheres
3. Identifying the number of faces, edges, and vertices on a geometric shape
4. Identifying plane geometric figures from the faces of geometric shapes

in later chapters the learner will—

1. Make a congruent model by tracing a plane figure
2. Decide by tracing whether two polygons are congruent
3. Decide by folding whether a plane figure has two congruent halves

Notes & Things

Geometry can be a joy if it is not burdened with a mass of formal definitions to be memorized and when it evolves from objects that are seen every day and can be touched, turned, and examined.

Our world is three-dimensional. It provides the reference for the study of shape, size, and space. The emphasis of this exploratory chapter is on the development of an intuitive understanding of geometric concepts of shape that are in existence in the child's own world. The approach depends on tactile and visual experiences. The vocabulary words develop because of an obvious need to communicate the ideas with another person.

The specific concepts of this chapter come from the construction of solid-shape models. This construction should yield the "discoveries" that a flat surface is contained on the face of some of the solid shapes and that the surfaces of solid figures are either flat or curved. The learner should come to know that an *edge* is formed whenever two faces meet and a *vertex* (a sharp point you can feel on some three-dimensional objects) is formed whenever three or more edges come together. Boxes (rectangular prisms), balls (spheres), and cans (cylinders) are the three major shapes to be explored. Cones, triangular prisms, and pyramids are also introduced and studied.

As a child looks only at one flat surface of a solid, he will see only a two-dimensional shape called a plane figure. This plane figure has either straight sides or curved sides. If straight sides come together, a corner is formed. Remember—for now, the vertex (three or more edges meeting) belongs to a solid, but the corner (two straight sides meeting) belongs to a plane figure. Correct use of the words *face* and *side* is important. In fact, this is one of the few sets of words that have to be controlled. We need the word *side* to talk about the line segments that make up a plane figure—the side of the square, the side of the triangle. The word *face* will eliminate problems concerning solids, too. The child will not forget about the top and the bottom of a box, for example, if we ask how many faces on the box rather than how many sides.

In order to keep everybody happy, there is one more set of words that should be used precisely. Shaded two-dimensional shapes such as

are called regions—circular region, rectangular region, triangular region.

Outlines of two-dimensional shapes such as

are plane figures with the obvious names circle, rectangle, and triangle. Picky, picky, picky! Even though we don't expect the children to memorize these labels, it is important that our language be correct and serve as a model for their use of words.

Upon completion of the chapter you can expect a certain degree of mastery. The student will be expected to *recognize* the following parts of a solid figure: face, edge, vertex. He should also be able to *sort* models into these classifications: rectangular and triangular prisms, spheres, cylinders.

things

large box filled with everyday objects (balls, boxes of all sizes, cans, paper cups shaped like cones, plastic pill bottles, cardboard cores from toilet tissue or waxed paper, mailing tubes, milk cartons, candles, wood scraps that form triangular prisms, any other appropriate junk available for pupils to handle)
commercial geometric solid shapes if available
chalk of several colors
transparent tape
scissors
straightedges

For the extra activities you will want to have available a spirit master of patterns.

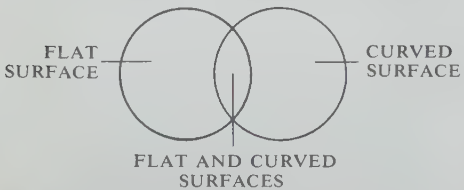
prebook activities 1 or 2 days

goal Providing experience in manipulating and analyzing geometric shapes

things Use the box of shapes described in "Notes and Things" and lots of floor space. Select 3 empty boxes, or 2 hula hoops, or 2 very large sheets of wrapping paper with 2 intersecting circular outlines drawn on it. If you use boxes, each box should have a label:

FLAT SURFACES	FLAT AND CURVED SURFACES	CURVED SURFACES

If you use hoops or circles, they must intersect.



first activity Organize manageable groups of children. They are to sort the shapes into 3 groups:

1. Shapes that sit flat no matter which face is touching the floor (Put them in one box or on the left side of the intersecting circles.)
2. Shapes that roll easily and will not sit flat no matter how you place them on the floor (Put them in another box or on the right side of the intersecting circles.)
3. Shapes that roll easily if you put them on the floor in one position but can sit flat if you put them in another position

   (Put them in another box appropriately labeled or in the intersection of the two circles.)

You are now able to establish the vocabulary words *flat surfaces* and *curved surfaces*. It is important that each child know that a solid can have both flat and curved surfaces, so emphasize that set of solid objects.

second activity Make a three-column chart on the chalkboard. Allow lots of space. Label the first column FLAT SURFACES. Label the second column FLAT AND CURVED SURFACES and the last column CURVED SURFACES.

Go on a shape hunt in the classroom. Pick up a book. Ask which set the book belongs to. Enter the word *book* in the first column. Show a piece of fruit, or a ball of clay, or a blown-up balloon, or a ball (when all else fails, wad up a sheet of paper), and ask which set this object belongs to. In the third column enter the name of the object you have shown. Then pick a piece of chalk and ask what set that object belongs to. (You may have to make sure it's a new piece, or you won't be able to demonstrate that it can sit flat.) Put the name of this object in the second column.

Divide the class into manageable groups. Have each group select a recorder—someone who will keep track of the objects found on the hunt. Give a different-colored piece of chalk to each recorder so that you'll know who entered what. (Accept pictures of objects as well as words, just in case the recorder isn't worth a hoot in spelling.) Now the groups can begin the hunt. They will classify as many objects in the classroom as they can find in a reasonable amount of time. (If you have a wild group where this hunt could get them off and running, you can make this a completely silent activity by having team members turn in slips of paper to the recorder instead of using oral labels. Oral decision making is by far the best, however, because one child will have an opportunity to learn from another. The decision-making process of several children, as to whether a given object should go into one classification or another, cannot be recreated in ordinary class discussion.)

You can have each group make its own chart if you prefer. Then the recorders should turn their papers in, and other children on the same team should serve as judges to evaluate another team's record as the class discusses the correctness of the classifications.

Hopefully, at least one group will observe that a table leg, for example, has both flat and curved surfaces. Discussing parts of a whole is valuable. After all, that table leg was a separate shape at one time.

One point can be given for each correct entry, and a one-point penalty can be assessed for each incorrect entry.

You can continue this activity by asking the pupils to list the names of objects they see on the way home that also fit into these three classifications.

goal Think about and explore ideas through a picture clue

page 145 This photograph of the modular houses featured at the Montreal World's Fair may stimulate discussion all by itself. In the houses, some youngsters will see a striking resemblance to the cliff homes of the Pueblo Indians. Others may consider the photograph quite bizarre. The chapter itself will plunge the learner into the observation of shapes in his world, but it is important to establish the perspective of the chapter through this photo.

Since people have significantly altered the offerings of nature, the primary source of ideas will be in the world of manufactured objects and this photo is worth some questions and answers. Does an apartment building put a lot of different families' homes together in one structure? Why doesn't every family live in a home on its own piece of land? Could this photograph be thought of as a picture of an apartment building? How is it different from most apartment buildings? Can you figure out what might be better about this kind of building? The notion of privacy and individuality may be far too sophisticated for youngsters this age. That's O.K. Nothing is lost if they don't understand this idea just yet. You still have established an entry into the chapter, as well as a magazine search. A classroom scrapbook of the shapes of people's homes will be a rich source of satisfaction and extended learning.

lesson Page 146

goal **Survey**—classifying and naming geometric shapes

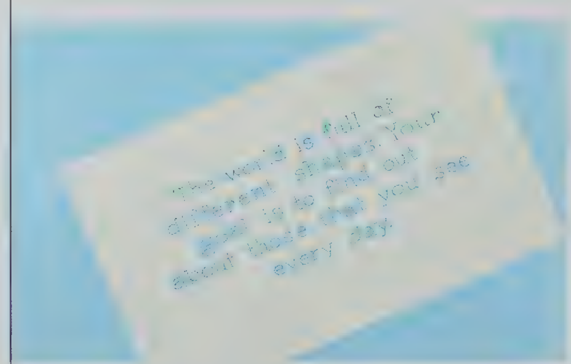
memo This is an exploratory chapter. It is based on the real world. Youngsters with a wide range of ability and mathematical skill should have the opportunity to work together through this entire chapter. There is much that they can learn from one another. The ability to multiply fortunately is not usually directly correlated with the ability to see and enjoy the environment.

page 146 Have fun talking about this page. Listen for the geometric vocabulary they use (square, rectangle, circle, curved surface, curved edges, flat surfaces, straight edges) and the ease with which the vocabulary is used. Do they recognize likenesses and differences?

Lots of different shapes can be put together in groups because they are so much alike. For example, look at the shape of a book. Then look at the shape of a box.

How are they alike? How are they different?
Straight edges, square corners, flat surfaces   Different-sized surfaces

Now look at the door to your room. It's larger than a book or box. BUT does it have the same shape? Yes



How much do you already know about shapes? Sort the following pictures into four groups. Each group will have shapes that are very much alike. a. 1, 7, 12 b. 2, 6, 9 c. 3, 5, 8, 11 c

13. Can you guess the geometry name for each of these four groups? Don't expect these exact names. Accept any good answers.

a) Rectang
b) Triangu
c) Cylinder

146

Ve Design Space for a Purpose

Vhat shapes are used?

147

lesson Pages 147, 148, 149, 150, 151

goal Developing an awareness of man's use of geometric shapes as a basis for design and construction

memo Pages 147 through 151 are designed for discussion—to relate geometry to our everyday lives. They are meant to help the student focus on his environment. Try to tie these discussions to your local community for a more meaningful experience.

page 147 What shape is the basis of each of these man-made constructions? Why that particular shape? Are there any advantages? There is no need to teach the labels now. Specific names for shapes will be developed on the pages that follow. This page is an overview.

Continue to page 148.

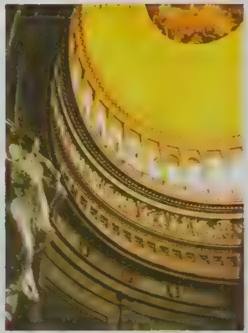
goal Examining how geometric shapes called SPHERES and **cones** are used to serve man's specific needs

page 148 Focus on why man chose each shape. What are the advantages of using that specific shape in each example? Can the pupils think of a different shape that might have been better? Can they think why this shape might not have been used?

Watch for confusion regarding shape and material used for construction. The materials available could have dictated the shape. Look closer to home. Examine some buildings in your own community.

Spheres and Cones We Live In

Accept any good answers. Examples are given.



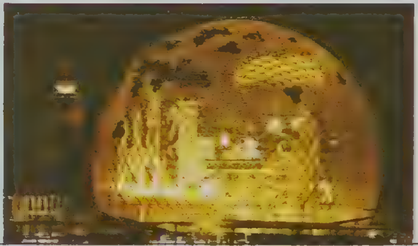
1. The great high ceiling of our capitol leaves one with a special feeling. Why do you think capitol buildings and churches are often built with large, high domes? To make us feel we are someplace special

2. An igloo also is shaped like a dome. What feeling do you think the Eskimos have inside their home? Why do you think they made them so small? For warmth

War
secu

3. A ball-shaped object is a sphere. The dome can be thought of as a half-sphere. What other objects can you name that are spheres or half-spheres? Spheres—globe, basketball, the earth, etc. Half spheres—bowl, hat, half a

4. The Indian tepee is cone-shaped. The tops of African huts are also cone-shaped. What advantages does this shape have? Rain can run off easily. Easier to build.



5. The U.S. exhibit at the Montreal Exposition was curved like part of a sphere. What advantages did this shape have for the exhibit? More space. Uniqueness left impression on people.

148

things old magazines with lots of pictures

Research project. Put your researchers to work again. Project: Find pictures of how man has used cones and spheres in his environment. Shutterbugs may even want to take snapshots of local examples.

These pictures can be displayed on a bulletin board, or the youngsters may want to make scrapbooks.

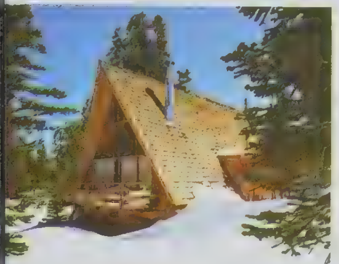
As other shapes are investigated, additions will be made to this collection.

Living in Spaces with Flat Surfaces

1. Most of our living spaces have flat surfaces. Both skyscrapers and telephone booths have a common shape that is called a rectangular prism. Is the room you are in now something like a rectangular prism in shape? Name other things shaped like rectangular prisms. Book, cereal box, etc.

Some modern homes have shapes that look like triangular prisms. Why do you think this shape is called a triangular prism? What part of a house sometimes looks like a triangular prism? The roof

The ends are triangles.



3. The old covered bridge includes two shapes. What are they? Rectangular prism Triangular prism

Which of the following describe most of the rooms in your home?

a The walls are flat surfaces.

b The corners in the room are square.

c The ceiling and the floor are about the same size and shape.

d Opposite walls are the same distance apart.

Time to add examples of man's use of flat surfaces to the picture collection. Continue building that scrapbook or bulletin board.

goal Examining living in spaces with flat surfaces; introduction to RECTANGULAR and TRIANGULAR PRISMS

memo There's no need for lengthy, complicated mathematical definitions. Should questions arise regarding prisms, you need only point out that the two ends of a prism are flat surfaces that have the same size and shape (are congruent) and that the surfaces that hold the ends in place also have straight edges and flat surfaces.

page 149 Some answers are given on the answer key. You'll want to listen to the pupils' ideas—they will probably be much better. Encourage free, imaginative thinking. Find the two ends of each prism. What shape are these ends in each example? Once again, look for models nearer to home.

goal Looking at man's use of geometric shapes in outdoor spaces

page 150 Man designs outdoor living spaces as well as indoor. The discussion will be more meaningful if related to the local shopping mall or center, the local park, the community surrounding the school.

Outdoor Living Spaces

1. Architects design outside space for outdoor living. What shapes would you see if you were sitting in this plaza? Rectangular prisms

2. Parks are arranged to provide special kinds of living spaces. What shapes might you see here? Rectangular prisms and cylinders

3. The natural setting in the woods is shown here. What shapes might you think of while sitting in the clearing among the trees? Half spheres, cylinders, and cones

4. What things near school should remind you of certain kinds of prisms? Can you think of things near home that look like rectangular prisms? Pla that look like spheres? Bushes, treetops, streetlights

Does that picture collection include both indoor and outdoor uses of geometric shapes? Don't overlook the newspaper for local examples.

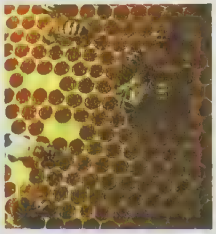
Animal Space

1. Animals build their living space in many different shapes. What shape is the oriole's nest like? Sphere
   Is a drop of water something like a sphere? Yes

2. The hollow tree is shaped like a cylinder. Cylinder
   What is the shape of a log? of a pile of logs? Triangular prism
   of a peppermint stick? of a pencil? Cylinder Cylinder
   What shape is the point of the pencil? Cone

3. What shape does the robin's nest look like? Half sphere
   What shape is the space for the robin before it is hatched? Sphere

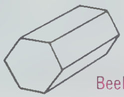
4. The beaver's den reminds us of many shapes.
   What part of the den reminds you of a cylinder? Entrance
   What part is like a sphere? Dome part



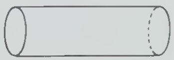
5. The larvae of the bee live in a shape like that of some pencils. How is the shape different from a cylinder? It has straight sides.

. Which animal houses do you think look something like each shape below?

Oriole's nest
Beaver's den



Beehive cells



Beaver's den
Woodpecker's home inside tree

Robin's nest
Beaver's den

151

goal Looking at nature as a source of man's ideas for geometric shapes

page 151 Geometric models in nature are sometimes crude. Could man have gotten his ideas from nature? Can the pupils think of any more examples? You may want to have the youngsters hunt for examples during recess or lunch time. Then continue the discussion.

This is a great place to start some individual research projects. The projects need not end in a written report. How about actual examples, such as honey still in the comb, abandoned birds' nests, and so on. Constructed models based on researched information would be great too.

lesson Page 152

goal **Progress Check**—identifying geometric shapes

page 152 Discussion or written? The decision will depend on the abilities of your pupils. Characteristics to consider:

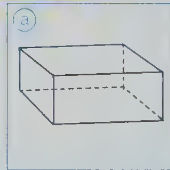
- Does any surface have square corners?
- What shape is each surface?
- How many surfaces does the shape have?
- Do the surfaces all have the same shape?
- Which surfaces have the same size? Which have a different size?

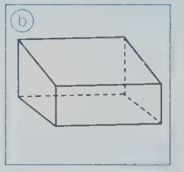
Discuss the shapes used on the page. What are some advantages of curved surfaces? of flat surfaces? When might man want to use a curved surface rather than a flat surface? Explore more models in the room, school, home, and community.

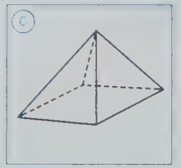
Skill: Identifying rectangular prisms

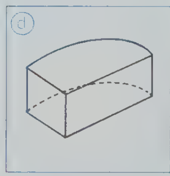
Look at the pictures on the right.

1. Why is ⓐ a rectangular prism?
   6 faces, square corners, opposite faces equal in size and the same distance apart
2. Why is ⓑ not a rectangular prism?
   Top and bottom not same distance apart
3. Why is ⓒ not a rectangular prism?
   Only has 5 faces
4. Why is ⓓ not a rectangular prism?
   Has a curved surface
5. What features do all rectangular prisms have? Name three. 6 faces, square corners, opposite faces equal in size and same distance apart
6. Why do you think most rooms are shaped like rectangular prisms? What would be some advantages and disadvantages of a room shaped like a cylinder? like a pyramid (figure ⓒ)?
   Easier to construct
   Cylinder:
   advantage—feeling of warmth
   disadvantage—less room since no corners, harder to fit furniture in
   Pyramid:
   advantage—interesting decor
   disadvantage—harder to make furniture fit





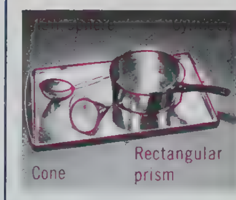


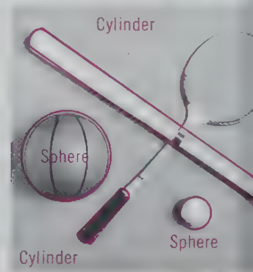


152

Shapes We Use

What are the shapes you see in each picture? Why do you think each object was shaped as it is?





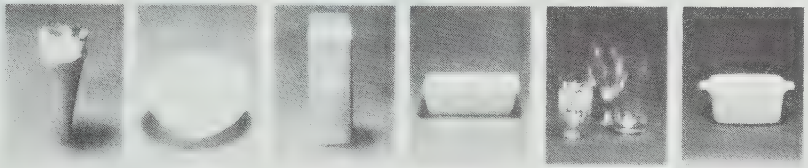
*In perspective

things any type of container; paint; construction paper; paste

Art project for everyone. Build a city or shopping center using an empty container as the base for each building, car, or whatever your young designers can create.

Be sure to display their creative efforts for everyone to see.

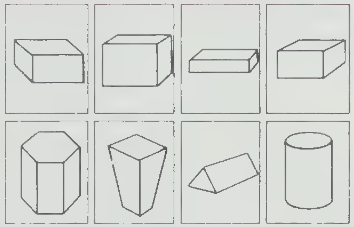
The Shapes of Containers



cones | Half sphere | Rectangular prism | Rectangular prism | cylinder | Rectangular prism

1. If each of the containers is filled, in what shape is the content of each? See above. Accept all good answers

2. What are some advantages of using a rectangular prism as a container for milk? Why is the cone a good shape for the ice cream you buy? Easier to store; easier to hold while pouring
Easier to hold while eating



3. Most packing boxes come in shapes like these. Why do you think those shapes are used more often for packing than these? Many different shapes will fit inside, Conserves space while storing or shipping. Easier to make.

4. Why do you think drinking glasses come in such a variety of shapes?
People like different sizes and shapes for their tables.

5. Cut and fold a piece of paper as shown on the right. Cut on solid lines and fold on dotted lines. Then try to shape the paper into a box. Does it look like the figures on this page? Yes

lesson Pages 153, 154

goal Looking at the characteristics and applications of certain shapes

memo Pages 153 and 154 involve both discussion and activity.

things paper
scissors
transparent tape

page 153 The shapes pictured on this page are called SOLIDS. Don't let the English use of **solid** confuse anyone. In English we might say, "He was as solid as a rock." In mathematics, the use of solid means a hard surface. In math, a solid is any shape that occupies space. If anyone fights the idea that an empty container is a solid, simply ask him to imagine that the container has been filled with water and then frozen. Now the shape is certainly a solid from both the math and the English point of view.

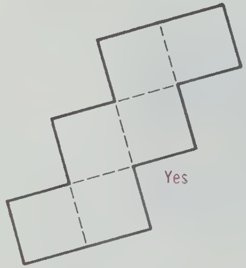
Exercise 5 is an activity. The end product should resemble a box. You may want the pupils to tape the edges after assembling the shape.

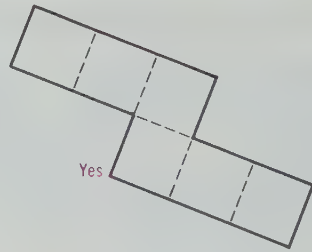
goal Exploring patterns that make cubes

memo Construct and learn! Make a cube yourself so that you can understand better the difficulties your students will encounter. This is a fun activity. Remember—cut along solid lines, fold perforated lines.

things paper
scissors
transparent tape
straightedge

page 154 You may want to make a spirit master of the pupil page in order to speed things up. If not, pupils will need a straightedge to help with the tracing. Make sure you try any patterns that are made up to verify that they will work.



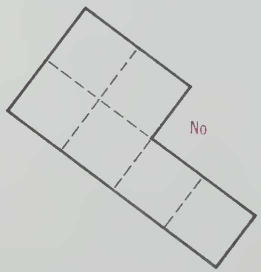


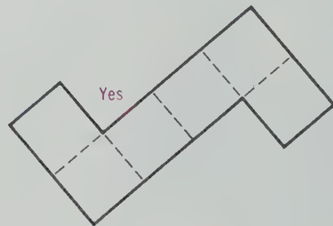
The figure that encloses these words is the pattern for a box. You will find that this closed box is a cube.

What is a cube? Trace the pattern on paper and cut it out. Then fold it carefully so that it forms a box. Tape the faces together so that the box is closed. What you have then is a cube.

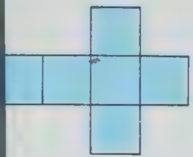
You see other patterns on this page. Some of them are patterns for cubes. Others are not. Which ones are the patterns for cubes? Trace and cut out the patterns to find out. You may be surprised.

Are there any other patterns that will make cubes? Try to draw some that will. Yes

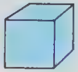




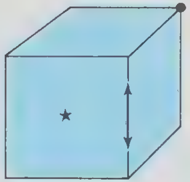
154



his is a pattern
or a cube.



This is a picture of a cube that could be made from the pattern.



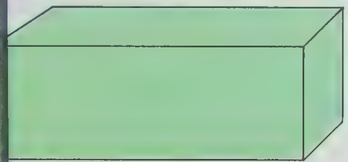
Look at a picture of a larger cube.

The ● marks a corner, or vertex.
The ★ marks a flat surface, or face.
The ↕ marks an edge.

How many faces does a cube have? 6

a How many vertices does a cube have? 8

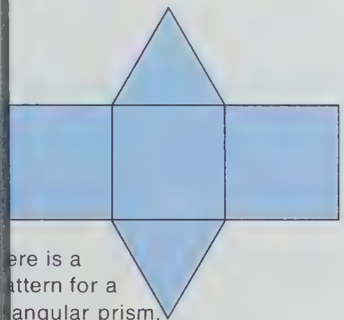
b How many edges does a cube have? 12



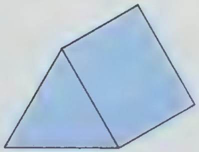
2. How many faces does a rectangular prism have? 6

a How many vertices? 8

b How many edges? 12



ere is a
ttern for a
angular prism.



This is a picture of the triangular prism that could be made from this pattern.

3. How many faces does a triangular prism have? 5

a How many vertices? 6

b How many edges? 9

4. What is needed to form an edge? Union of 2 faces

5. What is needed to form a vertex? 3 edges meeting at one point

6. What are the least number of faces needed to form a prism? 5 (Remember—a face is a flat surface.)

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Patterns for several geometric shapes are included in the Resource Section (pages 159a–d). These patterns can be used to find which do indeed form a triangular prism, cylinder, cone, or pyramid. Then these shapes can be transformed into buildings to make a street scene—young architects at work!

lesson Pages 155, 156

goal Focusing on the terms **vertex, face, edge;** summarizing the characteristics of rectangular and triangular prisms

memo Picky, picky, once again! A CUBE is a rectangular prism—a special rectangular prism. All of the faces are the same size and shape.

things models of rectangular and triangular prisms

page 155 Have the youngsters handle actual models. Ask what they feel and if there is anything special. Answers to the questions should simply "happen" with this hands-on experience.

goal Summarizing information in a table

memo You may want to use this page only with independent learners. It is not included in the minimal course.

things pattern of a cone cut from a shirt cardboard
cone-shaped paper cup

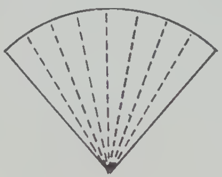
page 156 Summarizing the information serves two purposes: readiness for organizing information to be shown on a graph, and readiness for more advanced geometry work that comes at a later level. Then students will discover a pattern named Euler's formula:

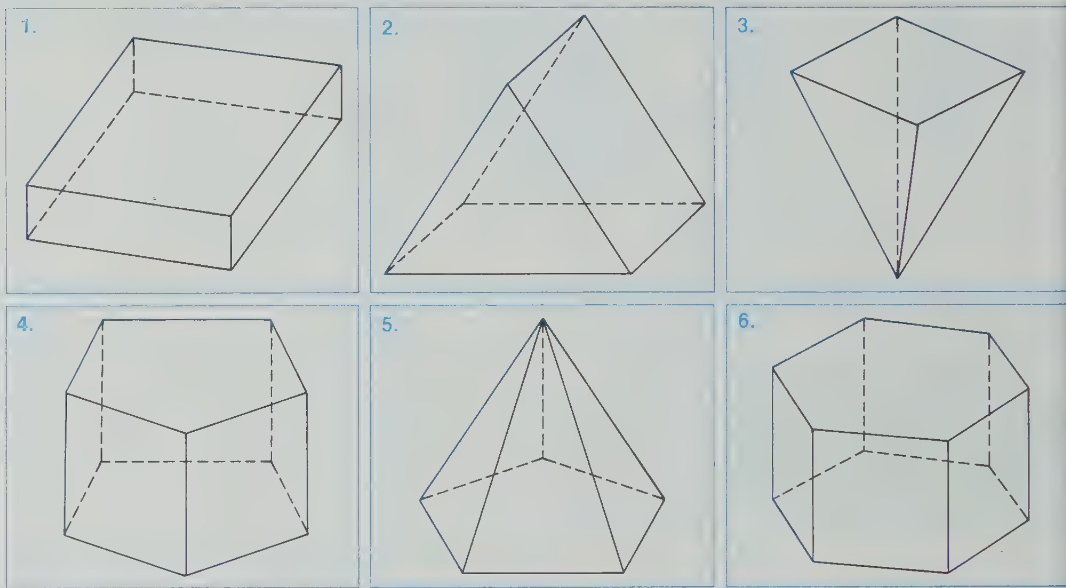
$$\text{number of vertices} + \text{number of faces} = \text{number of edges} + 2$$

Have a mathematical genius? Challenge him to find the pattern. Will it always work?

A vertex is defined as a sharp point you can feel wherever three or more edges come together. Usually cones are made of a pliable material. Where are the edges? Examine the paper cup.

Now take the pattern of a cone cut from lightweight cardboard (soft enough to fold but stiff enough not to roll). *How do I form a cone with this?* (By repeated folding) *What does each fold represent?* (An edge) Infinite edges meet at the vertex of a cone.





Six solid shapes are pictured above. The dotted lines show you what you could not see if these were pictures of models made of cardboard.

Fig.	No. of faces	No. of edges	No. of vertices
1.	6	12	8
2.	5	9	6
3.	5	8	5
*4	7	15	10
*5.	6	10	6
*6	8	18	12

Make and complete a chart like this.

Figure	Number of faces	Number of edges	Number of vertices
1.			
2.			
3.			
*4.			
*5.			
*6.			

See above.

Remember—the dotted line shows the part you could not see if these models were made of cardboard. Figures 4, 5 and 6 are hard, but give them a try.

1. Take paper and a sharp pencil.
Trace one of the faces of a cube.
What figure have you just drawn? Square or rectangle
Does the face of the cube fit the figure exactly? Yes
Do all the faces fit the figure? Yes

2. Now take a pyramid. Trace one of its faces.
What figure have you just drawn? Triangle (or square)
Does the face of the pyramid fit the figure? Yes
Do all the faces fit the figure? No

3. Select several objects you find in your room.
Trace as many different faces as you can.
Name any of the figures you trace.

4. So far, you have been tracing faces of solids with flat surfaces only.

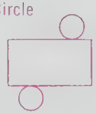
a Did you find any objects with curved surfaces to trace? No

b Can you trace the surface of a ball or any kind of sphere? No
Is the surface flat or curved?

c Can you trace the curved surface of a cone? Does it have a flat surface? No Yes
What shape is the flat surface? Circle

d Can you trace the curved surface of a cylinder? Does a cylinder have any flat surfaces? How many? No Yes 2
What shape are the flat surfaces? Circles

e Figure out how a tin can is made.
(Hint: It started out as flat pieces.)



Flat-surface shapes are many times called plane figures.

157

lesson Pages 157, 158

goal Deriving plane figures from solid shapes

things the box of shapes that contains boxes, cans, commercial geometric shapes, and so on

page 157 Real objects are more stable for tracing than models made from paper or cardboard.

Independent learners are on their own. You'll want to guide the others through the directions for this activity. Isn't it neat the way our system works??? Plane figures come from space figures. Geometry is indeed a part of the

real world!

goal Recognizing the characteristics of geometric shapes and figures

things models of the geometric shapes pictured

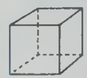
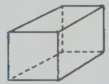
page 158 Some pupils may need to manipulate geometric models to help them answer the questions. That's O.K. The focus is on the concept for the present.

You may want to use models to help you answer these questions.

1. How many faces are needed to form an edge on a prism? 2

2. What is the least number of faces a prism can have? 5

3. Copy and complete this chart.

SOLIDS	a 	b 	c 	d 	e 
Number of square faces	? 6	? 2	? 0	? 1	? 0
Number of rectangular faces (including squares)	? 6	? 6	? 3	? 1	0
Number of triangular faces	? 0	? 0	? 2	? 4	0
Number of circular faces	? 0	? 0	? 0	0	2

4.  This shape could be part of which solids in the chart above? a, b, d

5.  This shape could be part of which solids? c, d

6.  This shape could be part of which solids? e

158

CHECKOUT

Which is it? Here are the names of the solids you will need for this page.

rectangular prism	cylinder	sphere
triangular prism	pyramid	cone

Skill: Identifying rectangular prisms, triangular prisms, cylinders, cones, and spheres

1. Name the geometric solid that best describes the shape of each of these objects.

Skill: Identifying number of faces of geometric models

2. Match the set of faces with the name of the solid.

Skill: Identifying number of edges and vertices of geometric shapes

3. a How many edges does a rectangular prism have? 12
   How many vertices? 8

   b How many edges does a triangular prism have? 9
   How many vertices? 6

   c How many edges does a sphere have? 0
   How many vertices? 0

lesson Page 159

goal **Checkout** – naming and identifying characteristics of geometric shapes

things the box of shapes

page 159 Everyone on his own. Spelling should be no problem – the words are given. Pupils can use actual models for problem 3.

For additional practice similar to problem 1, have the pupil make a slip of paper for each geometric shape listed on the page. Now have him match each slip of paper with an appropriate model in the box of shapes.

The patterns included in the Resource Section will help the youngster who has trouble with problem 2.

The youngsters who have trouble with problem 3 should use actual models and count. The information can be summarized in a chart.

things spirit masters

Make models of the various geometric solids. This activity will focus attention on the interior of each shape. Everyone is a member of the City Planning Commission, which is faced with the problem of selecting appropriate shapes for new buildings. What shape could a sports arena be? An auditorium for seating large groups of people? A building for storage? Which shape building would be easy to heat? The youngsters might even lay out a new civic center for their city.

RESOURCES

another form of evaluation

for Progress Check—page 152

True or false?

1. A rectangular prism has square corners. True
2. Opposite sides of a rectangular prism are not always the same distance apart. False
3. The top and bottom of a rectangular prism are different sizes. False
4. All sides of a rectangular prism are flat. True
5. Name 3 objects shaped like a rectangular prism; like a cylinder; like a pyramid. Answers will vary.

for Checkout—page 159

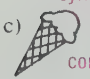
Here are the names of the solids you will need for this page.

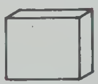
rectangular prism	cylinder	sphere
triangular prism	pyramid	cone

1. Name the geometric solid that best describes the shapes of these objects.

a)  cylinder

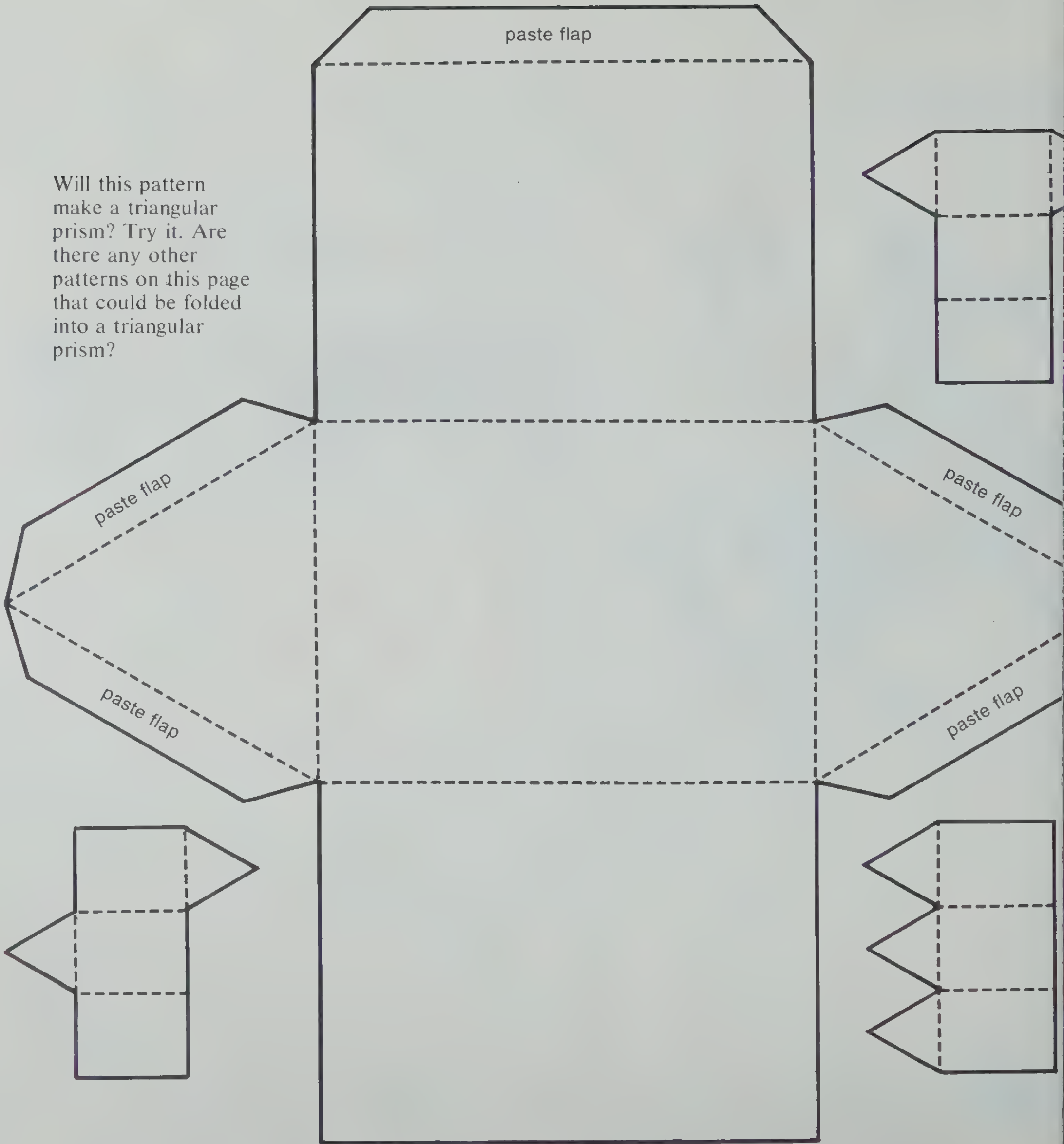
b)  sphere

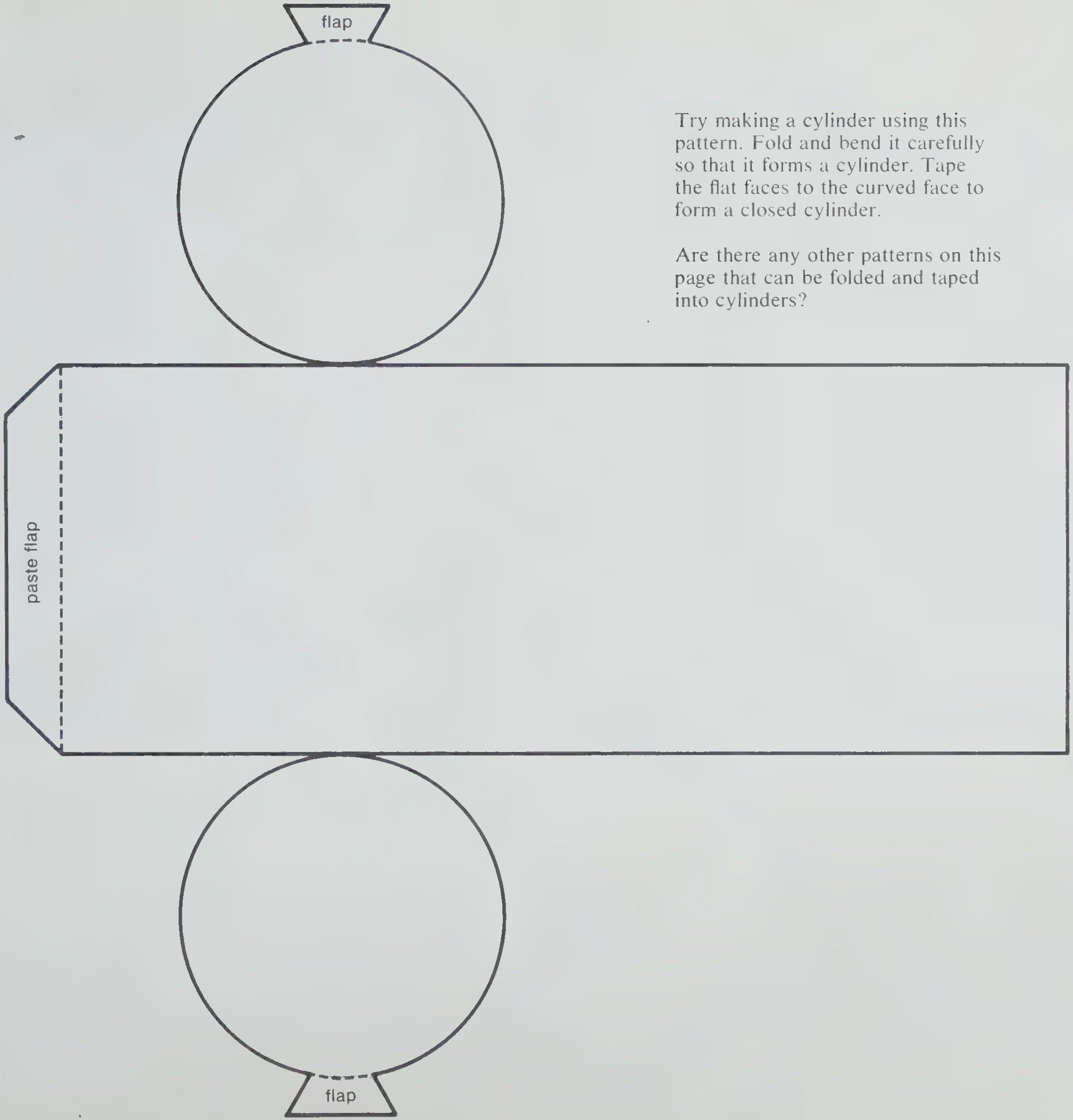
c)  cone

d)  rectangular prism

2. Match each description with the name of the solid.
   a) It has no edges. sphere
   b) It has 3 rectangular faces and 2 triangular faces. triangular prism
   c) It has both curved and flat surfaces. cylinder
   d) It has 1 square face and 4 triangular faces pyramid

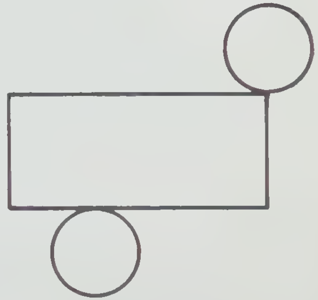
Will this pattern make a triangular prism? Try it. Are there any other patterns on this page that could be folded into a triangular prism?

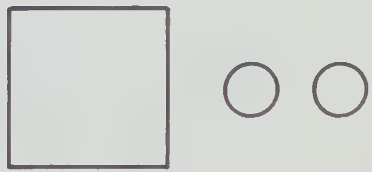




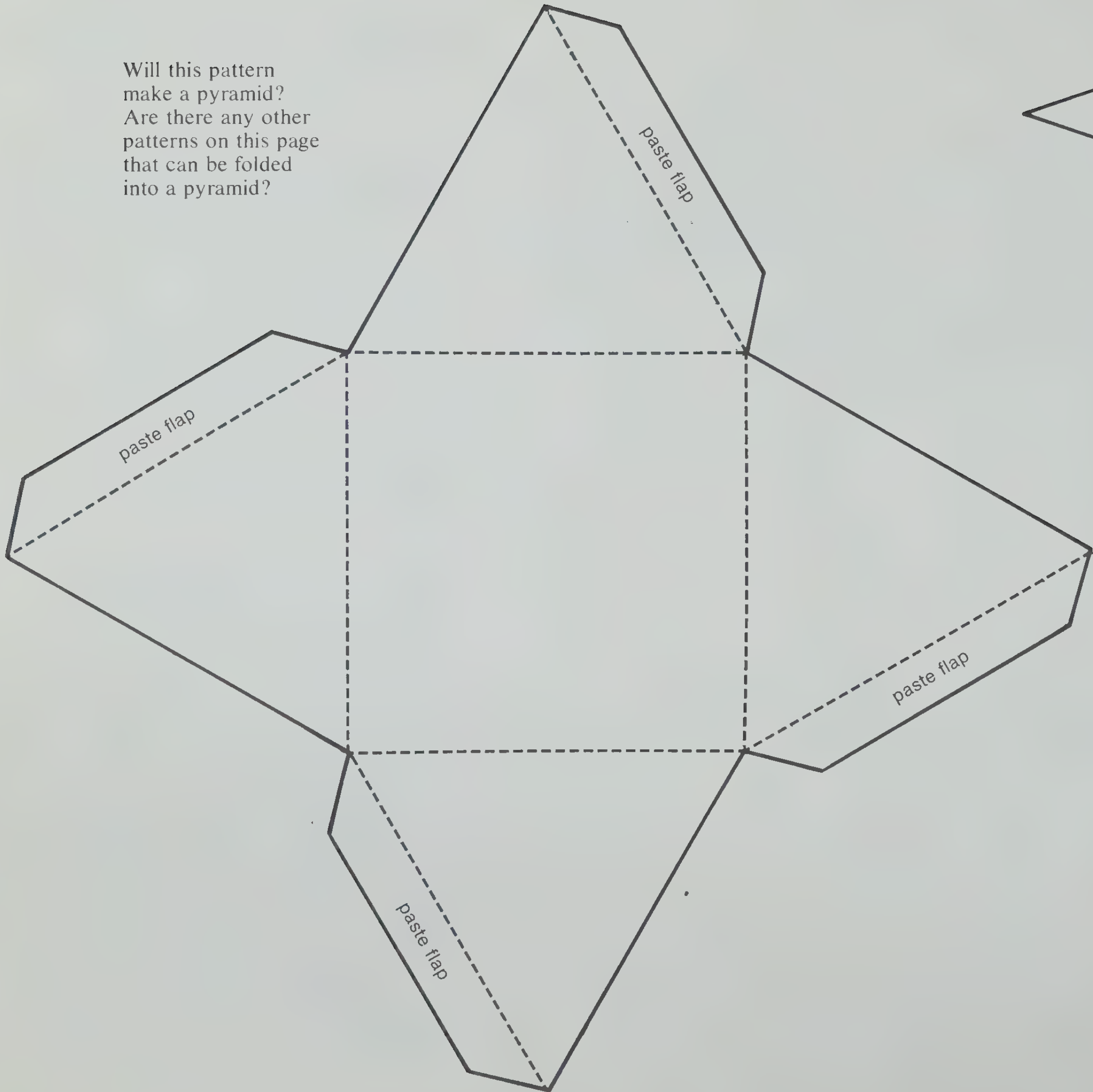
Try making a cylinder using this pattern. Fold and bend it carefully so that it forms a cylinder. Tape the flat faces to the curved face to form a closed cylinder.

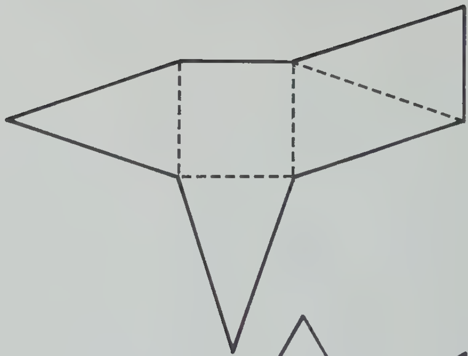
Are there any other patterns on this page that can be folded and taped into cylinders?

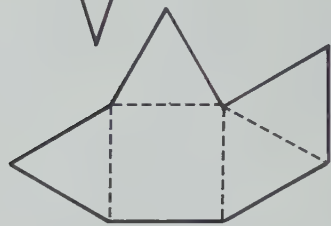


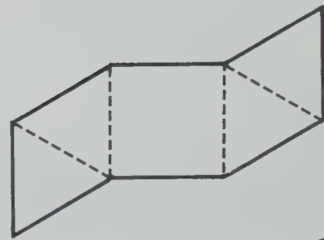


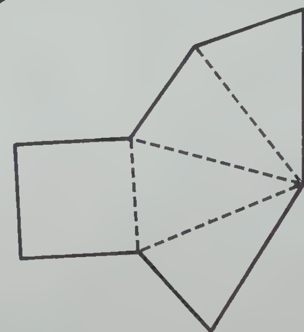
Will this pattern make a pyramid? Are there any other patterns on this page that can be folded into a pyramid?



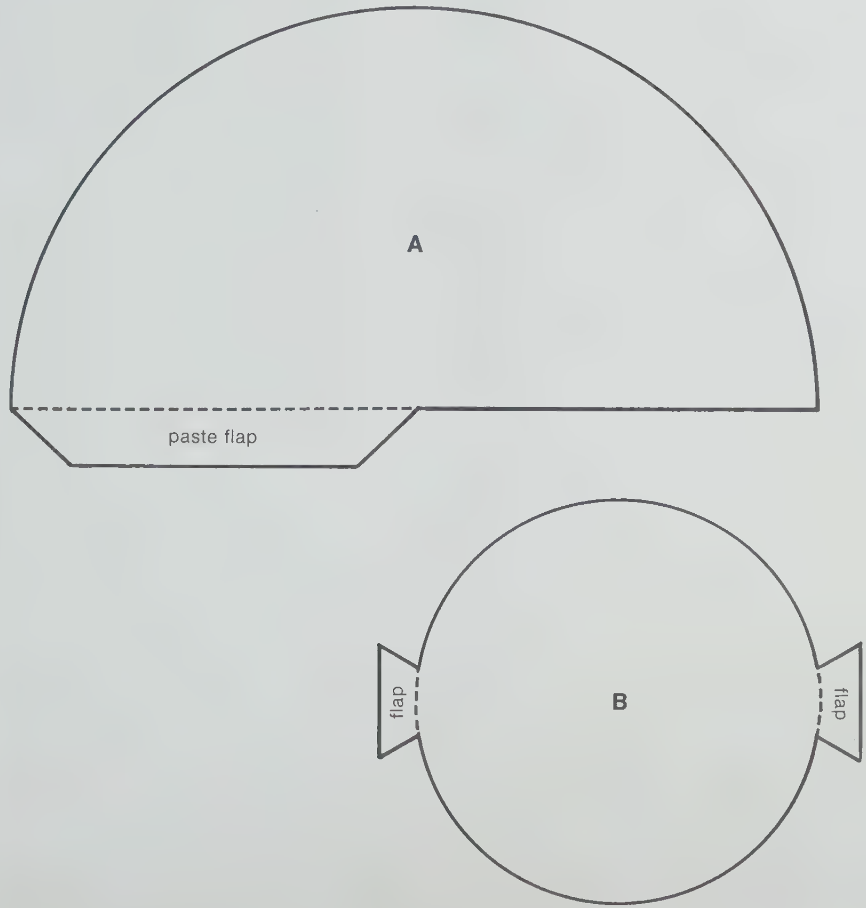








Try making a cone using this pattern. Bend part A carefully to form the curved surface of the cone. Tape it together. Tape the flat face, B, to part A to form the completed cone. Which other patterns on this page will form cones?



additional learning aids

concept – chapter objectives 1, 2, 3, 4

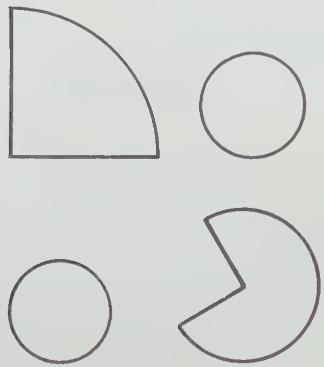
SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: G 1, 2
P 2

Mathematics Involvement Program, SRA (1971)
Cards: 81, 311, 122, 222, 232, 302, 342, 193, 203, 273, 44, 184, 45, 335, 176, 226

Skill through Patterns, level 4, SRA (1974)
Spirit masters: 7, 19

other learning aids (described on page 216e)
Geometry Figures and Solids, Good Time Mathematics, Polyhedra Model Kit, Shape Tracers, Soap-film Shapes



Do you think a pattern can be made for a sphere on a flat sheet of paper? Why?

8 COMPUTATION +, −, AND ×

before this chapter the learner has—

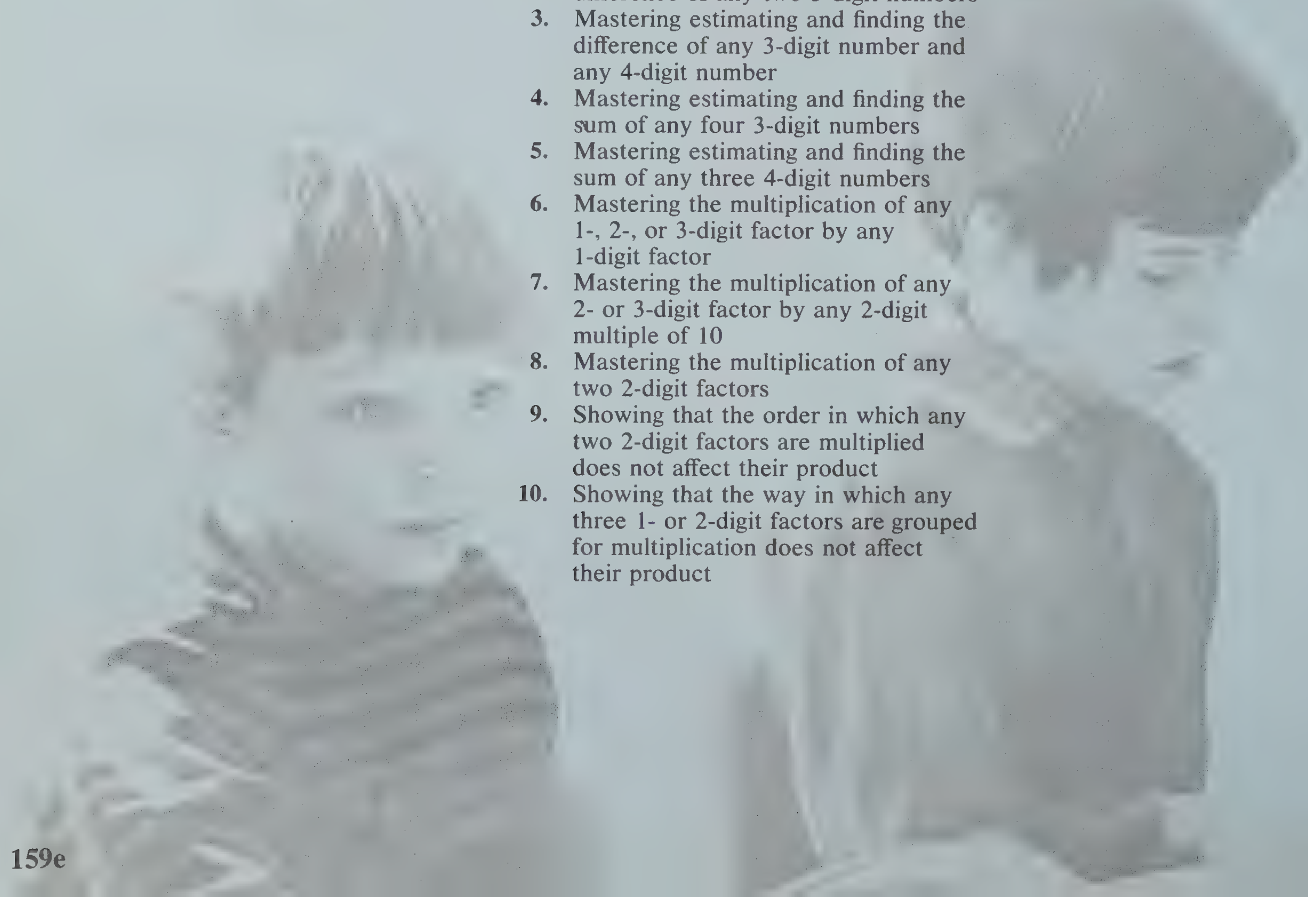
Developed the concepts and practiced the skills to be mastered in this chapter.

in chapter 8 the learner is—

1. Mastering estimating and finding the sum of any two 4-digit numbers
2. Mastering estimating and finding the difference of any two 3-digit numbers
3. Mastering estimating and finding the difference of any 3-digit number and any 4-digit number
4. Mastering estimating and finding the sum of any four 3-digit numbers
5. Mastering estimating and finding the sum of any three 4-digit numbers
6. Mastering the multiplication of any 1-, 2-, or 3-digit factor by any 1-digit factor
7. Mastering the multiplication of any 2- or 3-digit factor by any 2-digit multiple of 10
8. Mastering the multiplication of any two 2-digit factors
9. Showing that the order in which any two 2-digit factors are multiplied does not affect their product
10. Showing that the way in which any three 1- or 2-digit factors are grouped for multiplication does not affect their product

in later chapters the learner will—

1. Maintain the skills mastered
2. Apply and extend the concepts and skills mastered



Notes & Things

This chapter is different. Everyone can start together, but then look out! You will know in a very short time who has and who has not really mastered the skills of addition, subtraction, and multiplication.

This is a self-study chapter. Three major addition skills, two major subtraction skills, and three major multiplication skills have been isolated. These skills provide the thrust for this diagnostic, instructional review chapter. The grouping and order properties of multiplication are investigated apart from the sequence of skills.

This chapter, more than any other in the entire level, will give each learner a way to view his own skill development. Here is an objective basis for establishing his own personal learning goals. Nearly everything is here for a fresh start. The instructional review is based on models, with very few words. The practice material is carefully controlled so that the individual can study specific skills in addition, subtraction, and multiplication.

You, as the person responsible for math instruction, will also have the basis for objective evaluation of each pupil's progress. Much of the skills-study program for the rest of the year can be planned on the basis of this chapter. And it will be a means through which you can report to and plan with parents. Any of these skills that are not learned now will be a source of frustration for the learner in future study. But please, please do *not* give a letter grade for any diagnostic exercise. The motivation for building skills must come from self-evaluation, not from the threat of a bad grade.

The Resource Section at the end of the chapter contains many extra ideas for other approaches and more practice. Open up your bag of tricks too. Any additional effort spent now will be richly rewarded.

goal Think about and explore ideas through a picture clue

page 160 This photograph of grain harvesting will be viewed quite differently by youngsters who live in the city or on prairie land than by those who live in rural areas close to this sort of activity. But the geography and economics lesson that can be gained from this photo is appropriate for both groups.

An amazing range of numbers will be used by anyone who elects to do some research relating to farms. Try to get your researchers motivated with some direct questions. If we define a farm as a place where people raise crops or animals, what kind of farms can be identified? This photo shows part of a grain farm. What's grain? Would your life be different if there were no grain farms? What is grain used for? In what region of the country would you find grain farms? How large are they? How is grain sold? How much is it worth? (Have the newspaper handy for this one and be prepared to help the youngsters read the commodities market listing.) After the farmer sells it, where does it go? Can you trace the grain from the time it leaves the field until it gets to your house contained in a loaf of bread?

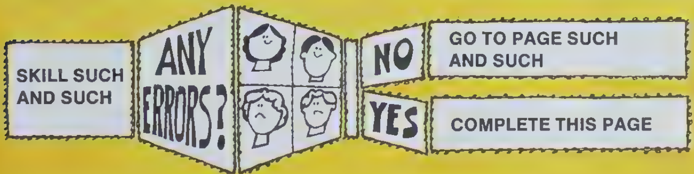
For one of your gifted learners the last question may provide a research project that will last for a long time. It's an exacting project and the resulting learning will open still another part of the world to that child.

Your goal is to check out on
skills you have had this year.
And learn a thing or two.
And then have time for FUN

This chapter is DIFFERENT

It's made so that you can work through the chapter by yourself.
But your teacher will want to give some advice.
It's made so that you can skip over parts you can prove you know.
It's made to help you with parts you don't know.

You'll be asked to work a couple of problems and then to check your answers. After that, you'll see a funny-looking chart that will look something like this:



This chart is a sort of road map. If you are honest with yourself and follow directions, you'll be finished in a short time. You will be proud to know what skills you have. You will be able to find those skills you need more practice with.

Everybody will start together. But that won't last long.

READY? SET! GO!

lesson Pages 161, 162, 163

goal Establishing the procedure to be used for this self-study chapter

memo All pupils complete pages 161, 162, and 163.

page 161 Chapter 8 is designed as a self-study unit of work. You will want to discuss its unique organization with everyone. The general rules are presented on the page. Emphasize these points:

- Each person will practice only the type of work he needs to practice.
- You, the teacher, will be available to help with directions and to help anyone who is not succeeding.
- Anyone who feels he needs more help should ask, not wait to be discovered.
- After the first day or two everyone will not be working on the same assignment at the same time.
- Some pupils will be directed to skip pages. That's part of the organization.

goal Examining the usefulness of estimation

memo This is a discussion page. Everyone will have good ideas to share.

page 162 In everyday situations we often need to know only **about how many.** An estimate is sufficient. Sometimes we need to know **exactly how many.** Then we require a computed answer—with no errors allowed. Estimation helps to check the computed answer too. A good estimate of an answer will signal whether or not the computed answer is **reasonable.**

The newspaper said that 76,500 people lived in the city. Do you believe it?

Are there exactly 76,500 people in that city? Probably not, bu
When? Might there be more? Might there At a point in time n
be fewer? Yes Yes

People move in. People move out. Babies are born. Some folks die. It would be a hard job to keep track of how many people lived in a city at any time.

The number is probably an *estimated* number. Our world is full of estimated numbers. Do you use them? How long does it take you to get home from school? Did you give an exact number or an estimate? Answers will vary.

What time will the game start? Exact or estimate? Answers will vary. Might be either one

How many math problems did you work in the last two weeks? You might not even be able to estimate that number. Accept reasonab

How much money will a new pair of shoes cost? If you really were buying a pair, you'd have to know the exact cost, including tax. Accept answers reasonable ran

How much money will you spend next week? Estimates will
Could you tell *exactly* how much money you spent last week? Probably not

Sometimes an estimated answer is all you need. But sometimes you need an exact answer. Estimated answers can help you find out if your exact answer is reasonable.

162

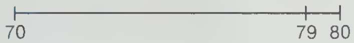
Before you can estimate an answer, you must round each
f the numbers you will compute. A round number is expressed
s an even number of tens, or hundreds, or thousands,
r ten-thousands, or . . . For example:

47 rounded to the nearest ten is 50.
167 rounded to the nearest hundred is 200.

ake another look at rounding. A number line can be a big help.

Is 32 closer to 30 or 40?

Is 79 closer to 70 or 80?



Is 25 closer to 20 or 30?

It's right in the middle, so round up to 30.

e ready to shift gears.

Is 180 closer to 100 or 200?

Is 435 closer to 400 or 500?

Is 1677 closer to 1000 or 2000?

Is 4500 closer to 4000 or 5000?
Halfway

ow see how rounding works
or estimation.

$$\begin{array}{r} 59 \\ +83 \\ \hline ? \end{array}$$ 142

Think

59 is closer to 60 than 50.
83 is closer to 80 than 90.
You know 60 + 80 is 140.
The answer should be about 140. Is it? Yes

163

goal Review of rounding numbers to the nearest ten, hundred, thousand

page 163 Rounded numbers are used when estimating. Estimating an answer should be done mentally.

Rounding to tens means that you will have zero ones. Rounding to hundreds means you will have zero tens and zero ones. You will work with the nearest hundred.

Learners who have had difficulty rounding seem to have the greatest success when using a number-line model.

lesson Page 164

goal Diagnosis of ability in and instructional review of estimating answers as a computational check

page 164 Take time to discuss the format of the page. Be blunt. The learner can either work the first problems to honestly determine what he can and cannot do or turn the book and copy the answers. *If you copy the answers, have you proven that you know this work?* An answer key will not appear on the Checkout. Encourage the pupils to be honest with themselves. Sometimes this is a bit hard for this age group to understand.

After the diagnostic problems have been completed and corrected, ensure that the flowchart is read correctly and each learner is headed in the right direction.

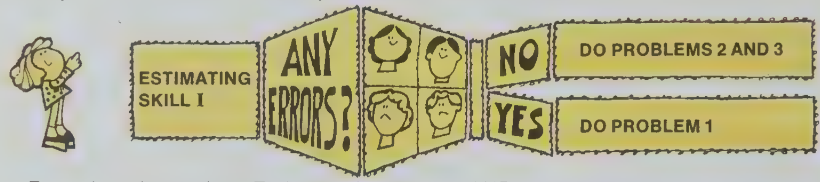
Estimate the answer. Do not compute.

a 59 people were there. 43 more came. How many in all? $60 + 40 = 100$

b 78 cars in the lot. 39 drove off. How many left? $80 - 40 = 40$

c There are 81 in each of 4 boxes. How many in all? $80 \times 4 = 320$

Check your answers with the key at the side of the page.



1. Round each number. Estimate the answer AND compute the exact answer too.

a
$$\begin{array}{rl} 63 & \text{rounds to } 60 \\ +29 & \text{rounds to } \underline{30} \\ \hline ? \text{ (exact)} & \quad ? \text{ (estimate)} \\ 92 & \quad 90 \end{array}$$

b
$$\begin{array}{rl} 77 & \text{rounds to } 80 \\ -42 & \text{rounds to } \underline{40} \\ \hline ? \text{ (exact)} & \quad ? \text{ (estimate)} \\ 35 & \quad 40 \end{array}$$

c
$$\begin{array}{rl} 29 & \text{rounds to } 30 \\ \times\ 3 & \qquad\quad \underline{3} \\ \hline ? & \qquad\quad ? \\ 87 & \qquad\quad 90 \end{array}$$

You generally do not round a 1-digit number when you multiply.

d $36 + 49 = ?$ 85
$40 + 50 = 90$

e $93 + 47 = ?$ 140
$90 + 50 = 140$

f $37 \times 5 = ?$ 185
$40 \times 5 = 200$

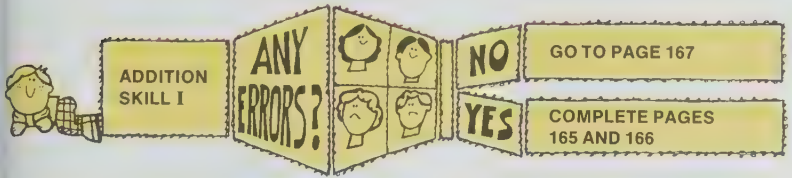
2. Think of a number.
Multiply it by 2.
Add 18 to the product.
Divide by 2.
Subtract the original number.
What is the result? 9
The result is always the same.
Check: try other numbers.

3. Multiply any number by 5.
Add 25 to the product.
Divide the sum by 5.
Subtract the number you started with.
Multiply that number by 3.
What is the result? 15
The result will always be the same.
Check: try other numbers.

Do these three addition problems.
First give an estimated sum. Then find the exact sum.

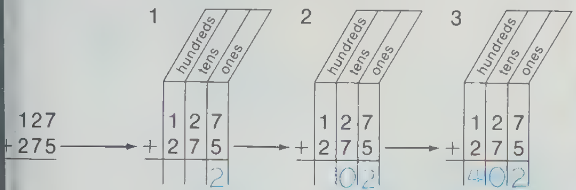
a 473	500	**b** 703	700	**c** 643	600
+ 389	+ 400	+ 498	+ 500	+ 675	+ 700
862	900	1201	1200	1318	1300

Check your answers with the key on page 166.



Tiger Transfer and Warehouse has tools in stock. They store the tools in their warehouse. They ship them out to customers when they have an order. They have a dock clerk who keeps a record of everything going in or out of the warehouse.

Here is a problem the dock clerk has to work. He really takes 3 steps to get the answer.



Is the dock clerk's answer a reasonable one? Yes
How do you know? By estimation
100
+ 300
400

See activity 1, page 188a.

lesson Pages 165, 166, 167, 168, 169, 170

goal Diagnosis of ability in and instructional review of addition of two 3-digit addends

memo Pages 165 through 170 focus on the addition skills to be mastered at this level.

things spirit master

page 165 The rules have been established. Everyone on his own.

Check errors carefully. Half of the battle is recognizing when it is necessary to rename. The following sets may be duplicated, then cut apart to better meet individual practice needs.

Set A Ring the columns that require renaming. Do not add.

29 + 30	57 + 62	64 + 35	84 + 48
99 + 19	44 + 44	76 + 49	31 + 49

Set B Ring the columns that require renaming. Do not add.

423 + 578	703 + 297	524 + 263
566 + 743	800 + 205	821 + 182

goal Practice in adding two 3-digit addends

page 166 Check these youngsters after the first two or three problems to make sure they are succeeding. If any are still having difficulty, check—

- Addition facts—especially with 2-digit sums
- Column addition of three 1-digit addends—skill needed when renaming
- Steps in renaming

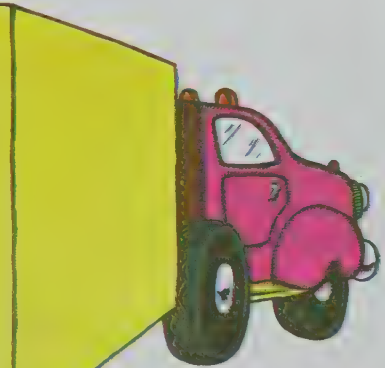
Emphasize estimating to check the reasonableness of an answer throughout this chapter.

You can check how reasonable the answer is by estimation. Round each addend to the nearest hundred and add.

$$\begin{array}{r} 127 \\ +\,275 \\ \hline 402 \end{array} \longrightarrow \begin{array}{r} 100 \\ +\,300 \\ \hline ? \end{array}$$

What do you get? 400 Is 402 close to your estimate? Yes
Is 402 a reasonable answer? Yes

Here is a record the dock clerk is filling out. Figure the new totals for him by adding the numbers in the second and third columns. Then check how reasonable your totals are by rounding and estimating. (Rounded numbers and estimates in parentheses)



TIGER TRANSFER AND WAREHOUSE
Hardware Inventory

Item	On hand	Received	Shipped	Total
1. Drill	127	275	——	402
2. Hammer	783	155	——	?
3. Hoe	341	782	——	?
4. Lawnmower	403	125	——	?
5. Pliers	789	957	——	?
6. Rake	374	451	——	?
7. Saw	213	323	——	?
8. Screwdriver	972	888	——	?
9. Shovel	215	346	——	?
10. Toolbox	154	237	——	?

(100)	(300)		(400)
(800)	(200)		(1000) 938
(300)	(800)		(1100) 1123
(400)	(100)		(500) 528
(800)	(1000)		(1800) 1746
(400)	(500)		(900) 825
(200)	(300)		(500) 536
(1000)	(900)		(1900) 1860
(200)	(300)		(500) 561
(200)	(200)		(400) 391

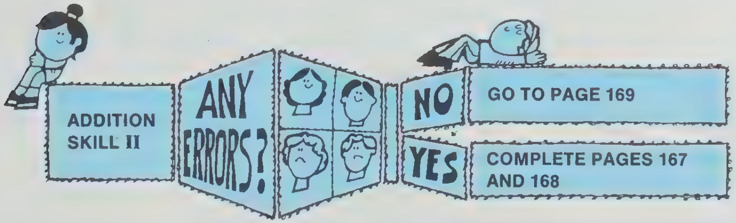
166

The largest numbers you have had to add in this book are 3-digit numbers. Do you think you could add larger numbers too? Try it.

Here are three problems. First give an estimated sum. Then find the exact sum.

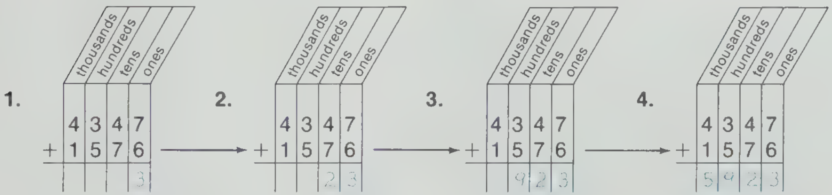
a	6219	6000	b	4763	5000	c	7279	7000
	+ 1582	+ 2000		+ 8819	+ 9000		+ 9321	+ 9000
	7801	8000		13,582	14,000		16,600	16,000

Check your answers with the key on page 168.



4347 pounds of nails are on hand in the warehouse. 1576 more pounds are received. How many pounds of nails does this total?

$$\begin{array}{r} 4347 \\ +\,1576 \\ \hline ? \end{array}$$



167

STOP See activity 2, page 188a.

goal Diagnosis of ability in and instructional review of addition of two 4-digit addends

things spirit master

page 167 Youngsters directed to the instructional review may need your help.

To check whether or not the pupil knows when to rename, duplicate the following sets. The skill is prerequisite to the renaming skill itself.

Set A Ring the columns that require renaming. Do not add.

5346 + 6954	3901 + 2058	7659 + 2530
6433 + 2959	3201 + 5906	1234 + 5678

Set B Ring the columns that require renaming. Do not add.

5639 + 4240	36 + 19	529 + 387	55 + 55
356 + 208	4361 + 5982	309 + 580	74 + 38

goal Practice in adding two 4-digit addends

page 168 Check accuracy after the first two or three problems. Assign additional practice as needed. If the first six problems are all correct, the learner should go on to page 169.

Is this a reasonable answer?

You can find out by estimating.
Round to the nearest thousand and add.

$$\begin{array}{r} 4347 \\ +1576 \\ \hline 5923 \end{array} \qquad \begin{array}{r} 4000 \\ +2000 \\ \hline ? \end{array} \ 6000$$

What do you think? Is 5923 reasonable? Yes

Compare this example with the one on page 166.
How are the two alike? How are they different?
You estimate first, then add. Page 166—you don't add thousands. Page 168—you add thousands.

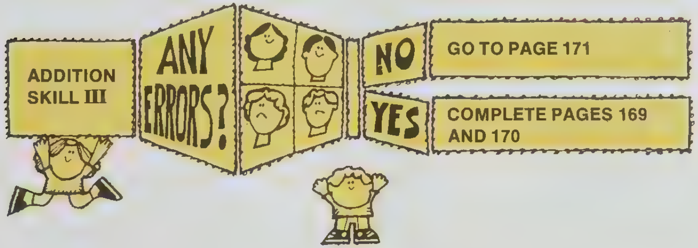
Try some problems like the example on this page.
Use estimation.
Decide whether your answers are reasonable.
(Estimated answer in parentheses)

1. 2345 + 6721 (9000) 9066	**2.** 5476 + 4231 (9000) 9707	**3.** 4987 + 3213 (8000) 8200
4. 1276 + 3982 (5000) 5258	**5.** 5143 + 2568 (8000) 7711	**6.** 8765 + 5934 (15,000) 14,699
7. 6848 + 7351 (14,000) 14,199	**8.** 2457 + 9876 (12,000) 12,333	**9.** 3843 + 5158 (9000) 9001
10. 5301 + 4999 (10,000) 10,300	**11.** 6770 + 1862 (9000) 8632	**12.** 6345 + 9724 (16,000) 16,069

Try these. Give an estimated sum and the exact sum.

a	436	400	**b**	7804	8000	**c**	880	900
	724	700		6123	6000		4892	4900
	351	400		+ 5291	+ 5000		76	100
	+ 832	+ 800		19,218	19,000		+ 369	+ 400
	2343	2300					6217	6300

Check your answers with the key on page 170.



One apartment building has 153 people living in it. Another has 222. A third has 154. And another has 76. How many people live in these four buildings?

Study this problem. It is done in several steps.

$$\begin{array}{r} 153 \\ 222 \\ 154 \\ +\ \ 76 \\ \hline \end{array}$$

1

	hundreds	tens	ones
		1	
	1	5	3
	2	2	2
	1	5	4
+		7	6
			5

2

	hundreds	tens	ones
	2	1	
	1	5	3
	2	2	2
	1	5	4
+		7	6
		0	5

3

	hundreds	tens	ones
	2	1	
	1	5	3
	2	2	2
	1	5	4
+		7	6
	6	0	5

goal Diagnosis of ability in and instructional review of adding columns of three and four addends with up to 4-digit numbers

page 169 HAVE PATIENCE!

Evaluate the skill carefully. It may be unrealistic for some learners even to attempt this work.

If your curriculum states that this skill is expected of all pupils, take time out with the group that is having trouble and try a completely different approach. Let the first problem serve as an example.

$$\begin{array}{r} 436 \\ 724 \\ 351 \\ +832 \\ \hline \end{array}$$

Break the problem into two problems.

$$\begin{array}{r} 436 \\ +724 \\ \hline \end{array} \quad \text{and} \quad \begin{array}{r} 351 \\ +832 \\ \hline \end{array}$$

Have the pupils find the two sums. Now don't tell them; rather, let them tell you what has to be done next. Making two separate addition problems plus a third—adding the two sums—is not sure-fire. The chances of increasing pupil error in recopying the numbers is great. Recopying errors is heartbreaking, especially when the sum is correct for the incorrect numbers. Estimation will help rule out silly answers. Perhaps having another way to add "big" problems will serve as a powerful motivation to be extra careful in recopying the numbers.

goal Practice in adding columns of three and four addends with up to 4-digit numbers

page 170 Once again, check progress after the third problem. If everything is O.K., have the pupils skip to problems 9 and 10. The correct answers for these depend as much on knowledge of place value and its importance in computing as on the addition skill.

Make additional assignments as needed. Do not spend over one additional day on practice.

Is this answer reasonable?

$$\begin{array}{r} 153 \\ 222 \\ 154 \\ +\ \ 76 \\ \hline 605 \end{array} \qquad \begin{array}{r} 200 \\ 200 \\ 200 \\ +100 \\ \hline ? \end{array}\ 700$$

Why do you suppose the estimated sum and the exact sum are so far apart? Rounded both 153 and 154 up to 200

Here is some practice.
Give both estimated sums and exact sums.

(Estimated answer in parentheses)

1. $\begin{array}{r} 368 \\ 92 \\ +547 \\ \hline \end{array}$ (1000) 1007

2. $\begin{array}{r} 476 \\ 108 \\ 354 \\ +285 \\ \hline \end{array}$ (1300) 1223

3. $\begin{array}{r} 2832 \\ 3705 \\ +4400 \\ \hline \end{array}$ (11,000) 10,937

4. $\begin{array}{r} 7661 \\ 6839 \\ +8009 \\ \hline \end{array}$ (23,000) 22,509

5. $\begin{array}{r} 8 \\ 24 \\ 821 \\ +954 \\ \hline \end{array}$ (1830) 1807

6. $\begin{array}{r} 728 \\ 931 \\ 640 \\ +351 \\ \hline \end{array}$ (2600) 2650

7. $\begin{array}{r} 766 \\ 130 \\ 952 \\ +417 \\ \hline \end{array}$ (2300) 2265

8. $\begin{array}{r} 6797 \\ 6318 \\ +9433 \\ \hline \end{array}$ (22,000) 22,548

9. $56 + 781 + 449 + 8 = ?$ (1300) 1294

10. $7551 + 899 + 6420 = ?$ (15,000) 14,870

170

Just for fun

ANNA is a girl's name. Spell it backward. ANNA

BOB is a boy's name. Spell it backward. BOB

These words are called palindromes. Palindromes can also be numbers. Let's focus on the numbers.

Take this number.	15	
Reverse it.	51	
Add.	66	AHA! A palindrome!

Take another number.	85	
Reverse it.	58	
Add.	143	It doesn't work!
Try again. Reverse it.	341	
Add.	484	It does work! Another palindrome!

Take a larger number.	123	
Reverse it.	321	
Add.	444	But will it always work?

Sometimes you can find a number palindrome by reversing only once. Sometimes you must reverse twice. Sometimes you must reverse even more times.

Do you think there are some numbers that cannot be made into a palindrome? You can't check all of the numbers in the world, but you can check some.

Pick several 2-digit numbers. Can you find one that is not a palindrome? Don't forget you may have to reverse the digits more than once. Don't give up too soon.

Now try some 3-digit numbers.

If you are really ready for adventure, try some 4-digit numbers.

See activity 3, page 188a.

lesson Page 171

goal Exploration of PALINDROMES; having fun with mathematics

memo This is an extension page that students of all abilities can handle—and get a lot of practice too!

page 171 You must agree that every learner, no matter how capable or slow, has just accomplished a big job. Be generous with your praise. Now relax and

enjoy

If for some reason you find certain pupils are hopelessly behind, you may want to save this page for them to use at another time. Don't forget about its high-motivation drill aspect, however. Everyone should have a chance at it sooner or later.

lesson Pages 172, 173, 174, 175

goal Diagnosis of ability in and instructional review of subtracting two 3-digit numbers

memo Pages 172 through 175 focus on the subtraction skills to be mastered at this level.

things spirit master

page 172 Continue as before.

Knowing when to rename and how to rename are prerequisite skills for mastering subtraction that requires renaming. The two sets of problems shown below can be duplicated on a spirit master. Then cut the sets apart to better meet individual practice needs.

Set A Ring the problems that require renaming.

$43 - 28$	$80 - 36$	$52 - 28$	$77 - 33$
$88 - 29$	$63 - 40$	$92 - 29$	$31 - 14$

Go back. Show the renaming. Do not subtract.

Set B Ring the problems that require renaming.

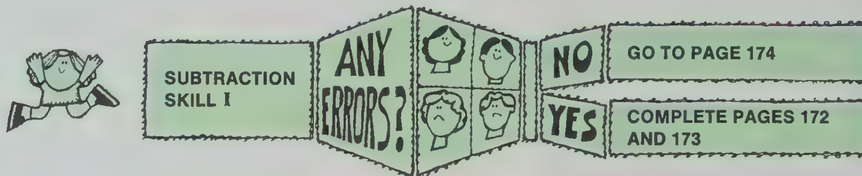
$576 - 293$	$809 - 256$	$403 - 249$
$376 - 248$	$600 - 229$	$472 - 198$

Go back. Show the renaming. Do not subtract.

Do these three subtraction problems. Give an estimated difference. Also give the exact difference.

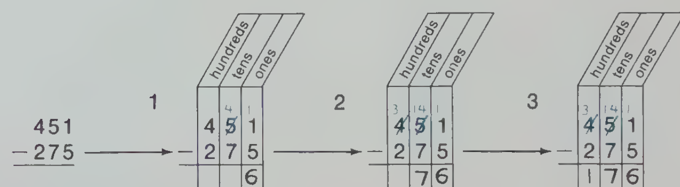
a	$639 - 246 = 393$	$600 - 200 = 400$
b	$543 - 458 = 85$	$500 - 500 = 0$
c	$800 - 271 = 529$	$800 - 300 = 500$

Check your answers with the key on page 173.



Tiger Transfer and Warehouse also handles a number of houseware items. Some of these are shown on the inventory given on the next page.

Here is a sample problem that the dock clerk must work for goods being shipped. It is taken from the inventory sheet. The dock clerk finds the answer in three steps.



Is this answer reasonable? How can you tell? By estimating
Yes
500
− 300
200

172

See activity 1, page 188a.

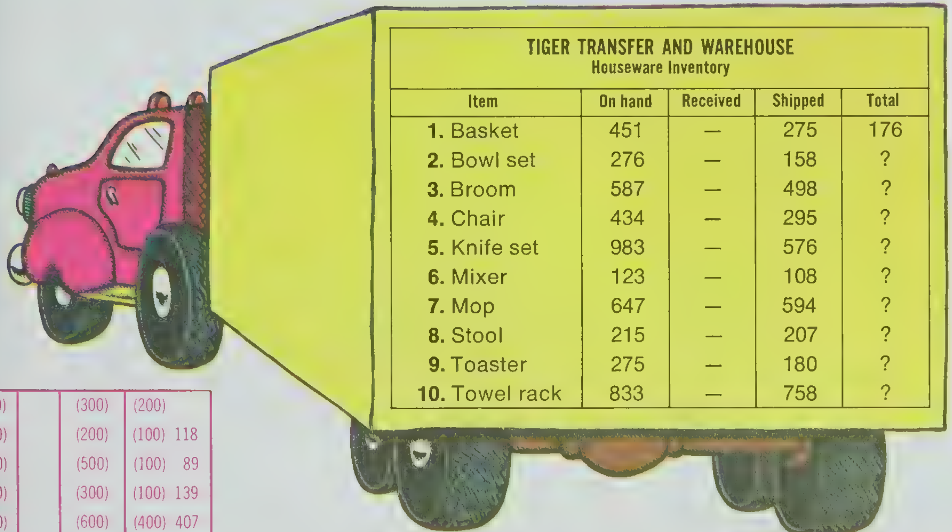
You can use estimation to tell whether the answer is reasonable. Round to the nearest hundred and subtract.

$$\begin{array}{r} 451 \\ -\ 275 \\ \hline 176 \end{array} \longrightarrow \begin{array}{r} 500 \\ -\ 300 \\ \hline ? \end{array}$$

What do you think? Is the answer 176 reasonable? Yes

Figure the new totals for the clerk. Subtract the numbers in the "Shipped" column from the totals in the "On hand" column. Then check how reasonable your totals are by rounding and estimating.

(Rounded numbers and estimates in parentheses)



TIGER TRANSFER AND WAREHOUSE
Houseware Inventory

Item	On hand	Received	Shipped	Total
1. Basket	451	—	275	176
2. Bowl set	276	—	158	?
3. Broom	587	—	498	?
4. Chair	434	—	295	?
5. Knife set	983	—	576	?
6. Mixer	123	—	108	?
7. Mop	647	—	594	?
8. Stool	215	—	207	?
9. Toaster	275	—	180	?
10. Towel rack	833	—	758	?

(500)	(300)	(200)	
(300)	(200)	(100)	118
(600)	(500)	(100)	89
(400)	(300)	(100)	139
(1000)	(600)	(400)	407
(100)	(100)	(0)	15
(600)	(600)	(0)	53
(200)	(200)	(0)	8
(300)	(200)	(100)	95
(800)	(800)	(0)	75

173

goal Practice in subtracting two 3-digit numbers

page 173 Check progress after the youngster has completed the first three problems. Continued failure could mean that the learner—

- Has not mastered the basic subtraction facts—particularly subtracting from a teen number
- Does not understand the basic steps involved in renaming
- Does not completely understand place value

Provide encouragement and praise when a job is well done.

goal Diagnosis of ability in and instructional review of subtracting a 2- and 3-digit number from a 4-digit number

memo Please, please, please, don't let a youngster attempt pages 174 and 175 if he has had little or no success on 172 and 173.

page 174 Zero will continue to be the digit that causes more headaches than any other. Estimates will help to eliminate some careless thinking, but they will not catch all the computational errors.

If renaming continues to be troublesome, work with children individually and try to convince them to complete all renaming before any subtraction is done. It's hard to do, but sometimes it is the only way to prevent mistakes. Use the following two sets for additional practice.

Set A Ring the problems that require renaming.

$$\begin{array}{r} 4261 \\ -\ 549 \\ \hline \end{array} \qquad \begin{array}{r} 5000 \\ -\ 460 \\ \hline \end{array} \qquad \begin{array}{r} 7231 \\ -\ 856 \\ \hline \end{array}$$

$$\begin{array}{r} 8579 \\ -2436 \\ \hline \end{array} \qquad \begin{array}{r} 2597 \\ -\ 865 \\ \hline \end{array} \qquad \begin{array}{r} 1205 \\ -\ 184 \\ \hline \end{array}$$

Go back. Show the renaming. Do not subtract.

Set B Ring the problems that require renaming.

$$\begin{array}{r} 3560 \\ -\ 291 \\ \hline \end{array} \qquad \begin{array}{r} 800 \\ -206 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ -37 \\ \hline \end{array} \qquad \begin{array}{r} 529 \\ -276 \\ \hline \end{array}$$

$$\begin{array}{r} 80 \\ -19 \\ \hline \end{array} \qquad \begin{array}{r} 424 \\ -242 \\ \hline \end{array} \qquad \begin{array}{r} 84 \\ -48 \\ \hline \end{array} \qquad \begin{array}{r} 1859 \\ -\ 379 \\ \hline \end{array}$$

Go back. Show the renaming.

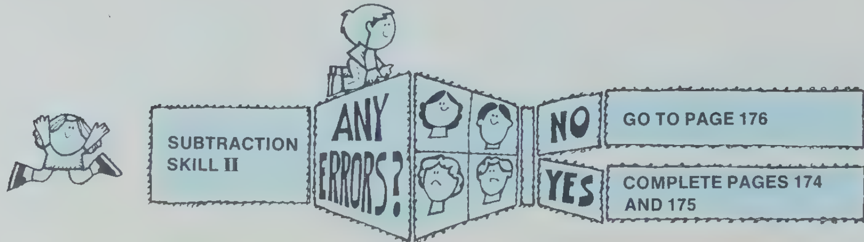
Try these. Give estimated differences and exact differences.

a $\begin{array}{r} 4280 \\ -\ 755 \\ \hline 3525 \end{array} \quad \begin{array}{r} 4300 \\ -\ 800 \\ \hline 3500 \end{array}$

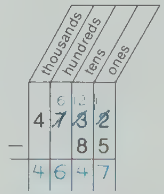
b 6007 − 62 5945
6000 − 100 = 5900

c 8612 − 922 7690
8600 − 900 = 7700

Check your answers with the key on page 175.



The attendance at the hockey game was 4732. 85 of the people had free passes. How many people paid for tickets?



Do you notice how the numbers are lined up for subtracting? Are they lined up the same way as they were in addition? Why is the 8 in 85 under the 3 in 4732? Yes 8 tens and 3 tens are recorded in the tens column.

Now consider this: Is the answer 4647 reasonable? Let's estimate to find out. What do you think we should round to? Why? Hundreds, or possibly tens
85 is close to 100, but rounding to tens gives an estimate closer to the actual answer. (How sure do you need to be about the actual paid attendance?)

174

See activity 2, page 188a.

$$\begin{array}{r} 4732 \\ -\ 85 \\ \hline 4647 \end{array} \longrightarrow \begin{array}{r} 4700 \\ -\ 100 \\ \hline ? \end{array}$$

4600

Well, is the answer reasonable? Yes

Here are some exercises. Be sure to check whether your answers are reasonable.

1. $\begin{array}{r} 5974 \\ -\ 885 \\ \hline \end{array}$ 5089

2. $\begin{array}{r} 7113 \\ -\ 672 \\ \hline \end{array}$ 6441

3. $\begin{array}{r} 3212 \\ -\ 633 \\ \hline \end{array}$ 2579

4. 157 − 89 = ? 68

5. 4250 − 315 = ? 3935

6. 7335 − 426 = ? 6909

7. 6389 − 693 = ? 5696

8. 4328 − 797 = ? 3531

***9.** 71,463 − 2494 = ? 68,969

***10.** 13,275 − 8066 = ? 5209

***11.** 643,545 − 6435 = ? 637,110

***12.** A human being lives about 27,010 days in a lifetime. If today is your ninth birthday, you have already lived 3287 days. How many days would this leave you? (Do you think everyone lives 27,010 days? What do you think 27,010 days really means in this problem?) About 23,723 No

It's probably an estimate based on an average.

goal Practice in subtracting two numbers that do not have the same number of digits

page 175 Mastery expectations for this level are covered by problems 1 through 8. Starred problems provide an added challenge and extend beyond the mastery expectations. Use your discretion in assigning these problems.

lesson Page 176

goal Examining some number puzzles

page 176 The youngsters have been working very hard, particularly those who have had much extra practice. Praise these pupils for their stamina and progress. The most capable have shot way out ahead, since their skills check out without the need for additional practice.

Challenge the capable to work independently. They have sufficient time and ability. Attack these puzzles as a group exploration with the remaining students. All learners should have some experience with them. Math can be fun!—at times, anyway.

Just for fun

Do these puzzles. Test several numbers to make sure they really work. Then try to figure out why they work. If you can figure them out—GREAT. If you can't figure them out, just enjoy the puzzle itself.

Think of a number from 1 through 8.
Add 9 to get a sum.
Double the sum. (Division by 2 undoes multiplication (doubling) by 2.)
Divide by 2.
Subtract the number you picked. (This removes your original number and leaves 9, which you added in the second step.)
The last number is always 9.

WHY?

Step	□ = your number	
Think of a number from 1 through 9.	□ + 1	(+1)
Add 1 and multiply by 3.	3□ + 3	(×3)
	3□ + 5	(+2)
Add 2 and multiply by 4.	12□ + 20	(×4)
	12□ + 21	(+1)
Add 1 and divide by 3.	4□ + 7	(÷3)
	4□ + 8	(+1)
Add 1 and divide by 4.	□ + 2	(÷4)
Subtract the number you picked.	2	(−□)
The answer is always 2.		

WHY? (This is a very hard one to answer the why.)

And just one more.
Don't worry about the "why" on this one.

Take any number. Write it down. Reverse the order of the digits to get a second number. Subtract the smaller number from the larger number. That answer will *always* be exactly divisible by 9.

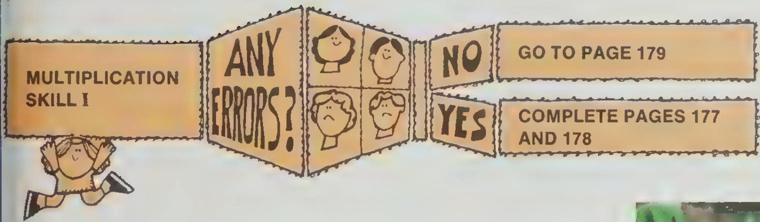
176

See activity 4, page 188b.

Can you work problems like these?
See if you can get the right answers.

a	b	c
$\begin{array}{r} 45 \\ \times\ 9 \\ \hline 405 \end{array}$	$\begin{array}{r} 743 \\ \times\ 5 \\ \hline 3715 \end{array}$	$\begin{array}{r} 409 \\ \times\ 7 \\ \hline 2863 \end{array}$

Check your answers with the key on page 178.



Myrtle is in school 35 hours a week. How many hours is this in 6 weeks?

$$\begin{array}{rl} 35 & \\ \times\ 6 & \\ \hline 30 & 6 \times 5 \\ 180 & 6 \times 30 \\ \hline 210 & \end{array}$$

We think of 35 as 30 + 5 and multiply 5 by 6 to get part of the product.

Why multiply 6 × 5 to get 30?

Why multiply 6 × 30 to get 180? To get the other part of the product

How do you get 210? 30 + 180 = 210
(Add the parts to get the product.)

GOT THE IDEA? Try doing the problems on page 178.

lesson Pages 177, 178, 179, 180, 181, 182

goal Diagnosis of ability in and instructional review of multiplying by a 1-digit factor

memo Mastery expectations in multiplication for this level are the focus of pages 177 through 182.

page 177 Cross your fingers that no child makes errors on this page because he doesn't know the multiplication facts. Ugh! If these errors occur, either stop and give still more fact practice or give that child a completed multiplication table to use as a reference. It is much better to use a table and copy the right product than to continuously reinforce an incorrect product by writing it again and again in problems such as those on this and the following pages.

goal Practice in multiplying by a 1-digit factor

page 178 You'll want to check for errors after the first two or three problems are completed. Multiplication-fact errors can be checked with exercise 21.

Other possible causes for errors include—

- Zero trouble when multiplying a multiple of 10 or of 100. Try working with patterns.

$$\begin{array}{r} 5 \\ \times\ 3 \\ \hline 15 \end{array} \qquad \begin{array}{r} 50 \\ \times\ 3 \\ \hline 150 \end{array} \qquad \begin{array}{r} 500 \\ \times\ 3 \\ \hline 1500 \end{array}$$

- Partial products are not aligned in columns to simplify addition. Here is a common error.

$$\begin{array}{r} 76 \\ \times\ 3 \\ \hline 18 \\ 210 \\ \hline 2118 \end{array}$$

Encourage the use of this form.

$$\begin{array}{rl} 76 & \\ \times\ 3 & \\ \hline 18 & (3 \times 6) \\ 210 & (3 \times 70) \\ \hline 228 & \end{array}$$

The pupil should not write these thinking steps.

It will also help to turn lined paper sideways to provide columns.

1. 43×6 — 258
2. 58×3 — 174
3. 91×7 — 637
4. 27×9 — 243
5. 49×4 — 196
6. 68×8 — 544
7. 156×5 — 780
8. 732×6 — 4392
9. 271×5 — 1355
10. 505×7 — 3535
11. 464×8 — 3712
12. 340×9 — 3060
13. 390×4 — 1560
14. 250×9 — 2250
15. 278×7 — 1946
16. 987×6 — 5922
17. 807×5 — 4035
18. 613×8 — 4904
19. There are 479 children in Morton's school. If each child sells 5 tickets to the school fashion show, how many tickets will this be? 2395
20. 789 people are sitting in a movie theater watching the show. Each has paid $3 to get in. How much money is this? $2367
21. Copy and complete this chart.

How many in all?	2 in each box	4 in each box	5 in each box	6 in each box	8 in each box	10 in each box
3 boxes	a 6	b 12	c 15	d 18	e 24	f 30
5 boxes	g 10	h 20	i 25	j 30	k 40	l 50
7 boxes	m 14	n 28	o 35	p 42	q 56	r 70
9 boxes	s 18	t 36	u 45	v 54	w 72	x 90

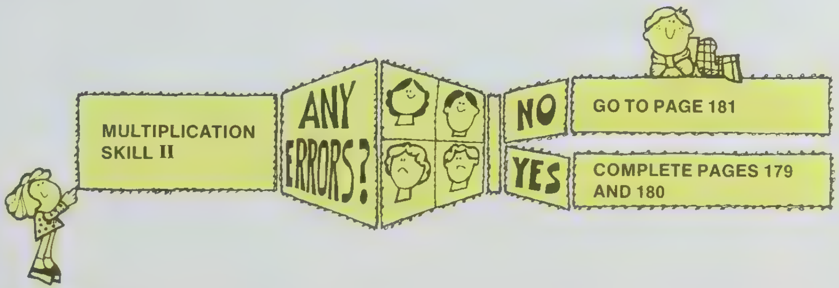
Now see if you can get the right answers to these problems.

a $\begin{array}{r} 72 \\ \times 80 \\ \hline \end{array}$ 5760

b $\begin{array}{r} 429 \\ \times\ 30 \\ \hline \end{array}$ 12,870

c $\begin{array}{r} 980 \\ \times\ 70 \\ \hline \end{array}$ 68,600

Check your answers with the key on page 180.



In the last section you learned that Myrtle is in school 35 hours a week. How many hours does she spend in school in 30 weeks?

$$\begin{array}{rl} 35 & \\ \times 30 & \\ \hline 150 & (30 \times 5) \quad \text{Why multiply } 30 \times 5? \\ 900 & (30 \times 30) \quad \text{Why multiply } 30 \times 30? \\ \hline 1050 & \longleftarrow \text{Where did that come from?} \end{array}$$

Here is some practice for you.

1. $\begin{array}{r} 36 \\ \times 40 \\ \hline \end{array}$ 1440

2. $\begin{array}{r} 51 \\ \times 70 \\ \hline \end{array}$ 3570

3. $\begin{array}{r} 29 \\ \times 30 \\ \hline \end{array}$ 870

4. $\begin{array}{r} 63 \\ \times 80 \\ \hline \end{array}$ 5040

5. $\begin{array}{r} 35 \\ \times 20 \\ \hline \end{array}$ 700

6. $\begin{array}{r} 76 \\ \times 90 \\ \hline \end{array}$ 6840

7. $\begin{array}{r} 614 \\ \times\ 80 \\ \hline \end{array}$ 49,120

8. $\begin{array}{r} 892 \\ \times\ 70 \\ \hline \end{array}$ 62,440

9. $\begin{array}{r} 425 \\ \times\ 60 \\ \hline \end{array}$ 25,500

10. $\begin{array}{r} 503 \\ \times\ 50 \\ \hline \end{array}$ 25,150

11. $\begin{array}{r} 972 \\ \times\ 40 \\ \hline \end{array}$ 38,880

12. $\begin{array}{r} 780 \\ \times\ 30 \\ \hline \end{array}$ 23,400

goal Diagnosis of ability in, instructional review of, and practice in multiplying by a multiple of 10

page 179 Check progress after problem 6 is completed. Does the youngster who continues to make errors know how to multiply by a multiple of 10? This problem is addressed at the top of page 180. Have this pupil skip the remainder of this page and take him directly to page 180.

Pupils who successfully complete problems 1 through 12 can go on to page 181.

goal Instructional review of and practice in multiplying by a multiple of 10

page 180 Your slowest learners will not be able to follow the development independently. They will require your help.

Emphasize—

- When multiplying by ten, the product must show tens. *How can a number be made to show tens?* (Zero in ones position)
- Each multiple of 10 has 10 as a factor hidden in it.

$70 = 7 \times 10$
$30 = 3 \times 10$

- We don't **see** this; we **think** it.

To multiply	**Think**	
5×30	5×3	That's no problem!
	$5 \times 3 = 15$	
	15×10	That's no problem either!
	$15 \times 10 = 150$	

This may not be obvious to learners as they follow the pattern at the top of the page.

There is a shortcut for multiplying tens.
The pattern at the right will give you a clue.

$1 \times 10 = 10$
$5 \times 10 = 50$
$9 \times 10 = 90$
$10 \times 10 = 100$
$15 \times 10 = 150$
$18 \times 10 = 180$
$100 \times 10 = 1000$
$125 \times 10 = 1250$

THINK 3 tens.

$1 \times 30 = 30$
No surprise.

$5 \times 30 = 150$
THINK $(5 \times 3) \times 10$.

$9 \times 30 = 270$
THINK $(9 \times 3) \times 10$.

$15 \times 30 = 450$
THINK $(15 \times 3) \times 10$.

Look at a more difficult problem.

$$\begin{array}{r} 469 \\ \times\ 80 \\ \hline \end{array}$$

This can be thought of as

$$\begin{array}{r} 469 \\ \times\ 8 \text{ tens} \\ \hline \end{array}$$

You certainly know how to do that. There is only one thing you MUST remember. You are multiplying tens. The answer you get in the shortcut must be multiplied by 10. That's easy enough. $3752 \cdot 10 = 37{,}520$

Try the shortcut on these.

1. $\begin{array}{r} 73 \\ \times 20 \\ \hline 1460 \end{array}$
2. $\begin{array}{r} 49 \\ \times 30 \\ \hline 1470 \end{array}$
3. $\begin{array}{r} 163 \\ \times\ 40 \\ \hline 6520 \end{array}$
4. $\begin{array}{r} 52 \\ \times 50 \\ \hline 2600 \end{array}$
5. $\begin{array}{r} 348 \\ \times\ 60 \\ \hline 20{,}880 \end{array}$
6. $\begin{array}{r} 96 \\ \times 70 \\ \hline 6720 \end{array}$
7. $\begin{array}{r} 587 \\ \times\ 80 \\ \hline 46{,}960 \end{array}$
8. $\begin{array}{r} 607 \\ \times\ 90 \\ \hline 54{,}630 \end{array}$

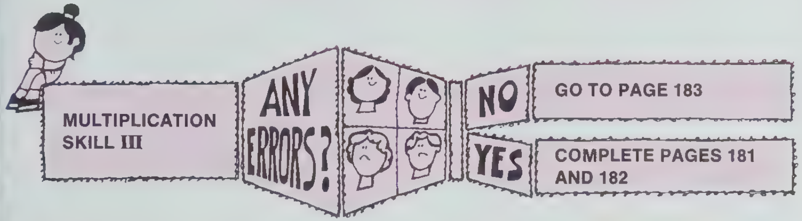
180

STOP See activity 5, page 188b.

Try these problems. Give both estimated and exact answers.

a	78	80	**b**	14	10	**c**	86	90
	× 64	× 60		× 92	× 90		× 57	· 60
	4992	4800		1288	900		4902	5400

Check your answers with the key on page 182.



Myrtle goes to school 36 weeks out of the year. How many hours of school is this? (Remember—she goes 35 hours a week.)

Myrtle did the problem this way →

$$\begin{array}{rl} 35 & \\ \times\,36 & \\ \hline 30 & 6 \times 5 \\ 180 & 6 \times 30 \\ 150 & 30 \times 5 \\ 900 & 30 \times 30 \\ \hline 1260 & \text{Is this reasonable?} \end{array}$$

Her mom checked her work. She did it this way →

$$\begin{array}{r} 35 \\ \times\,36 \\ \hline 210 \\ 1050 \\ \hline 1260 \end{array}$$

Both Myrtle and her mom had estimated the answer. →

$$\begin{array}{r} 40 \\ \times\,40 \\ \hline 1600 \end{array}$$

The estimate and the exact answer aren't very close. Can you figure out why? At least your estimate tells you that you haven't made a big mistake. An answer of 12,600 or 126 would be a big mistake!

Rounded 35 and 36 up to 40. This accounts for a larger estimate.

When you do multiplication, you can use the form that Myrtle or her mom used.

goal Diagnosis of ability in and instructional review of multiplying two 2-digit factors

page 181 Those who need additional practice can use either the long or the short form—the one that each individual feels most comfortable with. Watch for addition errors in the addition of partial products.

The instructional review continues on page 182.

goal Instructional review of and practice in multiplying two 2-digit factors

page 182 Pupils need not write the **think** steps shown in color. Their focus should be on writing the partial products and on keeping the place value organized enough to be able to finally add those partial products.

Compare your estimate and exact product on this one.

EXACT PRODUCT (ONE WAY OF WRITING THE PROBLEM)

79	
× 34	
36	4 × 9
280	4 × 70
270	30 × 9
2100	30 × 70
? 2686	

(ANOTHER WAY)

79	
× 34	
316	4 × 79
2370	30 × 79
? 2686	

ESTIMATE (FOR BOTH FORMS)

80
× 30
? 2400

In finding the estimate, what place do we round 79 to? Why? What place do we round 34 to? Why? 34 is close to 30.
Tens 79 is close to 80. Tens
What is the exact product? Is it close to 2400? Yes
2686

Here is some practice. Find both the exact product and the estimated product. (Estimated answer in parentheses. Partial products not shown in answers.)

1. 46 × 31 (1500) 1426	2. 51 × 65 (3500) 3315	3. 29 × 72 (2100) 2088	4. 94 × 62 (5400) 5828	5. 87 × 54 (4500) 4698	6. 38 × 97 (4000) 3686
7. 74 × 62 (4200) 4588	8. 92 × 43 (3600) 3956	9. 59 × 24 (1200) 1416	10. 68 × 81 (5600) 5508	11. 48 × 32 (1500) 1536	12. 87 × 54 (4500) 4698

182

This page is for everyone

So far this year you have done 182 pages of mathematics (but you're not through yet). If there are 26 kids in your class, how many pages in all have been done? (Or have you done that many?) 4732

How many in all were done by the 6 people in the first row? 1092

How many in all were done by the other 20? 3640

How many in all were done by the 26? 4732

$$\begin{array}{r} 182 \\ \times\ \ 6 \\ \hline 1092 \end{array} \qquad \begin{array}{r} 182 \\ \times\ 20 \\ \hline 3640 \end{array} \qquad \begin{array}{rl} 182 & \\ \times\ 26 & \\ \hline 1092 & 6 \times 182 \\ 3640 & 20 \times 182 \\ \hline 4732 & \end{array}$$

In the third problem, why did we multiply 6×182? To get part of the product

Why did we multiply 20×182? To get the other part of the product

How did we get 4732? $1092 + 3640 = 4732$ — add the parts to get the product.

Try doing this problem yourself. Use the shortcut.

$$\begin{array}{r} 563 \\ \times\ 54 \\ \hline ? \end{array}$$

What answer did you get? Let's go through the problem together step by step.

$$\begin{array}{rl} 563 & \\ \times\ 54 & \\ \hline 2252 & \text{How do we get 2252?} \quad 4 \times 563 \\ 28150 & \text{Where does 28,150 come from?} \quad 50 \cdot 563 \\ \hline 30402 & \text{Explain 30,402.} \quad 2252 + 28{,}150 = 30{,}402 \end{array}$$

Do you think you can work problems like this by using the shortcut? **Try it.** Give both estimated and exact answers.

183

lesson Pages 183, 184

goal Development of the short form for multiplying by a 2-digit factor

memo This page is for **everyone.** The degree to which this short form is practiced and mastered will be determined by your expectations and the curriculum of your school.

page 183 Using the long form with this type of problem often becomes unmanageable for the pupil. If a pupil has used the long form exclusively, it will be hard for him to change. Consider having a peer tutor. Also help by standing by to give encouragement during the transitional change.

goal Practice using the short form in multiplying by a 2-digit factor

page 184 Much practice is provided. Use your discretion in making assignments—problems 1 through 4 will test the youngsters' ability to operate. Note that problems 16 through 18 are challenges and may **not** be for all pupils.

Multiply. Use the shortcut. (Partial products not shown in answers.)

1.	2.	3.	4.	5.	6.
48	96	818	775	927	84
× 25	× 37	× 94	× 62	× 98	× 67
1200	3552	76,892	48,050	90,846	5628

7.	8.	9.	10.	11.	12.
67	708	707	393	675	961
× 51	× 77	× 89	× 71	× 48	× 65
3417	54,516	62,923	27,903	32,400	62,465

13. Mr. Zee worked 8 hours a day, 5 days a week, 50 weeks a year. How many hours does he work in one year? 2000

14. Mr. Zee is the kind of guy who needs 8 hours of sleep every night. If he gets that much sleep every day, how many hours does he sleep in a week? in the 52 weeks during a year? 2912
56

15. Mr. Zee watched TV 4 hours every day in the year. How many hours does he spend watching TV in a year? 1460

If you are brave, try these. You don't have to do them.
Answers will vary. Accept any reasonable answers.

16. About how many hours of TV do you watch in a year's time?

17. How many hours do you spend in school each day? each week?

18. How many hours do you spend in school during a school year? (Assume 180 days in one school year.)

Before you go on, everyone should study these pages.

Copy the problems and multiply. (Partial products not shown in answers.)

a	$\begin{array}{r} 9 \\ \times 7 \\ \hline 63 \end{array}$	$\begin{array}{r} 7 \\ \times 9 \\ \hline 63 \end{array}$	**b**	$\begin{array}{r} 3 \\ \times 12 \\ \hline 36 \end{array}$	$\begin{array}{r} 12 \\ \times 3 \\ \hline 36 \end{array}$	**c**	$\begin{array}{r} 15 \\ \times 23 \\ \hline 345 \end{array}$	$\begin{array}{r} 23 \\ \times 15 \\ \hline 345 \end{array}$	**d**	$\begin{array}{r} 57 \\ \times 97 \\ \hline 5529 \end{array}$	$\begin{array}{r} 97 \\ \times 57 \\ \hline 5529 \end{array}$

Look at each pair of answers. What can you say about changing the order of the numbers you multiply? The order in which you multiply two numbers does not change the product

Are you wondering why this is important to know?
Try working this problem.

$$\begin{array}{r} 7 \\ \times 65{,}432 \\ \hline 458{,}024 \end{array}$$

Or would you rather work this one?

$$\begin{array}{r} 65{,}432 \\ \times \quad 7 \\ \hline 458{,}024 \end{array}$$

Look at this problem.
Do you know what the parentheses ( ) mean?

$$3 \times (2 \times 4) = \underline{\ ?\ }$$

Parentheses always say "Do me first." So you multiply 2×4 first.

$3 \times (2 \times 4) = 3 \times 8$
$3 \times (2 \times 4) = 24$

Do the problem another way. You do it yourself.

$$(3 \times 2) \times 4 = \underline{\ ?\ } \quad 24$$

Which two numbers did you multiply first? 3×2
Did you get the same answer? Yes

185

lesson Pages 185, 186

goal Examining the order and grouping properties of multiplication

memo Both the commutative (order) and associative (grouping) properties of multiplication are found on this page. The labels for these properties are not introduced, however. You are free to introduce this vocabulary if you wish, but emphasize that the learner be able to simplify his work by **using** these properties rather than memorize vocabulary.

page 185 Everyone should examine pages 185 and 186. You can choose to do this as a group, or you can allow capable learners the freedom to continue independently while you work with the others.

goal Practice in using the order and grouping properties to simplify computation

page 186 Pupils who are able should continue independently.

Here are some more problems for you to do.

a	$(7 \times 9) \times 3 = \underline{?}$ 189	$7 \times (9 \times 3) = \underline{?}$ 189
b	$(15 \times 25) \times 4 = \underline{?}$ 1500	$15 \times (25 \times 4) = \underline{?}$ 1500
c	$(25 \times 17) \times 31 = \underline{?}$ 13,175	$25 \times (17 \times 31) = \underline{?}$ 13,175
d	$(46 \times 21) \times 38 = \underline{?}$ 36,708	$46 \times (21 \times 38) = \underline{?}$ 36,708

Look at each pair of answers. What can you say about changing which two numbers you multiply first?
You get the same product no matter which two numbers you multiply first.

Sometimes it's nice to know ideas like these.
Can you do the following problem in your head?
No fair using pencil and paper.

$$4 \times 7 \times 25 = \underline{?}$$

You can do this problem mentally if you know what to do. How much is 4×25? What is 7×100? 100 700
Does this give us the right answer? Yes
Can we work the problem this way? Yes

$4 \times 7 \times 25 = \underline{?}$	This is what we start with.
$7 \times 4 \times 25 = \underline{?}$	What are we doing in this step? Changing the order of the factors. 4×25 is easier than 7×25.
$7 \times (4 \times 25) = \underline{?}$	Explain this. Does this tell how we just found the answer? Yes Putting parentheses around 4×25 means multiply these numbers first.

If you have three numbers to multiply, you can multiply them in any order you wish.
Sometimes hard problems become easy this way.
Try doing these exercises in your head.

1. $5 \times 9 \times 2 = \underline{?}$ 90 **2.** $9 \times 3 \times 3 = \underline{?}$ 81 **3.** $6 \times 25 \times 4 = \underline{?}$ 600

4. $2 \times 7 \times 4 = \underline{?}$ 56 **5.** $2 \times 7 \times 50 = \underline{?}$ 700 **6.** $20 \times 5 \times 9 = \underline{?}$ 900

186

Just for fun

Find the answer to each of these problems.

$(1 \times 9) + 2 = \underline{?}$ 11

$(12 \times 9) + 3 = \underline{?}$ 111

$(123 \times 9) + 4 = \underline{?}$ 1111

$(1234 \times 9) + 5 = \underline{?}$ 11,111

$(12{,}345 \times 9) + 6 = \underline{?}$ 111,111

$(123{,}456 \times 9) + 7 = \underline{?}$ 1,111,111

$(1{,}234{,}567 \times 9) + 8 = \underline{?}$ 11,111,111

$(12{,}345{,}678 \times 9) + 9 = \underline{?}$ 111,111,111

DOES THE PATTERN CONTINUE? No

Here's another multiplication pattern.

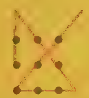
$6 \times 7 = \underline{?}$ 42

$66 \times 67 = \underline{?}$ 4422

$666 \times 667 = \underline{?}$ 444,222

$6666 \times 6667 = \underline{?}$ 44,442,222

Make an array of nine dots like this. Draw no more than 4 line segments that pass through each of the 9 points once and only once. No fair lifting your pencil!



187

See activity 6, page 188b.

lesson Page 187

goal Exploration of number puzzles

memo This is an extension page that meets all levels of ability.

page 187 Three puzzles are provided—of varied levels of difficulty. Pupils can be selective. Share the results as a group. Capable learners should be challenged with all three puzzles.

lesson Page 188

goal **Checkout**—adding and subtracting with up to 4-digit numbers; multiplying a 2- or 3-digit factor by a 1- or 2-digit factor

page 188 Note that a specific page reference is given for each particular type of problem. The skills are identified on the answer key. The chapter itself provides instructional review and practice.

Check. Was any error simply carelessness with facts? Those facts must be mastered. They are necessary tools for computation. Assign a tutor to each pupil directed back to instructional review.

CHECKOUT

This should be painless.

Skill: Adding two 3-digit numbers — Skill: Adding two 4-digit numbers

Compute.

1. $765 + 586 = ?$ You did this on page 165.
1351

2. $8964 + 1147 = ?$ You did this on page 167.
10,111

Skill: Adding four addends

3. This is like the hardest one on page 169. $62 + 8105 + 681 + 1500 = ?$
10,348

Skill: Subtracting 3-digit numbers

4. $900 - 509 = ?$ You had no trouble with this on page 172.
391

And this was on page 174. $4062 - 158 = ?$
Skill: Subtracting 4-digit numbers
3904

Skill: Multiplying

These started on page 177.

3-digit by 1-digit — 3-digit by multiple of 10 — 2-digit by 2-digit — 3-digit by 2-digit

5. $562 \times 7 = ?$ 3934

6. $861 \times 40 = ?$ 34,440

7. $57 \times 63 = ?$ 3591

8. $452 \times 32 = ?$ 14,464

And they ended on page 184.

Skill: Using commutative property of multiplication

9. You found on page 185 that 52×46 had the same product as 46×52. But what is the product?
2392

Skill: Using associative property of multiplication

10. Your last job was to multiply $20 \times 15 \times 5$. You made the job easier by picking whichever two numbers you wanted to multiply first. Do that job again.

$$20 \times 15 \times 5 = (20 \times 5) \times 15 = 100 \times 15 = 1500$$

188

STOP

things 2 wood cubes

Write the following numerals on the cubes:
- 37, 44, 76, 158, 329, 618
- 6, 7, 9, 38, 42, 50

For more practice, have the pupil roll the two cubes, then find the product of the two numbers that land faceup.

See activity 7, page 188b.

RESOURCES

another form of evaluation

for Checkout—page 188

Compute.

1. $\begin{array}{r} 497 \\ +748 \\ \hline 1245 \end{array}$ You did this on page 165.

2. $\begin{array}{r} 7368 \\ +2954 \\ \hline 10{,}322 \end{array}$ You did this on page 167.

3. This is like the hardest one on page 169. $\begin{array}{r} 307 \\ 4165 \\ 72 \\ +5904 \\ \hline 10{,}448 \end{array}$

4. $\begin{array}{r} 843 \\ -567 \\ \hline 276 \end{array}$ You have no trouble with this on page 172.
And this was on page 174. $\begin{array}{r} 6407 \\ -\ 738 \\ \hline 5669 \end{array}$

These started on page 177.

5. $\begin{array}{r} 437 \\ \times\ \ 6 \\ \hline 2622 \end{array}$ 6. $\begin{array}{r} 928 \\ \times\ 30 \\ \hline 27{,}840 \end{array}$ 7. $\begin{array}{r} 56 \\ \times 42 \\ \hline 2352 \end{array}$ 8. $\begin{array}{r} 374 \\ \times\ 95 \\ \hline 35{,}530 \end{array}$

And they ended on page 184.

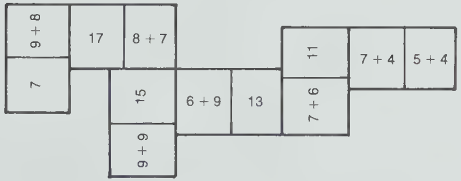
9. On page 185 you found that 38 × 64 had the same product as 64 × 38. What is the product? 2432

10. Your last job was to multiply 4× 23 × 25. You made the job easier by picking whichever two numbers you wanted to multiply first. Do that job again. 2300

activities

1. things index cards

Draw lines with a ruler dividing cards into halves. Write either an addition fact or a sum on each half. Place the cards facedown in a random arrangement. Turn one card faceup. Each player in turn selects a card and turns it faceup. Match each sum with an appropriate fact or each fact with an appropriate sum.

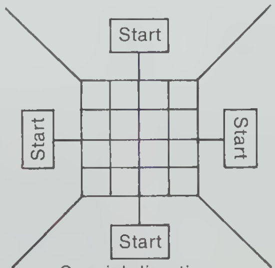


Continue until all the cards have been played.

Similar sets of cards can be made for subtraction, multiplication, and division facts.

2. things large sheets of paper; dice

Have each group of 4 pupils prepare a game board as shown.



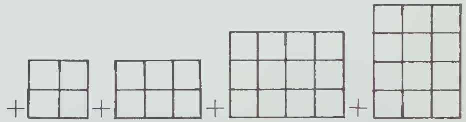
Special directions go here on each of the sides.

The numbers 1 through 16 are written in a random arrangement in the 16 cells. Label the areas outside the square with various special directions. For example: Subtract 5 points from your score, subtract 10 points from your score, take another turn, and so on.

Each player begins at a starting position and in turn gently rolls one die. The number that lands faceup on the die is added to the number of the cell in which the die stops. If the die overlaps 2 or more cells, the number of all the cells touched by the die are added to the number that lands faceup. Special directions must be followed. The first player to reach 100 (or some other predetermined number) wins.

3. things 3 sets of numeral cards 0 through 9

Pupils make their own grids for the appropriate type of practice.



The cards are shuffled and placed in a stack facedown. The dealer draws the number of digits needed to form a problem in the grid. These are displayed to the group. Each person may arrange the digits as he wishes, then add to find the sum.

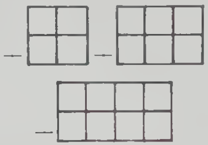
Added challenge: Arrange the cards to find the greatest sum; the least sum.

4. things 2 sets of numeral cards 0 through 9

Pupils make their own grids for the appropriate type of practice.

The numeral cards are shuffled and placed in a stack facedown. The dealer draws the number of digits needed to form a problem in the grid. These are displayed to the group. Each person may arrange the digits as he wishes, then subtract to find the difference.

Added challenge: Arrange the cards to find the greatest difference; the least difference.



5. things target; 2 beanbags

Make a target and tape it to the floor. Label the 9 squares of the target with multiples of 10 in random order. Each player tosses 2 beanbags. He multiplies the 2 numbers on which the beanbags land. The product is his score. Continue until one player has 5000 points or any other predetermined number.

10	60	50
70	20	80
40	90	30

6. Individual activity (Reproduce these directions for the pupil.)

things spirit master from activity 5, page 45b.

1. Write a 1- or 2-digit number in each □
2. Multiply across. Write the products in the ○
3. Multiply down. Write the products in the ◇s.
4. Multiply diagonally. Write the products in the □s.
5. Find the product of the numbers in the ◇s.
6. Find the product of the numbers in the ○s.
7. Find the product of the numbers in the □s.
8. Find the product of the numbers in the □s.

   This is a whopper!
9. What's **special** about the products for 5, 6, 7, and 8? (They should all be the same.)

7. things newspaper; rulers

Individual project. Measure the width of a column of print on a newspaper page. Use this measure to mark a square of print. Count the number of letters printed in this unit square. Use the information you have found to estimate the number of letters printed on the entire page.

Have each pupil use a different page for this project. Then have them compare the results. Do all the pages contain approximately the same number of letters? What kinds of pages contain more letters? fewer letters?

additional learning aids

concept – chapter objectives 9, 10

SRA products

Mathematics Involvement Program, SRA (1971)
  Cards: 374, 15, 285, 126
Visual Approach to Mathematics, level 3, SRA (1967)
  Visuals: 15, 16, 17, 18, 19, 20, 21

operation – chapter objectives 1, 2, 3, 4, 5, 6, 7, 8

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
  Spirit masters: P 3, 5, 6
  W 10, 17, 18, 19
Computapes, SRA (1972)
  Module 2, Lesson: AS 36
  Module 4, Lessons: MD 23, 24
Computational Skills Development Kit, SRA (1965)
  Addition cards: 13, 15, 16
  Subtraction cards: 8, 9, 10, 11, 12, 13, 14, 15
  Multiplication cards: 5, 8, 9, 11, 14, 15, 16
Cross-Number Puzzles (Whole Numbers), SRA (1966)
  Addition cards: 4, 6, 7, 8, 14, 17
  Subtraction cards: 7, 8, 9, 10, 11, 12, 13
  Multiplication cards: 5, 8, 14, 15, 16
Skill Modes in Mathematics, SRA (1974)
  Level I, Molecules: A, B, C
Skill through Patterns, level 4, SRA (1974)
  Spirit masters: 24, 26, 27, 40, 41, 42, 45
  46, 49, 50, 51, 52, 53, 59, 62, 63, 64, 70, 71

other learning aids (described on page 216e)
Good Time Mathematics, Numble*, Veri-Tech Senior (addition, subtraction, multiplication books)

*Trademark of Sigma Scientific, Inc.

9 DIVISION

before this chapter the learner has—

1. Named a number that comes between two given numbers having up to 4 digits each
2. Worked with the division facts
3. Mastered subtracting any 3-digit number from any 4-digit number
4. Found the product of a 1-digit factor and a 2- or 3-digit factor

in chapter 9 the learner is—

1. Estimating the quotient for a 1-digit number and a 2- or 3-digit multiple of 10 or multiple of 100
2. Finding the quotient and remainder (if any) for any 2- or 3-digit number and any 1-digit number
3. Checking the accuracy of a completed division computation by multiplication
4. Mastering the division facts

in later chapters the learner will—

1. Master estimating and finding the quotient and remainder (if any) for any 3-digit number and any 1-digit number
2. Estimate and find the quotient and remainder (if any) for any 4-digit number and any 1-digit number

Notes & Things

This is an exploratory chapter on what properly can be called long division. You will find several things that are different about the approach. The concept is explored by spending a great deal of time on estimating—how many 6s can be subtracted from 126, for example. Estimation and repeated subtraction are teamed and used as the approach to division. This approach will give everyone a chance for success. A person who isn't so good at estimating may do a lot of subtraction before he has finished. But he does have a way to finish. He will not fail to get an answer. Experience will let him know that he can experiment with the number that he will subtract. If the number is too large, he erases and tries a smaller one. If the number is far too small, he does a lot of extra subtraction. (And who really wants to spend hours subtracting?)

The child is given lots of time to improve his estimation and subtraction skills before the computational form for long division is even introduced.

This approach to division also makes the top stacking of partial quotients desirable. Top stacking has value in and of itself, no matter what computational approach is taken. It is one of the few meaningful ways in which the youngster will be able to keep track of the place value of the quotient. If you have taught older children, you know how frustrating it is when a long-division problem has been completed and everything is correct in the multiplication and subtraction steps but the pupil has forgotten a zero in the quotient. The correct answer of 1602 turned out to be 162. The answer has to be marked wrong.

Side stacking is another method that helps keep track of place value. If you prefer side stacking, use it. Explain frankly to the pupils the small differences between the two methods and go from there.

The operation of long division demands that the pupils use all these skills. They have to estimate, multiply, subtract, and then add to find the final quotient. They will review and practice each of these skills just before they need them for the next step in the development. It will probably seem as if you will never get to the computational form for division. Have patience. All the steps of division will be practiced before the form is introduced. All of the prior work will pay off, because the learners will be ready to put their work into some nice, neat form.

Checking division is also introduced. Pupils will be encouraged to find the error—not merely acknowledge that something is wrong.

For the extra activities you will want to have these things available:

10 same-size boxes (milk cartons)
spirit master of puzzle
spirit master of paths

goal Think about and explore ideas through a picture clue

page 189 This photograph will get your sports-minded youngsters thinking. Women runners are shown but your intent will be to get the youngsters to think about sports in general. The track has been divided into running lanes. If the track is straight, there is no problem, but what happens if the track is oval? Would the distance for the person in the outside lane be the same as that for the person in the inside lane if they all started and finished at the same place? How is this problem solved?

If you have track enthusiasts you can continue with questions about how distances are divided for relay teams. Questions about competitive swimming can follow, but you want to get to the point where the youngsters are asked to investigate how many different sports involve the operation of division. This may lead to something as simple as the divisions of a 100-yard football field into 10-yard parts or as complex as the division of winnings from a victory at baseball's world series.

lesson Pages 190, 191

goal Survey—ability to divide a 2- or 3-digit number by a 1-digit number

memo Pages 190 and 191 provide readiness for a new concept. You'll want to discuss them with everyone.

page 190 Use the page to kick off the discussion. Pupils should actually experience the tedious process of subtracting one group of six at a time to appreciate why shorter ways to do such time-consuming tasks were invented. You may even want them to use manipulatives or to act out the situation. The subtractive method of computing division will provide less capable pupils with a way to operate in any division situation.

Problems 2 and 3 will help you identify pupils who may already be able to divide. Their skill can be verified with the Progress Checks. For others, let these problems set the learning goals of the chapter.

Continue the discussion to page 191.

---

1. The members of the Hideway Club decided to build a clubhouse. Someone gave them 78 pieces of old lumber. They could carry 6 boards each trip if they worked together as they moved the lumber to their building site. About how many trips do you think it would take them to move the 78 boards?

The club members started to solve the problem this way:

	78	
	− 6	1 trip
Boards remaining	72	
	− 6	1 trip
Boards remaining	66	
	− 6	1 trip
Boards remaining	60	
	− 6	1 trip
Boards remaining	54	

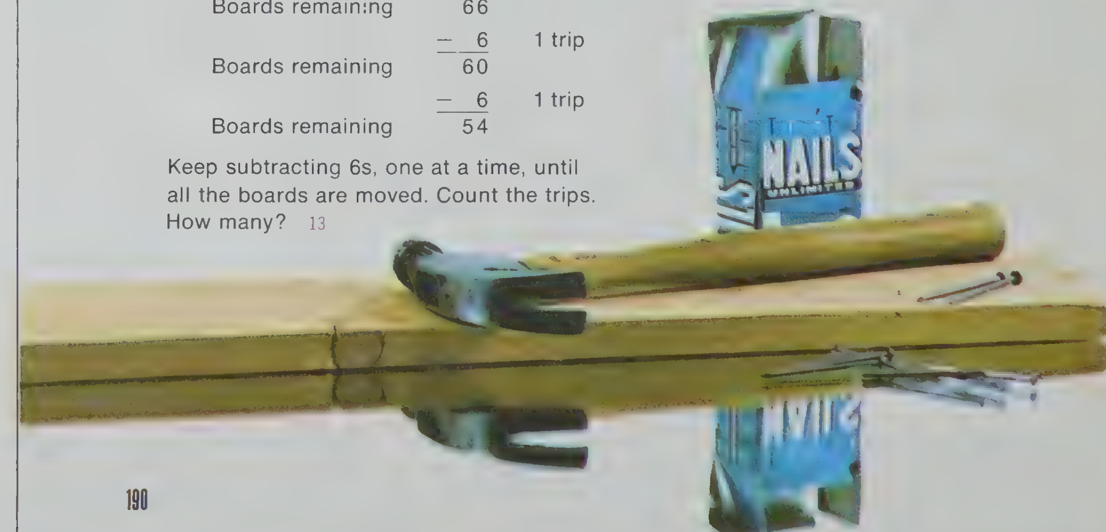
Keep subtracting 6s, one at a time, until all the boards are moved. Count the trips. How many? 13

2. How many sixes are there in 78? 13 You found out by subtracting. You could have used the operation of division.

Subtracting the same number again and again is a very long method of division.

3. Or do you already know how to divide? What's 837 ÷ 6? 139 R3

In this chapter
you will learn about division.
You will find a shorter method
to solve a math sentence like 78 ÷ 6 = ■.



You solved a division problem by using repeated subtraction.
Try another one.

The City Cab Company received a shipment of 68 new tires.
How many cabs could get 4 new tires?

THINK	How many 4s in 68?	68	
		− 4	1 four.
		64	Any more fours? Yes
		− 4	1 four.
		60	Any more fours? Yes

If you subtract 1 four at a time, it will take you forever. Start over.

THINK	How many 4s in 68?	68	
	Are there 6 fours?	− 24	6 fours are 24.
		44	Any more fours? Yes

Many more 4s remain. Right? Start over.

THINK	How many 4s in 68?	68	
	Are there 8 fours?	− 32	8 fours are 32.
		36	Any more fours? Yes

That's better, but try one more time.

THINK	How many 4s in 68?	68	
	Are there 10 fours?	− 40	10 fours are 40.
		28	How many fours remain? 7
		− 28	7 fours! Right?
		0	Zero remains.

How many 4s in 68? 17
Using 10 fours worked well, didn't it? Yes, it made quick work of the problem

191

goal Development of readiness for the division algorithm

page 191 Some pupils may themselves think of subtracting more than one group at a time. Wonderful! Watch for pupils who try to subtract too great a number of groups.

lesson Pages 192, 193

goal Practice in prerequisite skills for division

page 192 This is an exploratory activity. Allow complete freedom in the selection of how many groups to subtract at one time for problem 1. Pupils will have to use the subtraction method to determine their answers.

Exercises 2 and 3 are diagnostic. The skills tested are necessary for success in division work. Pupils who make errors here need 10 minutes of daily practice on multiplication facts until those facts are mastered. Strange as it may seem, youngsters will probably rely more heavily on the multiplication facts for division than on the division facts.

1

Find the answers to these problems.

a	How many 3s in 57?	19	b	How many 4s in 60?	15	c	How many 5s in 90?	18
d	How many 3s in 51?	17	e	How many 4s in 84?	21	f	How many 2s in 92?	46
g	How many 5s in 75?	15	h	How many 6s in 78?	13	i	How many 7s in 77?	11
j	How many 8s in 96?	12	k	How many 9s in 99?	11	l	How many 7s in 91?	13

2

The problems so far show that you need to know the multiplication facts. Do you need to review multiplication?

find out

a	3×5	15	b	2×9	18	c	4×5	20	d	3×7	21	e	8×6	48	f	6×4	24
g	7×7	49	h	4×9	36	i	3×4	12	j	8×5	40	k	4×7	28	l	5×6	30
m	7×9	63	n	6×6	36	o	8×8	64	p	7×5	35	q	8×3	24	r	9×6	54
s	8×9	72	t	9×9	81	u	6×9	54	v	8×7	56	w	7×4	28	x	3×9	27

3

Replace the box to make these true.

a	$\blacksquare \times 3 = 21$	7	b	$\blacksquare \times 7 = 42$	6	c	$8 \times \blacksquare = 16$	2	d	$\blacksquare \times 9 = 27$	3	e	$8 \times \blacksquare = 56$	
f	$\blacksquare \times 9 = 81$	9	g	$5 \times \blacksquare = 45$	9	h	$\blacksquare \times 3 = 18$	6	i	$\blacksquare \times 5 = 25$	5	j	$8 \times \blacksquare = 32$	
k	$7 \times \blacksquare = 49$	7	l	$9 \times \blacksquare = 18$	2	m	$6 \times \blacksquare = 54$	9	n	$\blacksquare \times 9 = 63$	7	o	$3 \times \blacksquare = 24$	

192

What is the largest number that will make this true?

Example: □ $\times 7 < 30$

We can replace the box with 0, 1, 2, 3, or 4.

$0 \times 7 < 30$, $2 \times 7 < 30$, $3 \times 7 < 30$.

But 4 is the largest replacement that makes a true sentence.

Your answer: $4 \times 7 < 30$.

1

Try these. Find the largest number that makes these true.

a	□ $\times 5 < 24$	4	**b**	□ $\times 7 < 64$	9	**c**	□ $\times 6 < 47$	7
d	□ $\times 8 < 34$	4	**e**	□ $\times 6 < 58$	9	**f**	□ $\times 9 < 65$	7

2

10 fives is another way of writing 10×5. Both are multiplication situations. Find the answers.

a	10×3	30	**b**	20×2	40	**c**	10×5	50	**d**	30 fours	120	**e**	40×6	240
f	70×3	210	**g**	60×5	300	**h**	50×7	350	**i**	40×4	160	**j**	40 fives	200
k	100×3	300	**l**	200×4	800	**m**	500 threes	1500	**n**	800×5	4000	**o**	700×6	4200
p	600×9	5400	**q**	400 eights	3200	**r**	900×8	7200	**s**	800×9	7200	**t**	300×9	2700
u	10×9	90	**v**	90×7	630	**w**	100 nines	900	**x**	800×7	5600	**y**	80 sevens	560

goal Practice in prerequisite skills for division

page 193 The pupil's attention is now focused on the **largest** possible number of groups that can be removed at one time. Emphasize the **less than** sign. We are in the process of building all the skills needed for the division algorithm. Don't rush. A solid foundation will lead to success when you put it all together.

Problem 2 reviews multiplying a multiple of 10 or 100. This skill has been used in two previous chapters. Help those who are having trouble to focus on the basic multiplication facts involved. Pupils should know the difference between adding zero and annexing (or "adding" as the children will say) a placeholder zero.

$$6 + 0 + 0 = 6, \text{ not } 600$$

These students may require additional review.

$7 \times 3 = 21$	ones $\times$ ones = ones
$7 \times 30 = 210$	ones $\times$ tens = tens
$7 \times 300 = 2100$	ones $\times$ hundreds = hundreds
$70 \times 3 = 210$	tens $\times$ ones = tens
$700 \times 3 = 2100$	hundreds $\times$ ones = hundreds

lesson Pages 194, 195

goal Development of readiness for the division algorithm

page 194 Real-world situations establish the conditions necessary for division. To solve these situations by subtraction is perfectly sound—but sometimes lengthy.

Please do not rush into the development of shorter forms. Be patient. By building much background and understanding, the pupils will be ready for the division form.

Let's go back to division situations and try another problem.

1. Warren is in charge of food for the class party. He plans to get 96 ounces of juice and some paper cups. Each cup holds 4 ounces of juice. How many 4-ounce cups of juice can Warren serve? Can he serve 10?

THINK How many 4s in 96?

$$\begin{array}{rl} 96 & \\ -40 & \text{10 fours.} \\ \hline 56 & \quad \text{56 remains. Are there 10 more fours?} \\ -40 & \text{10 fours.} \\ \hline 16 & \quad \text{16 remains. Are there 10 more fours?} \\ -16 & \text{4 fours.} \\ \hline 0 & \quad \text{Any more fours?} \end{array}$$

   a How many 4-ounce cups can be served in all?

   b Are there any ounces of juice that remain?

   c The math sentence to show this situation is $96 \div 4 = ■$.

2. Warren needs to serve more cups of juice from 96 ounces. So he bought some smaller cups, each holding 3 ounces of juice. How many 3-ounce cups of juice can he serve?

THINK How many 3s in 96?

$$\begin{array}{rl} 96 & \\ -30 & \text{10 threes.} \\ \hline 66 & \quad \text{Are there 10 more threes?} \\ & \quad \text{You complete the computation.} \end{array}$$

   a How many 3-ounce cups can be served in all?

   b Are there any ounces of juice remaining?

   c The math sentence for this problem is $96 \div 3 = ■$.

194

Try these yourself

1. How many 4s in 92? 23 2. How many 5s in 195? 39 3. How many 7s in 553? 79
4. How many 6s in 438? 73 5. How many 3s in 135? 45 6. How many 9s in 747? 83
7. How many 8s in 656? 82 8. How many 7s in 378? 54 9. How many 6s in 198? 33

10. Michael decided to sell candy to earn money. He can buy one large bag of candy that has 250 pieces for $2.50. He thought he could put 7 together and sell each small package for 10¢. How many small packages can he fill from the large bag?

THINK: How many 7s in 250?

	250	
Try 20.	− 140	20 sevens.
	110	Any more sevens?
Try 10 more.	− 70	10 sevens.
	40	Any more sevens?
	− 35	5 sevens.
	5	Any more sevens?

How many 7s in 250? How many pieces of candy remain? What do you think he should do with the candy that remains? What can you do with the number that remains when you write the answer?

11. 384 boys played basketball on Saturday. The manager wanted 8 boys on each team. How many teams will play? 48
Did everyone get on a team? Yes

12. Mrs. Smith bought 175 yards of terry cloth for her sewing classes. Each student is to make a bathrobe that takes 2 yards. How many students can make a robe? Is there any cloth remaining? Yes—1 yard.
87

goal Introduction to the possibility of remainders in division

page 195 Division situations in the real world frequently pose a problem. Things don't come out nice and even. The notion of having a **remainder** must be discussed. When we are dealing with real situations, the remainder must be dealt with practically. If there is candy left over, the candy can be eaten by one person. No problem. If a class is to be divided into teams and one person remains, there is a problem! When cloth is divided and some remains, there is another problem.

The remainder may have no meaning for a child in practice or drill problems that deal only with number. He'll gladly go along with putting the remainder in the answer any way you tell him to. But appreciate the lack of understanding that may result if division is not related to situations where the remainder is something more than **that last number that you can't divide anymore.**

lesson Pages 196, 197, 198, 199, 200

goal Development of readiness for the division algorithm

page 196 The pupil is given the freedom to operate at his own level provided he multiplies and subtracts accurately. Yet he is gently prodded toward fewer subtractions by taking out larger groups. Stress that there is no one right way; there are simpler, shorter ways. The ability to estimate will determine the number of subtractions made.

Watch for confusion with problems 5 and 6. Some students may be surprised to find a 2-digit total (86 and 99) yielding a 2-digit subtraction answer. The other totals are all 3-digit and yield the same type of answer. Further, problem 6 has a remainder.

Try to finish this problem.

How many 8s in 597?

$$\begin{array}{r} 597 \\ -320 \\ \hline 277 \\ ? \end{array}$$

40 eights.
Any more eights? Yes

Could you have subtracted more than 40 eights in the first step? Yes What is the answer? 74 R5

Now do these problems.

1. How many 5s in 445? 89
2. How many 9s in 666? 74
3. How many 3s in 132? 44
4. How many 4s in 196? 49
5. How many 2s in 86? 43
6. How many 7s in 99? 14 R1

The fewer subtraction steps you use, the faster you can do the problems. Look at the two methods used for this problem.

717 ÷ 8 = ■ **THINK** How many 8s in 717?

METHOD 1

717	
− 160	20 eights.
557	Any more eights?
− 240	30 eights.
317	Any more eights?
− 240	30 eights.
77	Any more eights?
− 64	8 eights.
13	Any more eights?
− 8	1 eight.
5	Any more eights? No. Only 5 remains.

METHOD 2

717	
− 400	50 eights.
317	Any more eights? Y
− 240	30 eights.
77	Any more eights? Y
− 72	9 eights.
5	Any more eights? No. Only 5 remains.

Is the answer the same for both methods? Yes – 89 R5 Could this problem be done with only *two* subtraction steps? Yes

TRY IT

717	
− 640	80 eights
77	
− 72	9 eights
5	

REMEMBER

Remember—there are no rules about how many subtraction steps you should use. But you will want to try to use as little time and paper as possible. There are things a lot more fun to do than to sit around and subtract numbers. Some people are lucky and can guess just about what the answer should be.

1. How many 7s in 513? Bill guessed 20.
   Sue guessed 40.
   Jay guessed 50.
   Who had the best guess? Where any of the guesses wrong?
   Jay   No. Bill's and Sue's guesses were pretty far from a good estimate, however.

Guessing an answer does not depend on luck. If you take time to think, your guess can be based on a reason. Then you don't guess at all. You estimate the answer.

2. How many 4s in 251? Are there 10? 20? 30? 40? 50? 60? To make an estimate you would think: What number times 4 is close to 251? You'd pick 60, because 60 × 4 is 240. That's closer to 251 than 50 × 4, or 200.

3.	How many 9s in 190?	a 10 nines?	b 20 nines?	c 30 nines?
	Which estimate did you pick? Why?		20 × 9 = 180—closer to 190	30 × 9 = 270
4.	How many 6s in 489?	a 50 sixes	b 70 sixes	c 80 sixes
5.	How many 7s in 654?	a 80 sevens	b 90 sevens	c 100 sevens
6.	How many 9s in 805?	a 80 nines	b 90 nines	c 100 nines
7.	How many 3s in 254?	a 60 threes	b 70 threes	c 80 threes
8.	How many 4s in 319?	a 70 fours	b 80 fours	c 90 fours

goal Introduction to estimation as a method of reducing the number of subtractions

page 197 The focus is on developing skill in estimating. With less capable pupils you may want to review examples such as these.

$$6 \times 5 = 30 \qquad 6 \times 50 = \underline{?}$$
$$6 \times 7 = \underline{?} \qquad 6 \times 70 = \underline{?}$$

Emphasize that 30 nines is simply another way of stating 30 × 9.

Watch for the youngster who is still having difficulty with this type of work. He may not know the basic facts involved, or he may be having difficulty with place value and place-value zeros. Order—placing a number between two numbers—may also cause some youngsters difficulty.

Many pupils may have the ability to work this page mentally, indicating only the answer. Great! Require no more paper-and-pencil work than the pupil requires. Continued quick oral drill on this type of work will help sharpen a necessary skill.

goal Practice in using estimation to reduce the number of subtractions

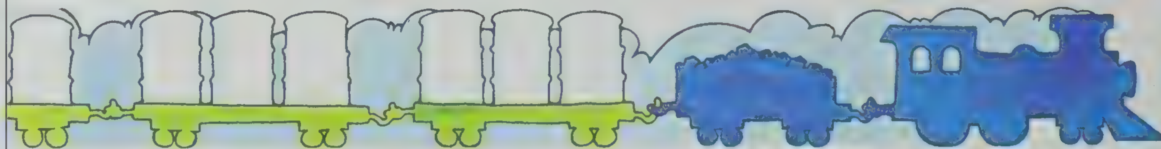
page 198 The possibility that there may be as many as a hundred groups to subtract is introduced.

SHOW HOW GOOD YOU ARE IN ESTIMATING

1. How many 2s in 183?
   a 70 twos b 80 twos c 90 twos

2. How many 5s in 499?
   a 80 fives b 90 fives c 100 fives

3. How many 8s in 742?
   a 80 eights b 90 eights c 100 eights



The company is shipping 441 tanks by railroad. They plan to put 3 tanks on each railroad flatcar. How many flatcars needed?

	441		
Step 1	− 150	50 threes.	
	291	Any more threes?	Yes
Step 2	− 150	50 threes.	
	141	Any more threes?	Yes
Step 3	− 120	40 threes.	
	21	Any more threes?	Yes
Step 4	− 21	7 threes.	
	0	Any more threes?	No

How many 3s in 441? Notice that 50 × 3 was subtracted in steps 1 and 2. Can those steps be put together? Yes 14

441		
− 300	100 threes.	
141	Any more threes?	Yes
− 120	40 threes.	
21	Any more threes?	Yes
− 21	7 threes.	
0	Any more threes? No!	

How many 3s in 441? 147

Subtracting groups of hundreds works well too, doesn't it? Yes

198

You really need estimating skills to divide.

HOW MANY

4s in 52?

For example:

$10 \times 4 = 40$
$20 \times 4 = 80$

Estimated answer:
There are between 10 and 20 fours in 52.

Here's another problem that is started for you.
The problem looks like this. $76 \div 2$

THINK

How many **2s in 76?**

$20 \times 2 = 40$	Too few.
$30 \times 2 = 60$	Close.
$40 \times 2 = \underline{?}$ 80	Too many? Yes

Estimated answer:
There are between ? (30) and ? (40) twos in 76.

YOUR TURN

Estimate answers only.

1. How many 2s in 150? 70 – 80
2. How many 4s in 150? 30 – 40
3. How many 3s in 100? 30 – 40
4. How many 6s in 100? 10 – 20
5. How many 5s in 200? 40
6. How many 9s in 200? 20 – 30
7. How many 7s in 300? 40 – 50
8. How many 8s in 400? 50
9. How many 9s in 540? 60
10. How many 7s in 490? 70

ESTIMATES

are used every day. Here's one example.

1. I have to wrap 5 presents. I need about 12 inches of ribbon for each one. Will one package of ribbon marked "6 feet" be enough? Or should I buy two? No
Yes

199

goal Further development of estimation skills by focusing on the concept of betweenness

page 199 The emphasis is on developing the ability to estimate so that it will become a skill—one that will immediately be used when the division algorithm is introduced. Actually pupils are practicing the first step in division.

Encourage those who have difficulty to use number-line models—and continue drilling multiplication facts.

	60	10×6	600	100×6
	54	9×6	540	90×6
	48	8×6	480	80×6
46 →	42	7×6	420	70×6
	36	6×6	360	60×6
	30	5×6	300	50×6
	24	4×6	240	40×6
	18	3×6	180	30×6
	12	2×6	120	20×6
	6	1×6	60	10×6
	0	0×6	0	0×6

How many 6s in 46? Locate 46 on the number line. 46 is between 42 and 48. 7×6 is close. 8×6 is too many. Use the same development with multiples of 10.

goal Application of division skills to real-world situations

page 200 It is important to accept that many times in the real world an estimate is a sufficient answer. Other times it is important to know the exact answer. The pupils are actually dividing even though they don't have the division symbols. Don't tell them yet. Let them make this discovery for themselves when the division form is introduced on page 201. Provide help with the word problems if necessary.

A can of Berpzi cola costs 25¢ in a vending machine. A man came to collect the money in the machine. He counted the quarters. There were 132. Estimate how many dollars that is.

There are 4 quarters in a dollar. So we need to find the number of 4s in 132.

First estimate.	$10 \times 4 = 40$	Too few.
	$30 \times 4 = 120$	Too few.
	$40 \times 4 = 160$	Too many.

The answer is between 30 and 40.
But what is the answer?

Compute.	132	
	− 120	$30 \times 4 = 120$
	12	Any more 4s? Yes
	− 12	$3 \times 4 = 12$
	0	Any more 4s? No

How many 4s in 132? 33

NOW TRY THESE PROBLEMS

1. Matt got 153 points on a test. If each problem was worth 3 points, how many problems did he get right? 51

2. Sara got 180 points on the test. She got all of them right. Remember—each problem was worth 3 points. How many problems were there in the test? 60

3. Jay got 174 points. How many problems did he get right? 58

4. Look out for this one! Debbie missed 5 problems. What was her score? 165

200

If you see a problem like 235 × 52,
you write it in another form to compute.
The same thing is true in division.
When you see 79 ÷ 4,
you will write it as $4\overline{)79}$ before you compute.

Rewrite these division problems so that they can be computed.

BUT DO NOT COMPUTE.

1. 85 ÷ 3 = ? $3\overline{)85}$
2. 94 ÷ 3 = ? $3\overline{)94}$
3. 745 ÷ 9 = ? $9\overline{)745}$
4. 395 ÷ 5 = ? $5\overline{)395}$
5. 845 ÷ 8 = ? $8\overline{)845}$
6. How many 4s in 594? $4\overline{)594}$

Now start on this computation. 83 ÷ 5 = ?

Think **How many 5s in 83?**
But you write $5\overline{)83}$

Now estimate. 10 × 5 = 50
20 × 5 = 100

The answer is between 10 and 20.

Now compute the answer.

$$\begin{array}{rl} 5\overline{)83} & \\ -50 & 10 \times 5 \\ \hline 33 & \text{Any more 5s?} \quad \text{Yes} \\ -30 & 6 \times 5 \\ \hline 3 & \text{Any more 5s? No. Only 3 remains.} \end{array}$$

The answer is ? (16) with a remainder of ? (3).
You need a place to write the answer.
Put it above the division symbol.
Write R to signal the remainder.
Put the remainder after the letter R.

$$\begin{array}{rl} 16\,\text{R}3 & \\ 5\overline{)83} & \\ -50 & 10 \times 5 \\ \hline 33 & \\ -30 & 6 \times 5 \\ \hline 3 & \end{array}$$

201

lesson Pages 201, 202, 203, 204

goal Introduction of the division algorithm

page 201 The pupils have been guided through every step necessary to perform these computations. They have been dividing for several pages, but without the customary form. Division has been developed not by types of problems, but rather by concept. They should have all the necessary skills and be ready to put them together by now.

Pupils who have not mastered the basic multiplication facts may still require the aid of number-line models or a multiplication table. It is far better for them to use these and experience success than to experience frustration and failure.

The purpose of this chapter is to develop concept. There is no emphasis on mastery.

goal Practice in dividing, using the division algorithm

page 202 You'll want to discuss where all those numbers come from. Do not require pupils who can work independently to write the **think** steps. Writing these is helpful only when you are working directly with the child. Then you can help him sort out those parts that pertain to finding the final answer.

Practice dividing after you look at this example.

$52 \div 4$ **THINK** How many 4s in 52? Write: $4\overline{)52}$

Estimate. $10 \times 4 = 40$
$20 \times 4 = 80$

The answer is between 10 and 20.

Use your estimate as you start to compute.

```
   3
  10
4)52
 -40  (10 × 4)   Write 10.
  12    THINK    Any more 4s?
 -12  (3 × 4)    Write 3.  Yes
   0    THINK    Any more 4s?
                 No
```

Then complete your answer.

```
13  ?
    3
   10
 4)52
  -40
   12
  -12
    0
```

Now practice

1. $7\overline{)84}$ 12
2. $5\overline{)130}$ 26
3. $9\overline{)118}$ 13 R1
4. $3\overline{)111}$ 37
5. $6\overline{)165}$ 27 R3
6. $7\overline{)135}$ 19 R2
7. $4\overline{)264}$ 66
8. $6\overline{)382}$ 63 R4
9. $8\overline{)184}$ 23
10. $3\overline{)169}$ 56 R1
11. $5\overline{)265}$ 53
12. $9\overline{)828}$ 92
13. Nick found a bargain: 100 foreign stamps for only 25¢. He decided to make 4 packages out of them and sell the packages for 10¢ each. How many stamps would there be in each package? 25

203

goal Application of multiplication and division skills in a real-world situation

page 203 **Bargains** are not always real bargains. Paper and a pencil often are not available in situations that we want to check. Encourage mental computation where possible. Some pupils may choose to multiply the regular price by the number of items; others may choose to divide the sale price. Either method is correct. Watch out for the jump ropes!

goal Extension of estimation skills; **Progress Check**—estimating quotients

page 204 Consider having pupils complete the estimation practice orally—rather than in written form. In actual division work, this step is thought, not written.

The Progress Check is independent work. Pupils who are having problems can make number-line models to use as an aid. This will allow them to continue working on the division concept as they work on mastering the multiplication facts.

	400	100×4		
	360	90×4	3600	900×4
	320	80×4	3200	800×4
	280	70×4	2800	700×4
	240	60×4	2400	600×4
	200	50×4	2000	500×4
	160	40×4	1600	400×4
	120	30×4	1200	300×4
96 →	80	20×4	800	200×4
	40	10×4	400	100×4
	0	0×4		

About how many 4s in 96? Find between which two products 96 is located. Now look at the factors. 96 is between 20×4 and 30×4. There are between 20 and 30 fours in 96.

CAN YOU ESTIMATE LARGER NUMBERS? FIND OUT

1. About how many 3s in 510?
   - $100 \times 3 = 300$ Too few.
   - $200 \times 3 = 600$ Too many.

   How many 3s? There are between ? (100) and ? (200) in 510.

2. About how many 2s in 845?
   - $100 \times 2 = 200$ Way too few.
   - $300 \times 2 = 600$ Still too few.
   - $400 \times 2 = 800$ That's better.
   - $500 \times 2 = 1000$ Too many.

   There are between ? (400) and ? (500) 2s in 845.

3. About how many 4s in 964?
   - $100 \times 4 = 400$ Too few.
   - $200 \times 4 = 800$ That's better.
   - $300 \times 4 = 1200$ Too many.

   There are between ? (200) and ? (300) 4s in 964.

PROGRESS CHECK

Estimate only. Skill: Estimating, 2- or 3-digit divided by 1-digit numbers

1. About how many 4s in 96? 20-30
2. About how many 3s in 41? 10-20
3. About how many 5s in 67? 10-20
4. About how many 6s in 83? 10-20
5. About how many 8s in 96? 10-20
6. About how many 6s in 890? 100-200
7. About how many 3s in 775? 200-300
8. About how many 5s in 744? 100-200
9. About how many 2s in 695? 300-400
10. About how many 4s in 998? 200-300

204

STOP See activity 1, page 216a.

See activity 2, page 216a.

1. How many 3s in 710? Write $3\overline{)710}$.

Estimate.

$10 \times 3 = 30$	Whoops! Start again.
$100 \times 3 = 300$	There are lots more than 100.
$200 \times 3 = 600$	That's closer.
$300 \times 3 = 900$	Too many.

Use your estimate to compute.

$$\begin{array}{r} 200 \\ 3\overline{)710} \\ -600 \\ \hline 110 \end{array}$$

(200 · 3) Write 200.
Any more 3s? Yes

Estimate again.
$20 \times 3 = 60$
$30 \times 3 = 90$
$40 \times 3 = 120$

Use your best estimate to continue.

$$\begin{array}{rl} 6 & \\ 30 & \\ 200 & \\ 3\overline{)710} & \\ -600 & \\ \hline 110 & \\ -\ \ 90 & (30 \times 3)\ \text{Write 30.} \\ \hline 20 & \text{Any more 3s? Yes} \\ -18 & (6 \times 3)\ \text{Write 6.} \\ \hline 2 & \text{Any more 3s? No} \end{array}$$

236 R2
What is the complete answer? Where do you write it?
Above the parts of the quotient

2. Copy this.
Finish the computation.
Write the answer.

Estimate. $100 \times 2 = 200$
$200 \times 2 = 400$
$300 \times 2 = 600$

$$\begin{array}{rl} 261 & \\ \hline 1 & \\ 60 & \\ 200 & \\ 2\overline{)522} & \\ -400 & \\ \hline 122 & \\ -\ \ \ ? & 120 \\ \hline 2 & \\ -\ \ \ ? & 2 \\ \hline 0 & \end{array}$$

Estimate again. $40 \times 2 = 80$
$50 \times 2 = 100$
$60 \times 2 = 120$

3. You try one. 230
How many 4s in 920? Write $4\overline{)920}$ and then compute.

$$\begin{array}{r} 230 \\ \hline 30 \\ 200 \\ 4\overline{)920} \\ -800 \\ \hline 120 \\ -120 \\ \hline 0 \end{array}$$

lesson Pages 205, 206, 207

goal Development of finding 3-digit quotients

memo Many steps are involved in finding the answer. You will want to discuss this page and possibly part of page 206. This may help prevent faulty learning when involved steps are to be taken.

page 205 Once again, have the child focus on recording the partial quotients only, not the thinking steps. If estimation is a weakness, use number lines to guide thinking.

goal Practice in finding 3-digit quotients

page 206 Complete the example with the learner. Emphasize the steps labeled **a** through **e**. Those who are able should go on independently.

Give as much help as is necessary. Remember, the focus in this chapter is on concept, not on mastery.

Mark's Record Shop has a sale on 564 phonograph records. A customer must buy 4 records to receive a discount during the sale. How many customers can buy records on sale?

The problem is to find how many 4s in 564. $4\overline{)564}$

Estimate and then go to work.

How many hundreds times four?

$100 \times 4 = 400$
$200 \times 4 = 800$

How many tens times four?

$20 \times 4 = 80$
$30 \times 4 = 120$
$40 \times 4 = 160$
$50 \times 4 = 200$

$$\begin{array}{r} 40 \\ 100 \\ 4\overline{)564} \\ -\underline{400} \\ 164 \\ -\underline{160} \\ 4 \end{array}$$

Now you're on easy street.
Finish up and write the answer on top.

$$\begin{array}{r} \underline{?}\ 141 \\ 1 \\ 40 \\ 100 \\ 4\overline{)564} \\ -\underline{400} \\ 164 \\ -\underline{160} \\ 4 \\ -\underline{\ \ 4} \\ 0 \end{array}$$

141 people could get a discount buying records.

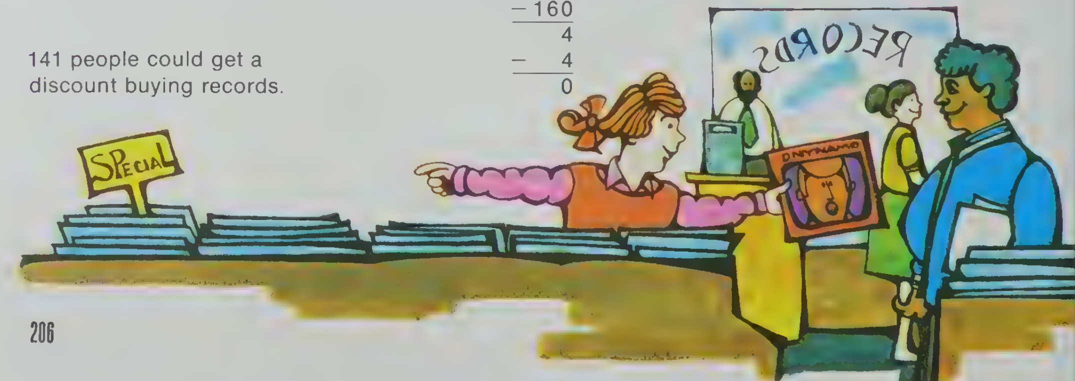
You try these problems, following the steps that are shown.

- **a** Write the problem.
- **b** Any hundreds? Estimate. Subtract your best estimate.
- **c** Any tens? Estimate. Subtract your best estimate.
- **d** Continue estimating and subtracting as long as you have to.
- **e** Write the answer on top.

1. $742 \div 3$ 247 R1 **2.** $865 \div 5$ 173

3. $549 \div 3$ 183 **4.** $268 \div 2$ 134

5. $672 \div 6$ 112 **6.** $444 \div 4$ 111



206

Did you have trouble?

Then work through this one.

950 ÷ 2 This has to be written $2\overline{)950}$

Estimate.	100 × 2 = 200	There are lots more than 100 2s.
	300 × 2 = 600	Still too few.
	400 × 2 = 800	That's better.
	500 × 2 = 1000	Too many.

Use your estimate to compute.

Estimate again.	50 × 2 = 100	
	60 × 2 = 120	
Use this one.	70 × 2 = 140 →	
	80 × 2 = 160	

$$\begin{array}{r} 475 \\ 5 \\ 70 \\ 400 \\ 2\overline{)950} \\ -800 \\ \hline 150 \\ -140 \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$$

475

What's the answer? How do you write it? 475

Where does it go? At the top

Try these

1. Each person who applies for a job at Smith Canning Company must fill out 3 forms. One month there were 870 forms done. How many people applied for a job?

   The problem: How many 3s in 870? Write $3\overline{)870}$.

   Any hundreds? Estimate. Subtract. Yes

   Any tens? Estimate. Subtract. Yes

   Continue estimating and subtracting.

   Write the answer on top.

$$\begin{array}{r} 290 \\ 90 \\ 200 \\ 3\overline{)870} \\ -600 \\ \hline 270 \\ -270 \\ \hline 0 \end{array}$$

2. $5\overline{)605}$ 121
3. $4\overline{)648}$ 162
4. $6\overline{)786}$ 131
5. $8\overline{)840}$ 105
6. $7\overline{)637}$ 91
7. $5\overline{)540}$ 108

goal Additional instruction in division resulting in 3-digit quotients

memo This page provides additional guidance and practice for those pupils who need a little more help. You are the best judge of which pupils need this help. Those who are ready to proceed can complete page 208 independently.

page 207 The page itself provides sufficient guidance for many pupils to work independently. You'll have to stick right with youngsters who aren't with it. A feeling of confidence with division is the desired end for this chapter.

lesson Pages 208, 209, 210

goal Introduction of the terms QUOTIENT and REMAINDER; practice with division

page 208 You'll want to be sure everyone can say and read those funny words **quotient** and **remainder.**

Problems 2 through 16 provide additional practice—too much for one day. Have each pupil select six problems. Use the remaining problems with pupils who are having difficulty. Those who are succeeding may proceed to the hassle with Madame Coruscant's relatives on page 210.

DIVISION

In division problems the answer has a special name. It is called a *quotient*. The number that remains after the last subtraction is called the *remainder*. Here is an example.

```
quotient ──→ 12 R1 ←── remainder
              2
             10
           8)97
            -80
             17
            -16
              1
```

Do these problems. Estimate the quotient and then compute. Problem 1 is done as an example.

1. $5\overline{)294}$ Estimate. $20 \times 5 = 100$
$30 \times 5 = 150$
$40 \times 5 = 200$
$50 \times 5 = 250$
$60 \times 5 = 300$

The quotient is between 50 and 60.

```
quotient ──→ 58 R 4 ←── remainder
              8
             50
          5)294
           -250
             44
           -  40
              4
```

Estimates are in parentheses.

(10-20) 12 **2.** $9\overline{)108}$	(40-50) 41 R3 **3.** $4\overline{)167}$	(60-70) 62 R1 **4.** $2\overline{)125}$	(10-20) 17 **5.** $5\overline{)85}$	(30-40) 30 R3 **6.** $7\overline{)213}$
(40-50) 49 **7.** $3\overline{)147}$	(100-200) 149 **8.** $5\overline{)745}$	(200-300) 231 R3 **9.** $4\overline{)927}$	(100-200) 163 **10.** $6\overline{)978}$	(40-50) 45 R5 **11.** $8\overline{)365}$
(100-200) 103 **12.** $9\overline{)927}$	(300-400) 318 R1 **13.** $3\overline{)955}$	(90-100) 91 **14.** $6\overline{)546}$	(200-300) 243 R3 **15.** $4\overline{)975}$	(200-300) 244 **16.** $3\overline{)732}$

MORE DIVISION

The example given in problem 1 is just about finished.

1. $$\begin{array}{r} 387\ \ 1 \\ ???\ R? \\ 2\overline{)775} \\ -600 \\ \hline 175 \\ -160 \\ \hline 15 \\ -\ \ 14 \\ \hline 1 \end{array}$$

Estimate. $300 \times 2 = 600$
$400 \times 2 = 800$

Estimate. $80 \times 2 = 160$
$90 \times 2 = 180$

2. $3\overline{)645}$ 215
3. $4\overline{)585}$ 146 R1
4. $7\overline{)923}$ 131 R6
5. $5\overline{)674}$ 134 R4
6. $2\overline{)875}$ 437 R1
7. $3\overline{)847}$ 282 R1

Use division to solve this problem.

8. Detective Sam Shovel has 6 junior detectives working with him. These 7 sleuths divide all the jobs they get equally among themselves. Here is a record of the number of cases they worked on every month for 3 months.

Month	Number of cases
Jan.	49
Feb.	91
Mar.	105

a How many cases did each detective work on in January? 7

b How many in February? 13 **c** How many in March? 15

209

goal Practice in division with 2- and 3-digit quotients

page 209 Use your discretion in assigning this page (or parts of the page) to meet individual needs.

goal Using division in a problem situation; **Progress Check**—dividing a 3-digit number by a 1-digit number

page 210 Use your discretion in assigning the mystery of Madame Coruscant's relatives. Once solved, you'll want to discuss the honesty of these people and the effects of increasing or decreasing the number of relatives. What will happen to the 6 remaining jewels when 9 relatives appear?

The Progress Check contains more problems than are necessary to check understanding of how to perform division computation. Have each pupil select six problems to compute. Agree on how these should be labeled on the paper for identification.

Continue to work with students who are not succeeding. Possible causes of errors are—

- Multiplication error—basic multiplication facts have not been mastered
- Difficulty in estimating—not using reasonable multiples
- Subtraction error—particularly a renaming error

Youngsters who are succeeding should move on to the challenge pages—211, 214, and 215.

1. Madame Coruscant came to see Detective Sam Shovel with a mystery. She has 96 priceless jewels that she wants to divide equally among her relatives. But strangers keep showing up, claiming to be long-lost cousins or relatives.

   On Monday she had 2 "relatives."
   By Tuesday she had 4 "relatives."
   By Wednesday she had 6 "relatives."
   By Thursday she had 8 "relatives."
   By Friday she had 9 "relatives."

   If she divided up her jewels, how many jewels would each "relative" get on Monday? on Tuesday? on Wednesday? on Thursday? on Friday? 48 24 16 12 10 (with 6 remaining)

2. Sam Shovel dug right into the case and unearthed the truth. Only 3 of the strangers were really relatives. How many jewels will each real relative get? 32

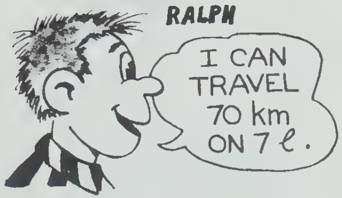
Divide. Skill: Dividing 3-digit by 1-digit number

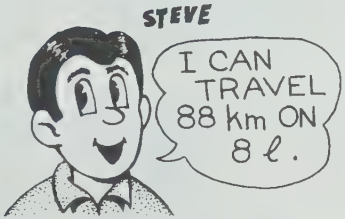
1. $4\overline{)927}$ 231 R3	2. $7\overline{)837}$ 119 R4	3. $6\overline{)802}$ 133 R4	4. $9\overline{)315}$ 35	5. $2\overline{)836}$ 418	6. $5\overline{)394}$ 78 R4
7. $3\overline{)593}$ 197 R2	8. $8\overline{)311}$ 38 R7	9. $7\overline{)298}$ 42 R4	10. $3\overline{)754}$ 251 R1	11. $4\overline{)953}$ 238 R1	12. $2\overline{)936}$ 468

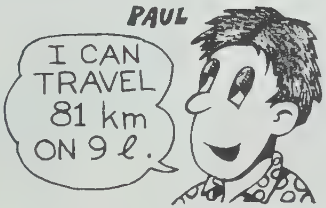
210

STOP See activity 3, page 216b.

Continue to text pages 211, 214, and 215 as directed in the guide.







1. "Kilometres per litre" means how many kilometres a car or other vehicle can travel on one litre of fuel. Ralph, Steve, and Paul were arguing about how many kilometres per litre each of their cars got.

   a How many kilometres per litre does Ralph's 10 car get? Steve's car? 11 Paul's car? 9

   b Whose car gets the most kilometres for one litre? Steve's

2. Sometimes the kilometres per litre a car gets depends on how fast the car is going. Steve tested his car. He got the following results. Compute the number of kilometres per litre for each speed.

   a Under 60 km/h
   "I can get 12 km/ℓ."
   How many litres needed to go 60 km? 5

   b 60-80 km/h
   "I can get 11 km/ℓ."
   How many litres needed to go 88 km? 8

   c Above 80 km/h
   "I can get 9 km/ℓ."
   How many litres needed to go 63 km? 7

211

lesson Page 211

goal Using division in a problem situation

memo Use this page with youngsters who need a challenge—your independent learners.

page 211 The content of the page can trigger a research project. Record the make and model of dad's or mom's car. Check and record the car's fuel consumption. *Is it the same for city driving and freeway driving?* Bring the results to class. Compare the findings. *How do these findings compare with the ones given in the book?*

In most metric countries fuel consumption is measured in ℓ/100 km. This will probably become the practice in Canada too. However, the mathematics involved is more complicated and therefore is not introduced to students at this level. But your brighter students may want to investigate the advantages and disadvantages of this method of measuring fuel consumption.

lesson Pages 212, 213

goal Introducing checking division computation by multiplication

memo An old idea extended. Discuss this before beginning to work.

page 212 You may want to review the relationship of multiplication to division even though this has already been taught. For example:

$$5 \times 9 = 45 \text{ and } 45 \div 5 = 9$$
$$45 \div 9 = 5$$

The fact that one factor (the quotient) may have 2 or 3 digits does not affect this relationship. Pupils are asked to check only the computations given. Challenge them to find the errors made in problem 4.

Here's an easy problem.

3)48

You will still estimate. $10 \times 3 = 30$
$20 \times 3 = 60$

The quotient is between 10 and 20.
Are you sure this quotient is *correct*? No, not sure.

```
   16 R0
    6
   10
3)48
 - 30
   18
 - 18
    0
```

However, the estimate shows that the answer is reasonable.

You can use multiplication to check your answer and make sure it is right.
If $48 \div 3 = 16$, then 16×3 should equal 48.
Does it?

```
  16
×  3
  48
```

It checks!

CHECK THESE.
IF AN ANSWER IS WRONG, MAKE IT RIGHT.

1

```
   24 R0
    4
   20
5)120
 - 100
    20
  - 20
     0
```

```
  24
×  5
 120
```

2

```
   97 R0
    7
   90
5)485
 - 450
    35
  - 35
     0
```

```
  97
×  5
 485
```

3

```
   143 R0
     3
    40
   100
3)429
 - 300
   129
 - 120
     9
   - 9
     0
```

```
 143
×  3
 429
```

4

```
   165 R0  (3)
     5
    60
   100
6)993
 - 600
   395   393
 - 365   360
    30    33
  - 30
     0     3
```

```
 165
×  6
 990
```

Practice dividing and checking. Be sure to estimate your quotient before you compute. Problem 1 is done as an example.

1. $3\overline{)298}$ Estimate. $90 \times 3 = 270$
$100 \times 3 = 300$

The quotient is between 90 and 100.

$$\begin{array}{r} 99\ \text{R}1 \\ 9 \\ 90 \\ 3\overline{)298} \\ -\,270 \\ \hline 28 \\ -\,27 \\ \hline 1 \end{array}$$

Good grief! The check won't work!

If $298 \div 3 = 99\ \text{R}1$, then $(99 \times 3) + 1 = 298$.

Multiply 99×3.

$$\begin{array}{r} 99 \\ \times\ \ 3 \\ \hline 297 \\ +\ \ \ 1 \\ \hline 298 \end{array}$$

Then add $297 + 1$.

It does check!

NOW TRY THESE.

2. $3\overline{)131}$ (43 R2)	3. $9\overline{)95}$ (10 R5)	4. $7\overline{)140}$ (20)
5. $8\overline{)245}$ (30 R5)	6. $7\overline{)119}$ (17)	7. $4\overline{)45}$ (11 R1)
8. $3\overline{)62}$ (20 R2)	9. $4\overline{)905}$ (226 R1)	10. $3\overline{)994}$ (331 R1)
11. $6\overline{)500}$ (83 R2)	12. $5\overline{)579}$ (115 R4)	13. $8\overline{)579}$ (72 R3)
14. $5\overline{)158}$ (31 R3)	15. $6\overline{)472}$ (78 R4)	16. $4\overline{)825}$ (206 R1)

213

goal Examining division computation that involves a remainder; checking the computation

page 213 Pupils are now directed to divide and then check their own computation by multiplication. Checking is introduced for a purpose—to catch errors. The error, or errors, may be in the division problem or the multiplication check, or perhaps in both. To discover that something is not correct isn't sufficient. Step 2 is to discover the error and then to correct it. Pupils who merely go through the motion of writing numbers in a given position are not helping themselves.

Please don't assign all the problems. Remember, checking doubles the number of computations. Be selective in making assignments.

lesson Pages 214, 215

goal Examining dividing by a 2-digit divisor

memo Pages 214 and 215 are only exploratory and involve skills that will be studied in later chapters. You may choose to omit these pages or use them only with your most capable students.

page 214 Dividing by a 2-digit number is included to provide a taste of what is to come. Pupils should not assume that division is possible with a 1-digit divisor only. No mastery is intended.

Can you divide by a 2-digit number?

$12\overline{)331}$ Estimate. $20 \times 12 = 240$
$30 \times 12 = 360$

The quotient is between 20 and 30.

Cookie Baker made 331 chocolate chip cookies. How many dozen is that?

$$\begin{array}{r} 27\text{ R7} \\ 7 \\ 20 \\ 12\overline{)331} \\ -240 \\ \hline 91 \\ -84 \\ \hline 7 \end{array}$$

Try this one. Candy Kane received a huge box of chocolates from her uncle in Europe. There were 380 chocolates in 10 layers. How many chocolates were there in each layer?

$10\overline{)380}$ Estimate. $30 \times 10 = 300$
$40 \times 10 = 400$
The quotient is between 30 and 40.

Maybe you already know the quotient. If not, you can compute it.

$$\begin{array}{r} 38\text{ R0} \\ 8 \\ 30 \\ 10\overline{)380} \\ -300 \\ \hline 80 \\ -80 \\ \hline 0 \end{array}$$

Check. $\begin{array}{r} 38 \\ \times 10 \\ \hline 380 \end{array}$ It checks. There are 38 chocolates in each layer.

You try this one. Estimate, divide, and check. Tina Bopper is giving flowers to all her friends. She has 131 flowers. If she gives each friend a dozen flowers, to how many friends will she give them? 10

$10 \times 12 = 120$
$20 \times 12 = 240$
The quotient is between 10 and 20.

$$\begin{array}{r} 10\text{ R11} \\ 12\overline{)131} \\ -120 \\ \hline 11 \end{array} \qquad \begin{array}{r} 12 \\ \times 10 \\ \hline 120 \\ +11 \\ \hline 131 \end{array}$$

214

For Experts Only

The first problem is done as an example.
Study it. Then do the others.

1. $5\overline{)7419}$

Estimate. $1000 \times 5 = 5000$
$2000 \times 5 = 10{,}000$

The quotient is between 1000 and 2000.

$$\begin{array}{r} 1483 \text{ R4} \\ 3 \\ 80 \\ 400 \\ 1000 \\ 5\overline{)7419} \\ -5000 \\ \hline 2419 \\ -2000 \\ \hline 419 \\ -400 \\ \hline 19 \\ -15 \\ \hline 4 \end{array}$$

Check:
Multiply 1483×5.
Then add 4.
Does the answer check?

2. $4\overline{)372}$ 93	3. $7\overline{)608}$ 86 R6	4. $3\overline{)1671}$ 557	5. $2\overline{)3620}$ 1810	6. $5\overline{)4461}$ 892 R1
7. $4\overline{)924}$ 231	8. $6\overline{)8333}$ 1388 R5	9. $8\overline{)1875}$ 234 R3	10. $5\overline{)9023}$ 1804 R3	11. $8\overline{)5419}$ 677 R3
12. $7\overline{)999}$ 142 R5	13. $6\overline{)8746}$ 1457 R4	14. $4\overline{)7987}$ 1996 R3	*15. $9\overline{)11{,}725}$ 1302 R7	*16. $3\overline{)10{,}651}$ 3550 R1
*17. $20\overline{)532}$ 26 R12	*18. $11\overline{)683}$ 62 R1	*19. $14\overline{)589}$ 42 R1	*20. $15\overline{)860}$ 57 R5	*21. $12\overline{)427}$ 35 R7

goal Finding 4-digit quotients

memo For experts only!

page 215 A page to challenge your most capable computers. These are definitely not for less capable students now.

lesson Page 216

goal **Checkout**—estimating, computing, and checking division

page 216 Independent work. Learners who need the extra help may use their number lines. The primary focus here is to check concept development.

The most common weaknesses include—
- Basic multiplication facts not mastered
- Inability to estimate—continue use of appropriate number lines
- Renaming errors in subtraction
- Renaming errors in addition when checking
- Not actually checking but merely writing the steps

CHECKOUT

Skill: Estimating

1. Estimate. Do not compute.

10 50 100
a How many 5s in 50? in 250? in 500? in 491? 90-100

10-20 20-30 30
b How many 4s in 68? in 90? in 120? in 400? 100

5 50 5-6
c How many 7s in 35? in 350? in 40? in 400? 50-60

8-9 80-90 70-80
d How many 9s in 80? in 800? in 640? in 4500? 500

Skill: Making judgments (using multiplication and division)

2. Which is the best buy?

a 3 for 33¢ or 10¢ each **b** 5 for 75¢ or 17¢ each

c 4 for 48¢ or 5 for 50¢ **d** 2 for 58¢ or 3 for 94¢

Skill: Dividing 3-digit by 1-digit numbers

3. Divide.

a $9\overline{)189}$ 21 **b** $8\overline{)376}$ 47 **c** $7\overline{)717}$ 102 R3 **d** $5\overline{)184}$ 36 R4

Skill: Checking division with multiplication

4. Are these problems correct? If any are wrong, correct them

a	**b**	**c**
61 R0	44 R~~0~~ 3	~~151~~ R0 115
1 Correct	4	5
60	40	10
$6\overline{)366}$	$4\overline{)179}$	100
− 360	− 160	$5\overline{)575}$
6	~~16~~ 19	− 500
− 6	− 16	75
0	~~0~~ 3	− 50
		25
		− 25
		0

STOP See activity 4, page 216 b.

See activity 5, page 216 b.

RESOURCES

another form of evaluation

for Progress Check—page 204

Estimate only.

1. About how many 7s in 84? 10–20
2. About how many 4s in 68? 10–20
3. About how many 5s in 73? 10–20
4. About how many 6s in 97? 10–20
5. About how many 3s in 58? 10–20
6. About how many 8s in 764? 90–100
7. About how many 4s in 874? 200–300
8. About how many 2s in 437? 200–300
9. About how many 5s in 738? 100–200
10. About how many 3s in 946? 300–400

for Progress Check—page 210

Divide.

1. $4\overline{)538}$ 134 R2	2. $7\overline{)379}$ 54 R1	3. $2\overline{)846}$ 423
4. $6\overline{)462}$ 77	5. $3\overline{)758}$ 252 R2	6. $5\overline{)493}$ 98 R3
7. $8\overline{)246}$ 30 R6	8. $4\overline{)922}$ 230 R2	9. $6\overline{)329}$ 54 R5
10. $9\overline{)687}$ 76 R3	11. $7\overline{)518}$ 74	12. $4\overline{)725}$ 181 R1

for Checkout—page 216

1. Estimate. Do not compute.
   a) How many 6s in 60? 10 in 360? 60 in 600? 100 in 584? 90–100
   b) How many 3s in 78? 20–30 in 97? 30–40 in 150? 50 in 600? 200
   c) How many 8s in 56? 7 in 560? 70 in 70? 8–9 in 700? 80–90
   d) How many 7s in 60? 8–9 in 600? 80–90 in 550? 70–80 in 6400? 90–100
2. Name the best buy.
   a) 2 for 64¢ or 35¢ each
   b) 4 for 27¢ or 7¢ each
   c) 3 for 48¢ or 4 for 68¢
   d) 4 for 56¢ or 5 for 67¢
3. Divide.
   a) $7\overline{)378}$ 54 b) $6\overline{)594}$ 99 c) $8\overline{)745}$ 93 R1 d) $9\overline{)642}$ 71 R3
4. Are the problems correct? If any are wrong, correct them.

a) 192	b) correct	c) correct
~~129~~ R0	42 R0	109 R3
2	2	9
90	40	100
100	$7\overline{)294}$	$5\overline{)548}$
$4\overline{)768}$	−280	−500
−400	14	48
368	−14	−45
−360	0	3
8		
−8		
0		

activities

1. things 10 same-size boxes (milk cartons); slips of paper

For practice in estimating 2-digit quotients, label the faces of the boxes as shown.

Write a division problem on each slip of paper. Make sure that the quotients are all 2-digit. The pupil selects a slip, estimates the quotient, and places the slip in the appropriate box. For example: $6\overline{)129}$. The pupil thinks to himself that there are more than 20 sixes, but less than 30. The slip is placed in the box between 20 and 30.

For practice in estimating 3-digit quotients, label the opposite faces of the boxes as shown and prepare a set of slips containing appropriate problems.

2. things deck of playing cards

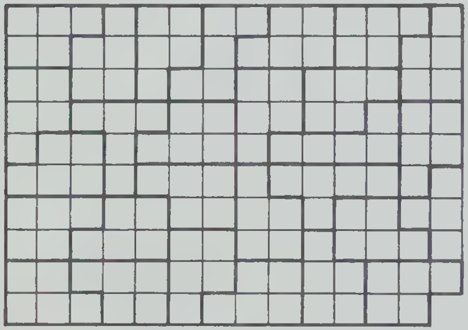
Remove the tens and face cards from the deck of cards. One player is selected to be the dealer. He deals 1 card to each player facedown and 2 or 3 cards to himself. The dealer arranges his facedown cards in a row, then turns the cards faceup to form a numeral.

Each player then uses his card as a divisor, and the dealer's numeral as the dividend, and estimates the quotient to the nearest multiple of 10 or 100. The dealer records each estimated quotient. A player may challenge any quotient he thinks is not reasonable. If the estimate is not challenged, the player earns points equal to his estimated quotient. Players predetermine the total number of points needed to win.

3. things graph paper

Use this activity with pupils who seem to lack understanding of what is happening in division. Be careful to select reasonable problems. On graph paper, have the pupil first make a border around as many squares as the number to be divided. Next have him show as many sets as possible within this border, each set containing as many squares as the divisor. Finally have him record the answer.

For example: $5\overline{)139}$



4. things spirit master

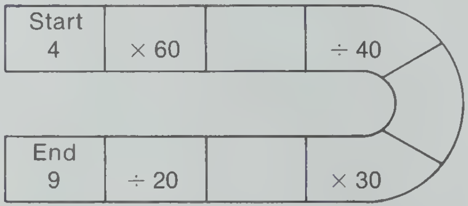
Prepare a spirit master as shown.

Look for a pattern. Clue: Compare the number in the middle to the number above or below. Now finish each row.

				450	350
8	5	6	4	9	7
240	150				

5. things spirit master

Prepare a spirit master of paths as shown.



Include some paths without any numbers so that the youngsters can make up problems of their own, leaving the answer spaces blank. Let them exchange papers and challenge a friend to complete the path.

Longer paths are possible. For example:
(3 × 80) (÷ 6) (− 4) (÷ 6) (× 3) (÷ 9)

additional learning aids

operation—chapter objectives 1, 2, 3, 4

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
  Spirit master: W 20
Arithmetic Fact Kit, SRA (1969)
  All division cards
Computapes, SRA (1972)
  Module 3, Lessons: MD 17, 18, 19, 20
  Module 4, Lessons: MD 29, 30, 31, 33, 34
Computational Skills Development Kit, SRA (1965)
  Division cards: 1, 2, 3, 4
Diagnosis: an instructional aid—Mathematics Level A, SRA (1973)
  Probe: L-4
Mathematics Involvement Program, SRA (1971)
  Card: 225
Skill through Patterns, level 4, SRA (1974)
  Spirit masters: 29, 56, 57, 67, 68, 71, 72
Visual Approach to Mathematics, level 3, SRA (1967)
  Visuals: 18, 19, 20, 21, 22

other learning aids (described on page 216e)
Orbiting the Earth (division), Rally with Remainders, Veri-Tech Senior (division book)

capture the flag

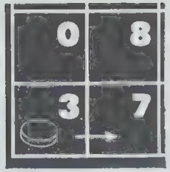
This game is for three or four people, preferably four.

Here is what you play with:

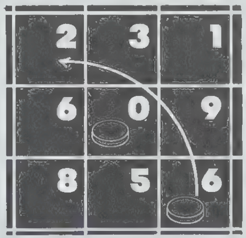
(1) A playing board that has five rows of five squares each. Each of these squares has one of the ten digits printed within it.
(2) Five tokens (one of which should be marked with a large X)
(3) A deck of forty cards. Each of the cards has one of the ten digits printed on it too. There are four cards for each digit.

Each player takes a token and places it on a corner square. There should be one player per corner if four are playing, one empty corner if only three are playing. The token marked X should be placed on the square in the center. This token is the flag. During the course of the game each player tries to capture the flag and escape with it to his own corner.

Everyone is dealt five cards from the deck. Then they take turns playing. When it is a player's turn, he discards a card from his hand and moves his token. His token can be moved one square in any direction—forward, backward, sideward, or diagonally—to a vacant square. But before a player can move a token from one square to another, the numbers printed within the two squares involved must have a difference equal to the number on the card that has just been discarded.



There is another kind of move that can be made. If a player's token is on one of the eight squares next to the flag and the square immediately behind the flag is vacant, the player can jump over the flag and land on the square immediately behind it. This is called "capturing the flag" because the player takes the flag and places it on top of his token. For a player to capture the flag this way, the number his token jumps from and the number it jumps to must have a difference equal to the number on the card just discarded.



A player can also jump over an opponent. If his token is next to an opponent's token and the square immediately behind the opponent is vacant, he can jump over his opponent and land on this vacant square. As with other moves, however, the number the player jumps from and the number he jumps to must have a difference equal to the number on the card just discarded. The jumped token remains on its square; it isn't captured. If it has the flag, though, the flag is captured by the token that made the jump.

When a player on his turn has discarded a card and moved his token, he may, if he is able and chooses to, discard another card and move again. He may move up to five times on one turn, each time discarding a card whose number is the difference between the number he moves from and the number he moves to. When the player is finished moving, he draws from the deck to restore his hand to five cards.

When a player on his turn cannot move at all or chooses not to do so, he passes up his turn without moving. He does this by discarding from one to five cards and drawing from the deck without touching his token. He restores his hand to five cards. Then play passes to the next person.

The game continues until one player captures the flag and gets it back to his corner (the one he started from) without losing it. He is the winner.

If a player carries the flag to a corner other than his own, he is momentarily free from losing the flag since no one can jump over him there. On his next turn, however, he must move out of the corner and stay out of it for at least one turn. If he cannot or will not do so, the flag is moved to the center of the board and play continues.

7	9	8	7	5
1	2	3	1	3
4	6	0	9	4
0	8	5	6	4
3	7	8	1	2

10 MATH SENTENCES

before this chapter the learner has—

1. Compared two numbers by using the symbol $>$, $<$, or $=$
2. Estimated and found the sum or difference of two 4-digit numbers
3. Found the sum or difference of two fractions with like denominators
4. Estimated and found the product of a 3-digit factor and a 1-digit factor
5. Estimated and found the product of two 2-digit factors
6. Estimated and found the quotient for a 3-digit number and a 1-digit number

in chapter 10 the learner is—

1. Identifying simple math sentences as true, false, or open
2. Finding solutions to open math sentences with one operation and one placeholder
3. Solving one-step word problems involving any one of the four arithmetic operations and writing math sentences to show that his solutions are true

in later chapters the learner will—

1. Master finding a solution to a simple open math sentence with one operation and one placeholder
2. Master solving a one-step word problem involving any one of the four arithmetic operations

Notes & Things

In this chapter math sentences are used to demonstrate one way that mathematical situations can be communicated. The pupil explores sentences that are inequalities, as well as the equalities that he has worked with in the past. The real purpose of the math sentence is to provide the learner with a means for understanding how a word problem has been solved and a means for checking the computation.

An adult solves a problem situation by computation. He often performs a computational check for accuracy. The adult theoretically has mastered computation, can read, and can mentally reason. Yet he rarely writes an open math sentence before solving the problem. Why expect a child to do it?

Word problems that can be included in texts for young lèarners must of necessity be quite simple. These youngsters have not yet mastered all computational skills, their reading skills can be limited, they lack extensive experience in reasoning, and number sentences are something abstract. Many times a child can read (or listen to) a word problem and somehow "know" the answer without knowing how he got it. Do not discourage this intuitive approach to problem solving. The learner can become discouraged if we insist that he first write an open math sentence to translate what the words say. Finding the answer—mentally whenever possible—is the ultimate goal.

The true math sentence can give children the necessary experience with math sentences. Let them puzzle together with all the puzzle pieces at hand. This will reinforce the learner's knowledge of math sentences and force him to critically examine his own reasoning. If he can't find a true sentence to confirm his answer, he will know that his answer is probably incorrect. It is his job to find the correct answer. It is your job to lead him to, not to tell him, the correct answer.

For the extra activities you will want to have these things available:

- 11 wood cubes
- washable crayon
- game board for true sentences
- spirit master of dots to color

goal Think about and explore ideas through a picture clue

page 217 Math sentences are of course a bunch of symbols that represent a lot of words, and we all live in a world of symbols. What better way to think about symbols than considering a walk to school or a walk in a city?

Everyone can collect information about the symbols that surround us. Newspapers and magazines will be the raw material needed for a scrapbook. Budding artists can do their own book of symbols complete with illustrations. If given time, your creative children will think of categories for the symbols and use the scrapbook as an organizer for their ideas. After all, we do have symbols on the streets, inside stores, at home, and at school. The job is to find as many as possible.

goal **Survey**—ability to recognize a math sentence and determine if a sentence is true or false

memo Communication of ideas is the key to this page, but please be aware that the RELATION SYMBOLS $=$, $\neq$, $>$, and $<$ are the key to the entire chapter.

These are new ideas. Don't expect all correct answers. Start with a discussion and clearly emphasize the learning goal for the chapter.

page 218 Before you start the numbered questions, take time out to review the relation symbols. There is no reason to expect the pupils to know the INEQUALITY SYMBOL, $\neq$. Develop its meaning by guiding a discussion something like this:

$4 \times 5 = 20$. Point to the $=$ symbol. *This symbol tells how 4×5 is related to 20. What words does the symbol stand for?*

$4 \times 6 \neq 25$. Point to the $\neq$ symbol. *This symbol also tells how the quantity on the left relates to the quantity on the right. What words do you suppose that symbol stands for?* (Is not equal to)

Review the other relation symbols in a similar way, making sure to stress that each symbol tells how the quantity on the left **relates** to the quantity on the right.

The pupils should be just a little bit confused when they finish the page. That's O.K. as long as they are a bit curious too.

Go directly to the next page.

Sentences help people communicate ideas—all kinds of ideas!

GOAL is to learn about math sentences so that you can communicate math ideas.

In your language book you find word sentences. In your math book you'll find sentences written with symbols. These are called math sentences.

All math sentences can be translated into word sentences. For example, $2 + 3 = 5$ could be written as the word sentence "Two plus three equals five."

Some word sentences can be translated into math sentences. For example, "Some number times two equals four" translates into a math sentence. $\blacksquare \times 2 = 4$

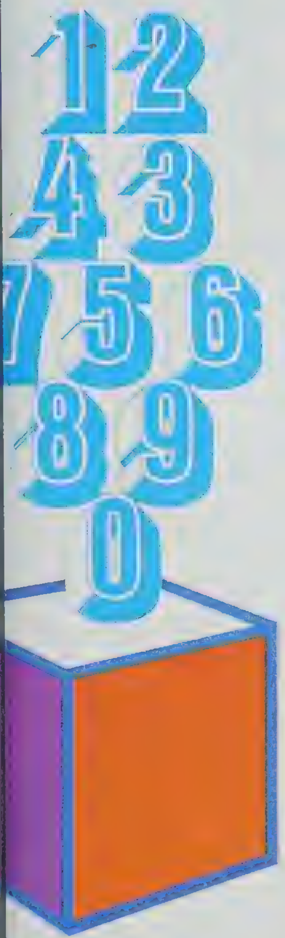
Don't expect correct answers from everyone on this page.

1. Which of the following are sentences?
   - a $5 + 6 \neq 20 \div 10$
   - b With a shout.
   - c $7 \div 2 = ?$
   - d Nancy wants to be a doctor.
   - e $10 + 25$
   - f $8 \times 3 < 25$

2. Sentences tell a complete idea, and they may be true or false. Which of these sentences are false?
   - a $20 = 5 \times 4$
   - b I am reading about math sentences.
   - c $5 + 6 > 20 \div 10$
   - d $13 \div 7 \neq 2$
   - e Bill can run one mile in one minute.
   - f $40 < 39$

218

See activity 1, page 235a.



There are a lot of different kinds of math sentences.

$8 - 5 = 3$
$9 + 5 = 14$ All of these are sentences.
$10 > 5$ How are they alike? All contain at least two numerals and a relation symbol.
$9 < 10$ How are they different? Different operations; different relation symbols; one uses ■; different numbers
■ $= 10$
$2 \times 4 = 8$
$2 \times 4 = 9$ ← What kind of sentence is this one? False

Look at this list.

$8 + 5$
$9 - 3$ None of these are sentences.
83 Can you figure out why? They're not complete; either a number or relation symbol or both are missing
$2 =$ What is the difference between
■ $\times 5$ this list and the first list? The first list contains sentences. This list doesn't.
> 4
$3 + \times 4$ ← Does this one make any sense at all? No

> Math sentences can have—
> one or more numbers involved;
> one or more than one operation symbol, or none;
> one or more than one placeholder, or none;
> correct or incorrect answers.
>
> But a math sentence *must* have a relation symbol.

219

See activity 2, page 235b.

goal Developing the definition of a math sentence

page 219 This is a discussion page. The definition of a MATH SENTENCE at the bottom of the page is certainly informal, but it should be adequate for the pupils' present needs.

The page can be extended with an activity that looks more like a language lesson than a math activity, but it should prove helpful to all children. See the Resource Section, page 235b.

If the kids are allowed to "argue" about the correctness of the entries, their reasoning may not only help their own thinking but also serve to instruct others.

goal Examining math sentences that state an INEQUALITY

memo It is extremely important that this page be handled in a light manner. Remember, this is a first-time experience. The majority of pupils can't be expected to know everything about number relations with so little practice. They will know soon.

page 220 A math sentence either states an **equality** or an **inequality.** An inequality means that one number is not the same quantity as the other—one quantity must be less than the other. To develop a broader understanding of this word, you may want to have the pupils complete the following statements.

Choose = or ≠

$3 + 2$ _____ 5

$5 + 5$ _____ 12

13 _____ 19

Can a different symbol be substituted for ≠ and keep the sentence true?

We agree $5 + 5 < 12$. What does this tell us about 12? . . . How does 12 compare with $5 + 5$? Continue with the next example.

Carry the work for exercise 3 one step further by asking the pupils to make up word problems for each of the situations. After they have the word problems, make up math sentences that summarize the stories.

Warn them to read the symbols carefully in exercise 4. Note for example: problem $\boxed{1}$ $225 > 150$ and problem $\boxed{4}$ $150 > 225$.

1

Write math sentences. Accept any reasonable answers. Examples are given.

a Write one with the math symbol for "equals." $3 + 2 = 5$

b Write another with the math symbol for "is greater than." $8 > 5 + 1$

c This time write one with the symbol for "is less than." $1 < 6 \div 2$

d Write a sentence with the symbol for "does not equal." $3 \times 2 \neq 3 + 2$

2

There are many more relation symbols than =, ≠, >, and <, but these are the ones you will be needing now. = means the symbols on the left name the same number as the symbols on the right.

≠, >, <, are inequality symbols.

What does *inequality* mean? Not equal, not the same

3

Which of these situations would signal an inequality?

a Bill had more than Tom.

b She had as many as he did.

c One mile is not the same distance as one kilometre.

d He didn't work as long a time as his brother.

4

All the following situations show inequalities. Choose one or more math sentences at the bottom of the page that best fit the situation.

a Sam's plant was 35 centimetres tall. Maria's was 40 centimetres. $\boxed{6}$

b In one day Tim collected 150 aluminum cans. Seth collected 225. $\boxed{1}$

c Sarah did 10 problems in one minute. Dilly did only 4 in one minute. $\boxed{5}$

d The Thames River is 210 miles long. The Ganges River is 1500 miles long. $\boxed{3}$ $\boxed{7}$ $\boxed{8}$

1 $225 > 150$ **2** $10 < 4$ **3** $210 < 1500$

4 $150 > 225$ **5** $10 > 4$ **6** $40 \neq 35$

7 $1500 \neq 210$ **8** $1500 > 210$

PROGRESS CHECK

Skill: Writing sentences

1. Choose one word, number, or symbol from each column. Try to write ten sentences. Sentences will vary.*

Open	up	$\neq$	92	day.
Today	$\times$	reserved	overcoat	20
$<$	little	26	$>$	now.
The	a	$-$	first	stay.
4	$\div$	3	7	7
9	the	5	to	work.
Button	are	the	$\boxed{n}$	strawberries.
30	56	men	door	5
We	$>$	your	eat	later.
15	is	people	$=$	31
21	$+$	is	involved	with.
?	$\neq$	here	$+$	12

We are here to stay.
Button up your overcoat later.
Open the reserved door now.
Today is the first day.
$21 \div 3 = 7$
$9 + 26 > 31$
$30 \neq 5 + 20$

2. Do all the combinations you wrote tell a complete idea? If they don't, you don't have a sentence. Yes

3. Are all the sentences true? They don't have to be true to be a sentence.

4. Does each of your math sentences have a relation symbol? If it doesn't have $=$, $\neq$, $>$, or $<$, you don't have a math sentence. Yes

* Examples are shown to the right of the chart. Accept any correct response that fulfills the qualification of one choice from each column.

221

goal Progress Check – identifying and writing math sentences

memo This page should be fun. Some silly sentences can be extracted from the chart, such as "Button up people involved strawberries." A good laugh will do everyone some good. The question still has to be asked as to whether the words form a sentence.

page 221 Work through one or two examples with the group. Make sure they understand directions. Possible sentences: $4 \times 3 = 12$ and $4 \neq 5 + 5$.

They can continue the project as independent work. Questions 2, 3, and 4 are designed to help pupils catch errors and recheck their thinking.

Group those children who experience much difficulty. Rework the page together. Look for all possible sentences. Lead but please don't tell. Encourage pupil participation. Have them record the suggested sentences.

STOP See activity 3, page 235b.

See activity 4, page 235b.

goal Investigation of TRUE and FALSE math sentences; exploration of OPEN SENTENCES

page 222 The distinction between a computation that is correct or incorrect (right or wrong) and a math sentence that is **true** or **false** will require some discussion.

To reinforce this point, have the pupils replace the placeholder (variable) in an **open sentence** with a correct replacement and with an incorrect replacement. An incorrect computation will yield a false sentence; a correct computation will yield a true sentence.

Discuss the sentences related to the pictures. The purpose of these open English sentences is to let the pupil know that more than one replacement can be used to form a true sentence. They will make the math sentences easier.

You'll want to discuss the results and then continue to page 223.

What about this English sentence?
Is it *true* or *false*?

He is taller than Jason.

We don't know whether it's *true* or *false* until we know who "he" is and who "Jason" is, and how ta each of them is. This sentence is *open*, because we don't have all the information.

How about this number sentence—true or false?

3 + ■ = 5

It could be true. It depends on what's in the box. Is it false? It could be, depending on what's in the box. We don't know whether 3 + ■ = 5 is true or false. 3 + ■ = 5 is an *open sentence*. An *open sentence* is neither true nor false.

Here are some words: *train, stumps, mountains, road, deer, trees, river, fence, snow*. Select one of the words to make each English sentence true.
Accept any reasonable answers. Examples are shown.

1. There is a winding ? . road, river
2. The sun shines on the ? . snow
3. The ? are tall. trees, mountains
4. The ? are near the fence. deer

Here are some numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
Select one of the numbers to make each math sentence true.

5. 4 > ■ 1, 2, 3
6. 3 + 4 ≠ ■ All numbers except 7
7. 10 < ■ 11, 12
8. 8 < ■ 9, 10, 11, 12
9. ■ < 12 All numbers 1 to 11
10. 0 < ■ All numbers 1 to

222

1. Which of the following sentences are true?

a All cats have blue eyes.

b $82 - 24 < 49$ c $36 + 3 \neq 58$

d Toronto is a Canadian city.

e There are 100 centimetres in 1 metre.

f $45 \div 9 = 5$ g $17 + 21 > 36$

h One mile is the same distance as one kilometre.

i A kilogram is a measure of weight.

2. Use $>$, $<$, or $=$ to make each math sentence true.

a $7643 \overset{>}{?} 7463$ b $16 \text{ thousand} \overset{=}{?} 16{,}000$

c $8 \times 8 \overset{=}{?} 64$ d $72 \div 8 \; ? \; 7$

e $18 + 3 \overset{=}{?} 21$ f $492 - 183 \overset{<}{?} 319$

3. Which sentences do you know are true?
None; all are open sentences.

a $14 + ? = 20$ b $? \div 5 = 30$

c $16 > ?$ d $10 \times ? < 70$

e $4 \times ? = 0$ f $10 \times ? = 10$

PROGRESS CHECK

Skill: Identifying true, false, and open sentences

Which sentences are true? Which are false? Which are open?

1. $18 + 12 = 15 + ?$ Open 2. $17 - 11 > 8 \times 8$ False 3. $64 - s = 6$ Open

4. $49 \div 7 = 16 - 9$ True 5. $27 \div 3 = 3 \times 3$ True 6. $16 > ?$ Open

Skill: Finding solutions for open sentences

Find the open sentences. Replace the symbol for the unknown number and make the sentence true.

7. $5 + 3 + \overset{13}{a} = 21$ 8. $7 + 4 = 12$ False 9. $10 \times \overset{6}{?} = 60$

10. $58 - \underset{18}{b} = 40$ 11. $36 \div 3 = ?$ 12 12. $13 + \overset{23}{g} = 36$

goal More practice with true and open math sentences; **Progress Check** – identifying and completing true, false, and open sentences

page 223 You may want to review the **greater than** and **less than** symbols – these somehow seem to get misread – before assigning exercises 1, 2, and 3 as independent work. If any of the pupils lack confidence, consider taking these exercises one at a time, correcting each set before going on to the next.

The Progress Check is independent work. Having the pupils show their computations will help you to determine whether an error is computational or whether the youngster really does not understand the question.

Some learners may not know how to find the unknown number. This is one way:

$$\begin{aligned} 5 + 3 + a &= 21 \\ 8 + a &= 21 \\ a &= 21 - 8 \\ a &= 13 \end{aligned}$$

STOP See activity 5, page 235b.

See activity 6, page 235b.

lesson Pages 224, 225

goal Using a graph and math sentences as a means of communication

memo Some students may need your help with this page. Much will depend on their previous experiences with reading graphs.

page 224 The answers for questions 1 through 4 could be obtained by counting units or by comparing columns on the graph, but math sentences are helpful to keep all the numbers organized. The best example of this is in finding the total amount collected.

English sentences help us communicate.
Math sentences do too.

Phil's class collected money for a class project. Study the chart that shows how much money the class collected each week.

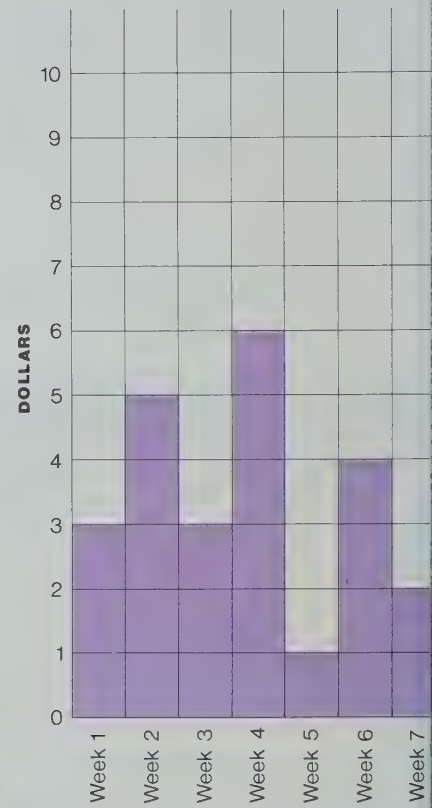
The graph gives us a good picture of what happened. But what could we tell someone about the project? Pick a math sentence at the bottom of the page that could help answer each question.

1. How much was collected each week? What was the total amount? c
2. What is the difference between the greatest amount of money collected in one week and the least amount collected in one week? b
3. How much more money was collected the first week than the last week? e
4. Their goal was to collect $20. Did they reach their goal? d

a $20 > \blacksquare$ **b** $6 - 1 = \blacksquare$

c $3 + 5 + 3 + 6 + 1 + 4 + 2 = \blacksquare$ **d** $20 < \blacksquare$

e $3 - 2 = \blacksquare$ **f** $6 - 5 = \blacksquare$



224

Pretend you got an invitation to a party. Lots of your good friends got invitations too. Nobody knew who was giving the party. The invitation told you only how to get to the party. Here is a copy of the invitation.

Come to a party Saturday night at 7:30

Just follow the map and directions and you'll know where to go.

Start at the school. Go y blocks east ---- 27 − y 2 = 25

Go n blocks north --- n 5 × 5 = 25

Go b blocks east ---- a 4 + 10 = 14 and 6 − a 4 = b 2

Go S blocks south --- 36 ÷ S 4 = 9

What was probably the main happening at the party? A movie

Was there a shorter way to get to the party? What was it? 3 blocks east 1 block north

Yes

goal Using math sentences as a means of communication

page 225 Math sentences as a means of giving directions for a party? WOW! But this is a fun activity for everyone. Be careful! Start at the school (left corner of the square) and count the whole block walked.

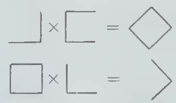
Why not challenge them to make up directions for reaching other sites on the map? These directions may be tested by exchanging with someone.

Here's a challenging activity for your sharpies. Numerals are symbols—right? Let's go one step further and replace these symbols with a symbol to write sentences. Arrange the numerals 1 through 9 in random order in a 3-by-3 grid.

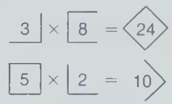
Choose two of these numbers at a time, multiply them, and record the products at random in a diagonal grid. For example:

3	6	2
7	5	8
4	9	1

The lines that define each cell of the grid form the new symbols with which to write sentences:



The above sentences can be completed like this from the grids shown:



lesson Pages 226, 227, 228

goal Matching math sentences to problem situations

page 226 The math sentences given all make use of the same numbers **but** each has different operation and relation symbols. The careless reader could experience some difficulty. The word problems must be examined for clues to the operation or relation. Translating English sentences into math sentences can be difficult for many learners. Give help where needed.

We can write a math sentence to fit situations.

Sam had 15 stamps → $15 - 3 = 12$

He gave 3 away

He had 12 left

What English words in the situation gave the subtraction clue? "gave away"

Four situations and four math sentences are written below. Pick the math sentence that fits the situation.

1. Tom caught 13 fish. Bill caught less. How many fish could Bill have caught? c

2. Sara had 13. How many did she have after she gave 2 of them away? a

3. Chris collected 13 pounds of scrap metal. Doug collected double that amount. How much did Doug collect? b

4. Judy had 13 box tops saved for the prize. Dan gave her 2 more. How many box tops did she have then? d

a $13 - 2 = \underline{?}$ 11 **b** $13 \times 2 = \underline{?}$ 26

c $13 > \underline{?}$ Any number less than 13 **d** $13 + 2 = \underline{?}$ 15

Now make each sentence true.

Would you add, subtract, multiply, or divide to compute the answers to the following problems? Write an open math sentence to show what you would do in each case.

Accept any appropriate math sentence. It is O.K. to use ■, ?, or a letter in any open sentence.

1. She had 6. He had twice as many as she did. How many did he have? $6 \times 2 = b$

2. They had 27. They gave 12 away. How many did they have left? $27 - 12 = b$

3. There were 24 in the bag. Three people were to share them. How many did each person get? $24 \div 3 = ?$

4. He ate 2 on Monday, 2 more on Tuesday, none on Wednesday, but 4 on Thursday. How many did he eat? $2 + 2 + 0 + 4 = ?$

5. He had made 1 sale. He bragged he doubled his sales. How many sales had he made in all? $2 \times 1 = m$

6. There should be 12 on each page. She wanted them in 3 rows. How many should be in each row? $12 \div 3 = ?$

7. One of them cost 49¢. They wanted 4 of them. How much did they pay in all? $4 \times 49¢ = ?$

8. He walked 15 blocks. He turned around and came back. How far did he walk in all? $15 + 15 = a$

9. She bought a red one that cost $2.98. She returned a blue one that cost $1.39. How much did she have to pay? $\$2.98 - \$1.39 = ?$

10. There were 10 rows of chairs. There were 24 chairs in each row. How many chairs in all? $10 \times 24 = ?$

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goal Writing open math sentences to fit problem situations

page 227 On page 226, the learner practiced **matching** a written math sentence to a problem situation. The emphasis here is on **writing** an open mathematical sentence to fit the problem situation. Note that the directions do not call for solving the sentences. Group those who need your additional guidance. Keep the spirit of the discussion light and work for a feeling of confidence rather than strive for mastery.

goal **Progress Check** – writing true, false, and open math sentences

page 228 You will have to be the judge of whether your students can complete this page independently or whether it is best handled as a group project. The completed chart summarizes the concepts developed in the chapter. Understanding the concepts is more important at this point than ability to operate independently.

PROGRESS CHECK

Skill: Writing true, false and open sentences

A math sentence can contain many different symbols. It must contain a relation symbol.

Here is a summary of the types of sentences you have studied. You supply at least three examples of each type. Accept any reasonable sentences.

Type of Sentence	Characteristics	Examples
Equality	Contains the relation symbol $=$.	1. You give 3 examples.
Inequality	Contains any one of the relation symbols: $>$, $<$, or $\neq$.	2. You give 3 examples.
True sentence	Must be mathematically correct.	3. You give 3 examples.
False sentence	Is mathematically incorrect.	4. You give 3 examples.
Open sentence	Contains a placeholder for missing numbers. It is neither true nor false.	5. You give 3 examples.

228

See activity 7, page 235b.

See activity 8, page 235b.

YOU

can find an answer to some story situations without writing a math sentence. If you want to make sure you are on the right track, however, you might write a math sentence to show yourself and others that your answer was right.

Here is a set of story problems about a hobby shop. You'll also see the answers that Terry thought were correct. Write a math sentence to check to see if Terry was correct. If he made an error, you give the correct answer. Accept any appropriate math sentences.

1. One case in the hobby shop had 6 shelves. Each shelf had 8 different model cars. How many model cars were in the case? **48** 6 × 8 = 48
2. Dan needed 200 picture mounts. Each package contained 50. How many packages would Dan have to buy? **4** 200 ÷ 50 = 4
3. Don had $3.00. He wanted to buy a model plane for 98¢, a craft book for $1.25, and a tube of glue that cost 49¢. Did Don have enough money? **No** $.98 + $1.25 + $.49 $2.72 Should be yes
4. How many metres of cord can Lois buy if it costs 9 cents a metre and she has $1.25? **9** $1.25 ÷ $.09 = 13 (R 8) Correct answer is 13
5. Len got back 16¢ change. He had given the clerk 75¢ for something priced at 59¢. Was this the correct change? **Yes** 75¢ 59¢ = 16¢
6. Jan bought 3 packages of seed beads for 89¢ each. How much did the 3 packages cost in all? **$2.58** 3 × 89¢ = $2.67 ← That's correct.

229

lesson Pages 229, 230, 231, 232

goal Using a math sentence to verify a computed answer

memo: Note that the math sentence is written after the solution is found.

page 229 Mental computation should be encouraged. However, we must know how to verify answers. Youngsters who have difficulty reading will need help locating clues. Make sure that the directions are clearly understood. They are to verify if Terry's answers are correct or incorrect by writing a true math sentence. An inequality signals that a computed answer is incorrect, but it does not correct the computation.

goal Solving word problems; writing a math sentence to back up the computation

page 230 Again, depending upon the abilities of your class, give help by reading the stories, but let the youngsters find math sentences through trial and error if necessary. Investigate the possibility of more than one math sentence to fit the problem situation. For example:

1. $19 + 12 > 30$
   $30 - 19 = 11$
   $30 - 19 < 12$
2. $15 \times 30 = 450$
   $450 \div 30 = 15$
   $450 \div 15 = 30$
3. $3 \times 29 = 87$
   $3 \times 29 > 84$
   $84 \div 3 = 28$

In problem 4, did Sidney study at least an hour? more than an hour? less than an hour? How much more than an hour did he study?

How about Jerry's 42 feet of rope in problem 5? Is this long enough to tie to the tree and still reach the ground?

TRUE OR FALSE?

This time it's your turn to find the answers.
Write a math sentence to back up your answer.
Accept any appropriate math sentences.

1 There are 30 students in Tim's class. 19 students brought their lunch. The rest bought their lunch. Tim is in charge of collecting the lunch money. He said that 12 students bought their lunch.
Is his statement true or <u>false</u>? $30 - 19 = 11$

2 Yesterday 15 students gave Tim their lunch money. Lunch costs 30¢. Tim said that he collected \$4.50.
Was his statement <u>true</u> or false? $15 \times \$.30 = \$4.50$

3 Monday was Peanut Butter and Jelly Day. Mae gave each student (including herself) 3 peanut butter and jelly sandwiches. She handed out 84 sandwiches in all. She said there are 29 students in the class.
Was her statement true or <u>false</u>? $84 \div 3 = 28$

4 Sidney studied for 20 minutes before the test and 45 minutes after the test. He said he studied for an hour.
Was his statement true or <u>false</u>? $20 + 45 = 65$

5 Jerry bought 16 feet of rope in the morning and 26 feet of rope after noon. His tree house is 40 feet off the ground. Jerry said, "Now I have enough rope to reach the tree house."
Was his statement <u>true</u> or false? $16 + 26 = 42$
$42 > 40$

230

Here is a copy of Cindy's homework. She made some mistakes.
Can you find them? Use math sentences to check her work.

Accept any appropriate math sentences

name *Cindy*

1. Peter had 24 comic books and Joseph had 47 comic books. Joseph threw away 22 of his comic books because they had bad plots. Who has more comic books now—Joseph or Peter? 47 − 22 = 25, 25 > 24

Joseph

2. Carla's mass is 30 kg . Together, she and her brother have a mass of 49 kg . What is the mass of Carla's brother? 49 − 30 = 19

19 kg

3. Tickets for the raffle cost 50¢ each. Carol has $2.75. How many tickets can she buy?
275 ÷ 50 = 5 R 25 Correct answer is 5 tickets.

6 tickets

4. Mara has just finished reading a book that is 232 pages long. Because she was in a hurry, she skipped 42 pages. How many pages did she actually read? 232 − 42 = 190 Correct answer is 190 pages.

185 pages

5. Arnie the porcupine had 148 quills. He gave 32 of them to a curious dog. How many quills does Arnie have left? 148 − 32 = 116 Correct answer is 116 quills.

126 quills

6. Which box of colored pencils is the best buy? They are all of the same quality.

10 PENCILS 49¢	20 PENCILS 80¢	50 PENCILS $2.00
49¢ ÷ 10 = 4¢R9	80¢ ÷ 20 = 4¢	$2.00 ÷ 50 = 4¢

A correct answer would also be the last box

7. Might there ever be a time when the cheapest is not the best buy?

No

Correct answer is yes. Poor quality is rarely a good buy

231

goal Using a math sentence to check a computed answer

page 231 Help youngsters who have difficulty with reading. In each situation there is more than one way to think about the problem and more than one way to solve it. Let each pupil use the math sentence he can justify.

Examine exercise 6. Are pencils of a poorer quality a good buy at a cheaper price? Quality as well as price are important factors when buying. With some purchases, such as fresh food, buying more than you can use would also be a poor purchase.

goal Using a math sentence to verify a computed answer

page 232 You may want to handle this page as a group project to determine appropriate math sentences. Let more than one sentence be given. Ask each person to tell why he thinks one sentence is better than another. In exercise 1, part **a** will need to be verified before they can do part **d.** Adding down, then checking by subtracting across to determine the amount saved in all is a standard bookkeeping technique.

You can't always solve a problem in your head. Sometimes large numbers make a problem difficult.

1. Steve works all summer doing odd jobs. He washes cars, mows lawns, babysits, runs errands, and does other things like that. Last summer he kept a record of how much he earned and how much he spent.

	Earned	Spent	Saved
July	$64.95	$49.50	?
August	98.70	56.30	?

Now answer these questions. If you don't need a math sentence to help you get started, then write one to show how you got your answer.

a How much did Steve save in July? in August?
$64.95 − $49.50 = $15.45 $98.70 − $56.30 = $42.40

b How much did he earn in the two months?
$64.95 + $98.70 = $163.65

c How much did he spend in the two months?
$49.50 + $56.30 = $105.80

d How much did he save in all?
$15.45 + $ 42.40 = $57.85 or $163.65 − $105.80 = $57.85

Could more than one sentence be written for the last question? Yes

2. Sometimes a math sentence helps you keep track of a lot of numbers needed to find just one answer to a problem. Try this problem.

Only 6 persons got off the bus at the first stop at C Street, 2 persons got off at D Street, 4 persons got off at E Street, and the last 5 persons got off at F Street. How many passengers were on the bus?

Write a true sentence to show your answer.
6 + 2 + 4 + 5 = 17

Math sentences can help you get organized. Work through these questions. Writing a math sentence for each question will help.
Accept any appropriate math sentences.

Mr. Mason owns the Bike Shop. One week he bought 8 boxes of bicycle spokes for $16.00. Each box contained 10 spokes.

1. How many spokes were there in the 8 boxes? $8 \cdot 10 = 80$
2. How much did each spoke cost Mr. Mason? $\$16.00 \div 80 = \$.20$
3. Do you think Mr. Mason will sell the spokes for the same amount of money he paid for them? Why? No He wants to make money.
4. There is usually a difference between the cost of an item and its selling price. Suppose Mr. Mason sells each spoke for 5¢ more than he paid for it.
   - a What would be the price of 1 spoke? $\$.20 + \$.05 = \$.25$
   - b If he sells all the spokes, how much will he get? $\$.25 \times 80 = \$20.00$
   - c How much more is this than he paid for them? $\$20.00 - \$16.00 = \$4.00$

These next questions are for thinking, not computing.

5. What are some of the things Mr. Mason will have to pay for with the money he gets from his sales? Think about all the bills he has to pay to keep his store open. These bills are called expenses. Upkeep of his shop, his employees' salaries, insurance—and he must buy the things he sells
   - a Would the expenses a grocery store pays be the same as those of a bicycle shop? Some of them
   - b Does a movie theater have expenses? Yes What might they be? Film rental, heat, electricity, employees' salaries, etc.
   - c Would a coin-operated laundry have expenses? Yes A bowling alley? Yes A restaurant? Yes

233

lesson Pages 233, 234

goal Using math sentences to sort information for complicated word problems

memo These multistep problems require some good thinking. They may be too complex for some pupils to handle. Why not go back to some practice using the pupil-made sentence charts that were made for page 221 or the word-problems box.

page 233 You'll have to decide how best to handle questions 1 through 4 with your class. Question 5 is strictly for discussion. Lead them to see why the shopkeeper must charge more than he paid for the spokes.

goal Practice with problem situations

memo This is problem solving at its best. Give everyone a chance, because this is a very real everyday situation. But be careful with the youngsters who are bewildered by it all. Make sure they are at least listening to the discussion.

page 234 To determine the best bargain, the youngsters will need to recompute each estimate. The cost per spoke and the cost per hour of labor are clearly indicated. An estimate is not a binding contract. Therefore, mistakes can be corrected. Mistakes such as these are often made in the real world. As customers, we should not merely accept a bill without checking the computations.

Sam broke some spokes on his bike and the chain too. He went to several repair shops to find out how much it would cost to have someone repair his bike. Sam took the information home to have his dad look it over.

His father found some

mistakes

Can you find them too?

Bill's Bikes	
Parts	
5 spokes 20¢ each	$1.00
Labor	
$2.60 an hour time : 1 hr.	2.60
TOTAL	~~$4.20~~ $3.60

WALLY'S CYCLES	
PARTS	
5 spokes 30¢ each	$1.50
LABOR	
$1.90 an hour 3 hours	~~$4.70~~ $5.70
TOTAL	~~$6.20~~ $7.20

ED'S BIKE SHOP	
Parts	
5 Spokes 25¢ each	$1.5
Labor	
replace Spokes $2.25 an hour labor 2 hours	$4.
TOTAL	~~$6.~~ $5.

If you were Sam, which place would you have do the repairs? Bill's Bikes

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goal **Checkout** – working with math sentences

CHECKOUT

Skill: Identifying true, false, or open sentences

1. Tell whether each sentence is true, false, or open.

a $13 > 14 \div 2$ True
b $125 \div 5 = 25$ True

c $n - 2 > 0$ Open
Any number > 2
d $63 \div 7 = 9 \times n$ Open
$63 \div 7 = 9 \times 1$

e $175 \div 7 = ?$ Open
$175 \div 7 = 25$
f $\frac{1}{4} < \frac{1}{3}$ True

g $8 + n = 25$ Open
$8 + 17 = 25$
h $2005 \times 6 = ?$ Open
$2005 \times 6 = 12.030$

i $\frac{1}{4} + \frac{3}{4} = \frac{4}{8}$ False
j $\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$ True

Skill: Completing open sentences

2. Make each open sentence above a true sentence. See above

Skill: Solving a one-step word problem and writing a math sentence to prove its solution

Find an answer to each of the problems. Then write a true math sentence to back up your answer.

3. **a** She had 15 goldfish. She sold 7. How many goldfish did she have left? 8
$15 - 7 = 8$

b She sold the goldfish for 25¢ each. How much money did she get when she sold the 7? $1.75
$7 \times \$.25 = \$1.75$

4. **a** 24 chairs in each row. There were 110 rows. How many chairs in all? 2640
$24 \times 110 = 2640$

b Only 950 adults came to the meeting. 467 of them were women. How many men were there? 483
$950 - 467 = 483$

5. **a** Bill bought 5 pens for 75¢. How much did each pen cost? 15¢
$75¢ \div 5 = 15¢$

***b** He sold each of the pens for 20¢. How much money did he earn? $.25
$20¢ \times 5 = \$1.00$
$\$1.00 - \$.75 = \$.25$

235

page 235 You will want to discuss directions for each problem set – then it's independent work. Skills are identified on the answer key.

In problem set 1, check for computational errors that could lead the pupil to make an incorrect statement.

With problem set 2, you will want to check whether the pupil knows how to solve an open sentence other than by guessing.

Ask the child who is having a reading problem how he can find the answer. Then let him explain.

STOP See activity 9, page 235c.

See activity 10, page 235c.

RESOURCES

another form of evaluation

for Progress Check—page 221

Answers will vary. Examples are given.

1. Write a math sentence using the symbol for addition. $35 + 6 = 41$
2. Write a math sentence using the number 5. $5 \times 4 = 20$
3. Write a math sentences using the symbol for "equals." $15 \div 3 = 5$
4. Write a math sentence using the symbol for "is less than." $5 < 16$
5. Are all your math sentences true? They don't have to be. Write a false math sentence. Then rewrite it and make it true. $16 - 9 = 9$; $16 - 9 = 7$ or $16 - 7 = 9$ or $18 - 9 = 9$
6. Do all your math sentences have a relation symbol? Yes You don't have a math sentence unless you have $=$, $\neq$, $>$, or $<$.

for Progress Check—page 223

Which sentences are true? Which are false? Which are open?

1. $4 + 7 = 3 + 9$ False 2. $6 < m$ Open
3. $7 \times 4 > 25$ True 4. $12 \neq 2 + 6$ True
5. $49 \div 7 = \underline{?} + 4$ Open 6. $17 - 5 = 11$ False

Find the open sentences. Replace the symbol for the unknown number and make the sentence true. 7, 8, 9, 10 and 12

7. $4 + 5 = n \div 2$ 18
8. $6 + 4 > p$ Accept 0–9
9. $15 - r \neq 7$ Accept any number less than 16 except 8
10. $2 \times 5 \times 6 = \underline{?}$ 60
11. $14 > 3 + 7$
12. $13 - 7 < h$ Accept any number greater than 6

for Progress Check—page 228

A math sentence can contain many different symbols. It must contain a relation symbol.

1. Which sentences are equalities? a, c
   a) $4 + 3 = 5 + 2$ b) $7 + 4 > 10$ c) $5 \times 6 = 25$
2. Which sentences are inequalities? b, c
   a) $3 + 5$ b) $4 \times 5 > 7 + q$ c) $19 \neq 7 + 8$
3. Which sentences are true? a
   a) $12 \div 2 > 9 - 4$ b) $4 \times 7 \neq 28$ c) $7 + 8 = s + 5$
4. Which sentences are false? a, b, c
   a) $20 \div 2 = 2 + 7$ b) $14 < 12$ c) $17 \neq 8 + 7 + 2$
5. Which sentences are open? a, b
   a) $4 \times p > 26$ b) $7 \times 5 = z$ c) $12 + 5$

for Checkout—page 235

1. Tell whether each sentence is true, false, or open.
   a) $14 - 5 = 7 + 3$ False b) $5 \times w = 40$ Open
   c) $3 \times 5 > 2 \times 6$ True d) $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$ True
   e) $45 \div 9 = k$ Open f) $25 \neq 5 \times 5$ False
   g) $18 - y < 10$ Open h) $81 \div 9 = 3 \times 3$ True
   i) $\frac{1}{4} > \frac{1}{2}$ False j) $749 - 368 = b$ Open
2. Make each open sentence above a true sentence. Answers may vary. Examples are given. b) $5 \times 8 = 40$ e) $45 \div 9 = 5$ g) y may be replaced by any number 9 through 18. $749 - 368 = 381$

Find an answer to each of the problems. Then write a true math sentence to back up your answer.

3. a) 6 bottles of pop in a carton. They need 36 bottles. How many cartons must they buy? 6; $36 \div 6 = 6$
   b) Each carton costs $1.09. How much money must they spend? $6.54; $1.09 × 6 = $6.54
4. a) He needed $20.00 to buy a dog. His grandmother gave him $5.00. How much more does he need? $15.00; $20.00 − $5.00 = $15.00
   b) He can earn 75¢ an hour mowing grass. How many hours must he work to earn enough money? 20; $1500 \div 75 = 20$
5. a) Sue's father drives 50 miles each day going to and from work. How far does he drive in 5 days? 250 mi; $50 \times 5 = 250$
   *b) Gas costs 34¢ a gallon. He can go 25 miles on 1 gallon of gas. How much does he spend on gas for 5 days? $3.40; $50 \div 25 = 2$ and $2 \times 5 \times 34 = \$3.40$ or $250 \div 25 = 10$ and $10 \times 34 = \$3.40$

activities

1. Challenge the youngsters to jot down a math sentence to communicate with numbers each of these English sentences.

1. Ann has 8 guppies. Cindy has 13. How many guppies do they have together? $(8 + 13 = s)$
2. Jay and Max together have 17 baseball cards. Jay has 8 cards. How many does Max have? $(8 + m = 17$ or $17 - 8 = m)$
3. 18 volleyballs must be packed in boxes. Each box holds 6 balls. How many boxes? $(18 \div 6 = b$ or $6 \times b = 18)$
4. Ted had 12 cookies. He gave Greg 4. He has how many cookies left? $(4 + t = 12$ or $12 - 4 = t)$

2. Our language often plays tricks on us. Sometimes we use words to communicate math ideas. Make a list of as many words or phrases as possible that signal a particular arithmetic operation. Then construct a simple sentence or story that uses one of the words as a clue. For example: She **added** 3 to the 4 she had. All four operations could be featured, but you may want to start with only addition and subtraction. Hope that there will be a demand to include multiplication and division.

Here are some words that might be listed as addition clues: **added, sum of, and, plus, increased by, combined, joined, get more, gain, total, all together.**

Subtraction clues include the following: **take away, minus, difference, decreased, diminished, less, remain, lose, lost, have left.**

Consider having a contest to see which group of children can find and use the most clues to an operation. Or have one child do his own research and let the other kids judge the correctness of his entries. Either approach may yield a bulletin-board display.

3. things game boards; washable crayons

Prepare various game boards as shown—or challenge pupils to create some themselves. Laminate the boards or cover them with a piece of clear plastic. Pupils use washable crayons or felt pens for marking. (The marks can be wiped off with tissue or a piece of cloth.)
Mark in +, −, ×, ÷, and = symbols to make true sentences. Circle the hidden sentences.

0	4	4 ÷	2 =	2	1	2	1	1	1
3	1	4	0	3	2 ×	5 =	1	0	3
6	3	0	2	4	4	1	3	4	5
6	5	9	1	9	2	4	1	5	5
6	0	9 −	9 =	0	1	8	1	8	7
3	4	1	0	7	0	3	4	3	3

4. things deck of cards; small cards

Remove the face cards from a deck of cards. Change the aces to ones. Each player will need 2 cards for each operation symbol and 1 equal-symbol card. Four playing cards are dealt to each player. The player then tries to form a true sentence with his cards. For example: $\boxed{3}\boxed{+}\boxed{4}\boxed{=}\boxed{5}\boxed{+}\boxed{2}$

Players predetermine the number of points earned for each true sentence and the number of points needed to win.

5. things 5 wood cubes; felt pen

Number the faces of the cubes with felt pen as follows:
- 1 cube—+, +, +, −, −, −
- 1 cube—all = symbols
- 1 cube—0, 1, 2, 3, 4, 5
- 1 cube—6, 7, 8, 9, 10, 11
- 1 cube—12, 13, 14, 15, 16, 17

Pupils work in small groups. The first player rolls the 5 cubes and challenges the next player to make a true sentence using the numbers and symbols that land faceup. If this is possible, the player earns a point; if not, the turn passes. The player with the most points wins.

Variation: Include an additional cube labeled =, =, ≠, ≠, <, >. (Be sure to write "less" under one symbol and "greater" under the other to avoid confusion.)

6. things 11 wood cubes; felt pen; container

Number the faces of the cubes with felt pen as follows:
- 3 cubes—0, 1, 2, 3, 4, 5
- 3 cubes—0, 6, 7, 8, 9, 10
- 3 cubes—+ symbol
- 1 cube—− symbol
- 1 cube—= symbol

The pupil shakes the cubes in the container and rolls them out. He tries to form the **longest** possible sentence from the set that lands faceup.

Pupils may work individually or in pairs. When working in pairs, they alternate as challenger to verify the other player's sentence.

7. things 3 boxes; small cards
(See page 235d.)

Label the boxes as follows: numerals, operations, relations. Prepare 2 cards for each operation sign, 3 cards for each relation symbol, and as many numeral cards as you like. Place the cards in the appropriate box.

A pupil may draw as many as 5 cards, one at a time, to make a math sentence. If no sentence is possible, he may return 1, 2, or 3 of his cards to the proper box and draw a replacement for each card he returns.

8. Extend activity 7 for more capable pupils by adding cards for variables to the numeral box and having these pupils predetermine a scoring system for true, false, and open sentences—also the total number of points needed to win.

9. things game board; index cards

Have your pupils prepare 3 sets of cards for the numerals from 0 through 18. Each pupil will need a game board as shown.

		____	=	____		1 point
____	+	____	=	____		2 points
____	−	____	=	____		3 points
____	×	____	=	____		4 points
____	÷	____	=	____		5 points

Each player is dealt the same number of cards. He may then use his cards to form as many true sentences as possible on his game board to earn the points indicated at the side of the board shown. Players predetermine the number of points to win.

10. things spirit master

Prepare a spirit master of examples as shown.

$3 + \square < 7$ Color the dots that make the sentence true red.

$3 + \square = 7$ Color the dots that make the sentence true blue.

$3 + \square > 7$ Color the dots that make the sentence true green.

additional learning aids

concept – chapter objective 1

SRA products

Mathematics Involvement Program, SRA (1971)
Cards: 123, 133, 124

other learning aids (described on page 288g)
True or False game

operation – chapter objectives 2, 3

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: P 8
W 15, 16

Diagnosis: an instructional aid – Mathematics Level A, SRA (1973)
Probe: L-14

Skill through Patterns, level 4, SRA (1974)
Spirit masters: 15, 18, 68

other learning aids – Heads Up*, Number Sentence Game,

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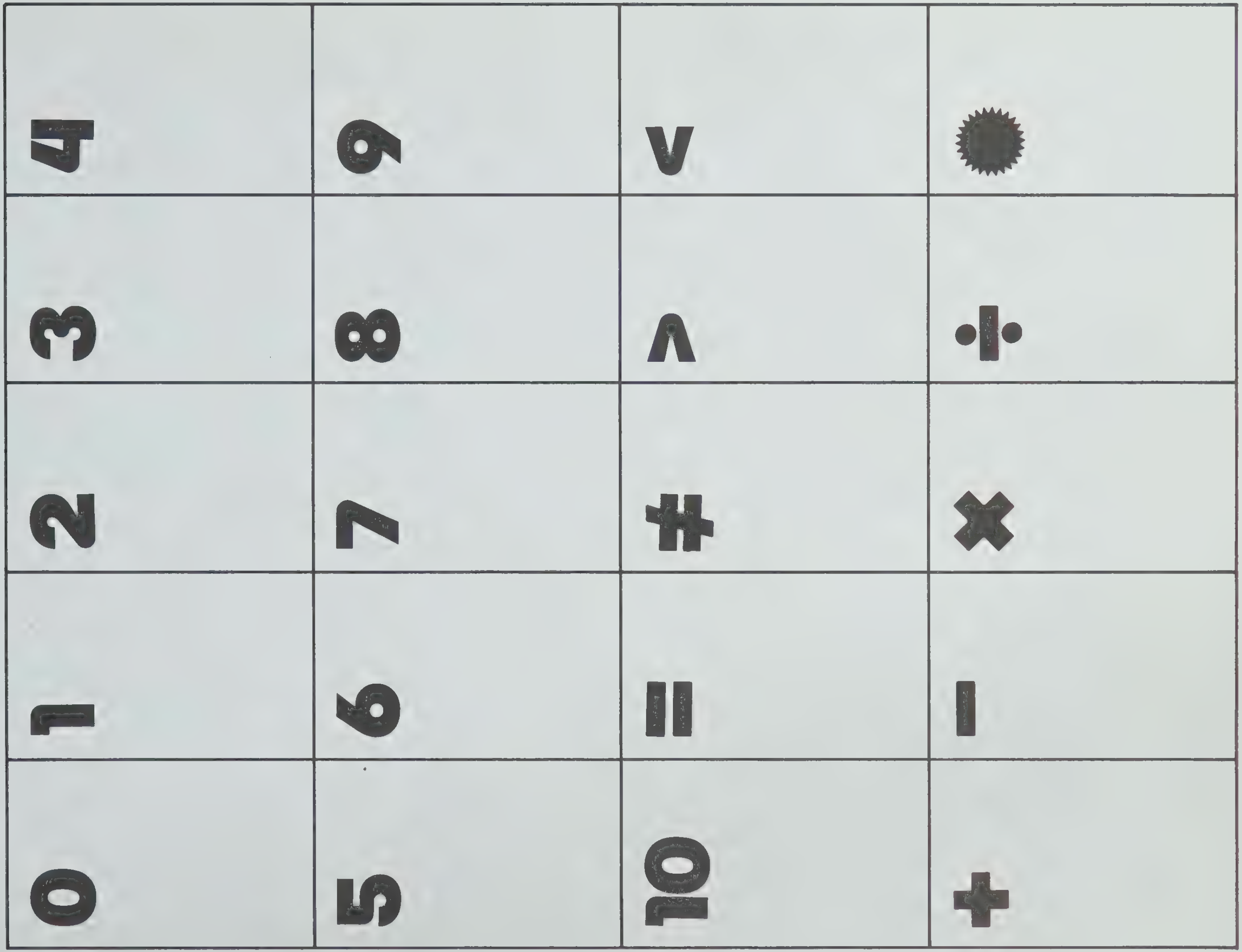


11 FRACTIONS

before this chapter the learner has—

1. Mastered naming a fraction associated with a fraction model—number line, region, or set
2. Mastered making a model to illustrate a fraction
3. Mastered comparing two fractions with like denominators
4. Mastered identifying fractions equal to 0 or 1
5. Mastered the addition and subtraction of two fractions with like denominators
6. Identified fractions that are greater than 1

in chapter 11 the learner is—

1. Renaming fractions
2. Renaming fractions in simplest form
3. Renaming appropriate fractions as whole or mixed numbers

in later chapters the learner will—

1. Master renaming fractions
2. Use renaming to add and subtract fractions with unlike denominators

Notes & Things

This chapter on fractions will give the learner enough review of and practice in the addition and subtraction of fractions with like denominators so that the majority will attain mastery.

The first experiences with renaming fractions also will be presented in this chapter. The focus will be on finding another name for a fraction rather than on the more formal approach of finding a set of equivalent fractions. The bulk of the work is intuitive. Each learner will examine the method for finding equivalent fractions, but the emphasis will remain on finding the simplest name for the answer to an addition or subtraction problem. The lack of formality means that the child will have to view every answer and ask himself if that answer is the simplest name. He will then discover for himself the value of inspecting to find if there is a common factor in the numerator and denominator. This informal approach allows fraction names for whole numbers and mixed numbers also to be included for renaming.

There is no pressure on the learner to use the greatest common factor in the renaming process. In fact, there is no isolated practice with the greatest common factor. The learner, through much experience, will come to see that taking time to find the greatest common factor will save a lot of work.

The approach featured in this chapter can be best summarized with some addition examples taken from pages 248 and 254 of the text itself.

$\frac{1}{10} + \frac{7}{10} = \frac{1+7}{10} = \frac{8}{10}$ Is $\frac{8}{10}$ the simplest name? Is there a common factor of the numerator and denominator?

The procedure for dividing both numerator and denominator by a common factor is then used so that the learner can progress to the next learning step.

$\frac{5}{6} + \frac{5}{6} = \frac{5+5}{6} = \frac{10}{6}$ ⨠ Common factor? $\frac{10 \div 2}{6 \div 2} = \frac{5}{3}$ Now rename so that you have a mixed number.

$\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \frac{7}{5}$ ⨠ Common factor? Rename as a mixed number.

$\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2} = \frac{2}{2}$ ⨠ Common factor? You know the number that kind of fraction names.

The instructional material is carefully controlled and sequenced. All computation problems are limited to reasonable fractions with a common denominator. The learner has a lot of examples to get involved with. His thinking is guided. He will be encouraged to give each answer to an addition or subtraction problem its simplest name. Mastery of renaming a fraction is not expected of all pupils, but you may be surprised to find how many are moving with complete confidence and accuracy. This renaming work is only the introduction to a much more extensive study of equivalent fractions that will be found in the next level.

things

scissors and 2 rectangular pieces of paper for each pupil

For the extra activities you will want to have these things available:

- clear acetate
- washable crayon
- 4-by-6 array game board
- spirit master of octagon
- 6 wood cubes

goal Think about and explore ideas through a picture clue

page 236 Very few words have to be said about this photograph. How full is the jar of milk? How full is the jar of orange juice? the jar of tomato juice? the jar of grapefruit juice?

It is surprising how many times fractions are actually used for descriptive or identifying purposes. (For example: *Use the jar that is half empty; a three-quarter length coat*) Pupils will provide other examples.

Your creative writers can listen and record the situations in which fraction names are used. Sometimes what they record may not refer to a number situation at all. "That's a half-baked idea" is one of many slang sayings that is far removed from any number reference. This activity will bring some surprising results.

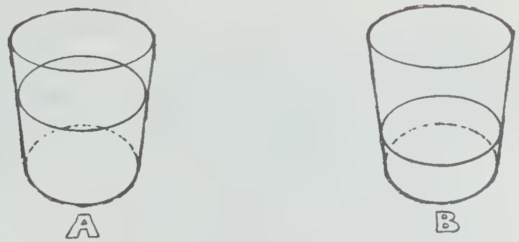
Both activities will serve to expand the learner's awareness of how frequently numbers are used in everyday situations.

lesson Pages 237, 238, 239

goal Survey - renaming fractions

page 237 Start this page with discussion of glasses A and B. Problem 1 gives a simple first look at renaming - not really so very hard. Problem 2 should help the children to understand that two eighths are actually equal to one quarter. Every child should work this one independently. Difficulties will show up, and can be handled, in subsequent discussion.

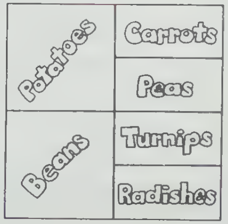
You have a good start in learning how to add and subtract fractions. Your goal now is to learn still more about fractions AND addition and subtraction.



Is glass A half full? Is it half empty? Yes Yes
Is glass B $\frac{1}{4}$ full or $\frac{3}{4}$ empty? Both

1. If I have a glass that is $\frac{1}{4}$ full of orange juice, and you have a glass that is $\frac{2}{8}$ full of orange juice, do we have the same amount? Yes
   - **a** If Muriel has a glass $\frac{4}{16}$ full, does she have as much as you have? Yes
   - **b** Does she have as much as I have? Yes
   - **c** Do we all have the same amount? Yes

2. Mr. Smith's vegetables are grown in a square plot. He hopes to get $\frac{3}{4}$ of his vegetables planted today, and the rest tomorrow. Name three different combinations of vegetables that he could plant to get $\frac{3}{4}$ of them in today.



Potatoes, beans, carrots, peas.
Carrots, peas, turnips, radishes, potatoes.
Beans, carrots, peas, turnips, radishes.

237

goal Introduction to renaming fractions

page 238 The youngsters have had previous experience in finding several names for a whole number. They have found names for 10:

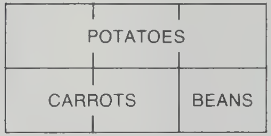
$$8 + 2 = 10 \qquad 2 \times 5 = 10$$
$$12 - 2 = 10 \qquad 20 \div 2 = 10$$

They also have found fraction names for 0 and 1. More than one name for some fractions should come as no surprise.

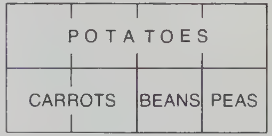
The garden plot diagrams reinforce graphically that 2/8 and 1/4 are the same, and similarly with 2/4 and 1/2, and 2/6 and 1/3.

Mr. Fielding and Mrs. Gardner have vegetable plots the same size and shape.

Mr. Fielding plants his plot with potatoes, carrots, and beans, like this:



Mrs. Gardner plants her plot with potatoes, carrots, beans, and peas, like this:



1. Does Mr. Fielding have $\frac{1}{2}$ of his plot in potatoes? Yes
   Does he have $\frac{3}{6}$ of his plot in potatoes? Yes
2. Does Mrs. Gardner have $\frac{1}{2}$ of her plot in potatoes? $\frac{2}{4}$ of Yes
   her plot? $\frac{4}{8}$ of her plot? Yes
   Yes
3. Who has more land in potatoes? Both have the same
4. Does Mr. Fielding have $\frac{2}{6}$ of his land in carrots? Yes
   Does he have $\frac{1}{3}$ of it in carrots? Yes
   Yes
5. Does Mrs. Gardner have $\frac{1}{4}$ of her land in carrots? $\frac{2}{8}$ of her land? Yes
6. Who has more land in carrots? Mr. Fielding
7. What part of his land does Mr. Fielding have in beans? $\frac{1}{6}$
8. What part of her land does Mrs. Gardner have in beans? $\frac{1}{8}$
9. Who has more land in beans? Mr. Fielding
10. What part of her land does Mrs. Gardner have in peas? $\frac{1}{8}$

238

1. You'll need a sheet of notebook paper, scissors, and a pencil.

   a Cut three strips from the bottom of your paper. Make each strip about 5 cm wide.

   b Fold the first strip into 2 equal parts. Crease the fold.

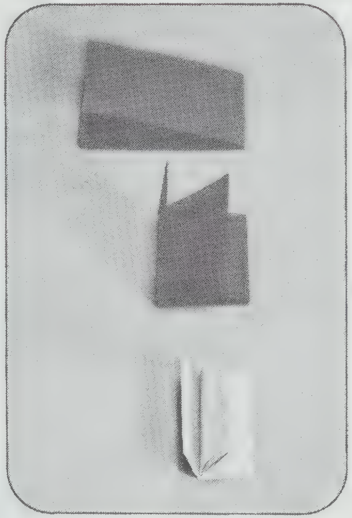
   c Open the strip. How many equal parts? Mark each part $\frac{1}{2}$. 2

   d Fold the next strip into 2 equal parts. Crease the fold. Fold it again into 2 equal parts.

   e Open the strip. How many equal parts? Mark each part $\frac{1}{4}$. 4

   f Fold the third strip into 2 equal parts. Fold it again into 2 equal parts. And once more into 2 equal parts. Crease the fold each time.

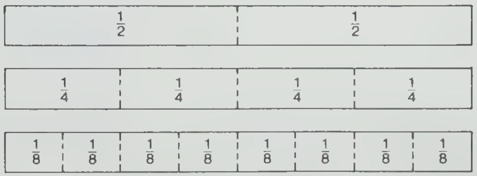
   g Open the strip. You should have 8 equal parts. Mark each part $\frac{1}{8}$.



2. Compare the three strips. Which is more?

   a $\frac{1}{2}$ or $\frac{1}{4}$ of a strip   b $\frac{1}{4}$ or $\frac{1}{8}$ of a strip   c $\frac{1}{8}$ or $\frac{1}{2}$ of a strip

Use the number strips if you need help.



3. Which is more?

   a $\frac{1}{8}$ or $\frac{1}{4}$ strip   b $\frac{1}{4}$ or $\frac{1}{2}$ strip

   c $\frac{1}{2}$ or $\frac{2}{4}$ Same   d $\frac{1}{2}$ or $\frac{4}{8}$ Same

239

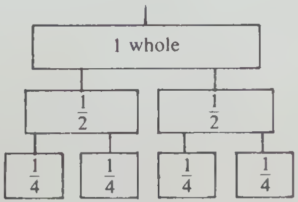
goal Learning about renaming fractions by making models

things for each pupil:
paper
scissors
pencil

page 239 Everyone should have the hands-on experience of making the models described. Why not dictate the directions and free them to **do.** All folds should be in the same direction—refer to the pupil page.

Folding the models will reinforce renaming 1 as a fraction. $\frac{2}{2} = 1$ $\frac{4}{4} = 1$ $\frac{8}{8} = 1$ Placing one strip under the other will help the pupil see other names for a fraction and how $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ compare.

Extend the paper-folding activity of the page to constructing a fraction mobile. There are many possible mobile shapes. Here is one of the most simple and obvious ones:



lesson Pages 240, 241, 242, 243, 244

goal Comparison of fractional parts of real things

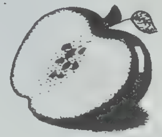
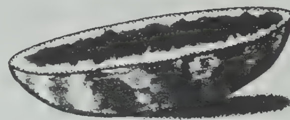
page 240 When comparing fractions with like denominators, it is assumed that the parts are all the same size. But talking about the real-world examples, as shown on the page, shows that the idea of equality cannot be assumed.

Certainly $\frac{1}{2}$ is equal to $\frac{1}{2}$ if you are comparing the same object or the same number of equal parts. (This idea builds readiness for the concept of a common denominator, which will be introduced in the next level.)

WHEN IS $\frac{1}{2}$ NOT EQUAL TO $\frac{1}{2}$?

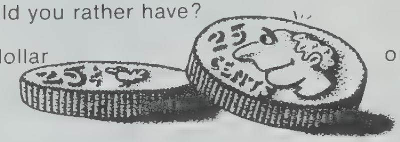
Here are some other questions that may help you answer that question.

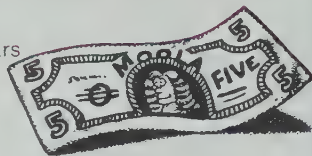
1. Which is larger?

$\frac{1}{2}$  or $\frac{1}{2}$ 

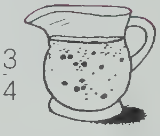
____

2. Which would you rather have?

$\frac{1}{2}$ of one dollar 

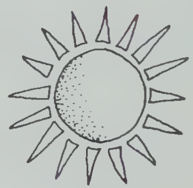
or $\frac{1}{2}$ of ten dollars __________ 

3. Which is smallest?

$\frac{3}{4}$  or $\frac{3}{4}$  or $\frac{3}{4}$ 

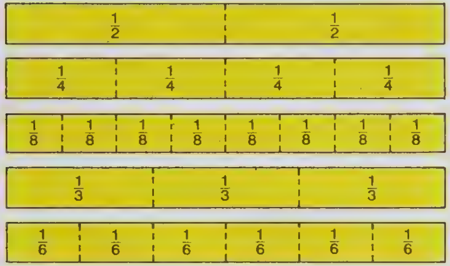
____

4. Which is longer, half an hour 

__________ or half a day? 

The questions were not meant to fool you. They should remind you that when we use fractions to compare, we must use equal units.

240



How many $\frac{1}{2}$s? 2 Does $\frac{2}{2} = 1$? Yes

How many $\frac{1}{4}$s? 4 Does $\frac{2}{4} = \frac{1}{2}$? Yes

How many $\frac{1}{8}$s? 8 Does $\frac{4}{8} = \frac{2}{4}$? Yes

How many $\frac{1}{3}$s? 3 Does $\frac{3}{3} = 1$? Yes

How many $\frac{1}{6}$s? 6 Does $\frac{2}{6} = \frac{1}{3}$? Yes

Use the number strips to help you answer these questions.

1. Which fraction is greater?

a $\frac{1}{6}$ or $\frac{1}{8}$	**b** $\frac{1}{3}$ or $\frac{1}{2}$	**c** $\frac{1}{4}$ or $\frac{1}{2}$	**d** $\frac{1}{8}$ or $\frac{1}{2}$
e $\frac{1}{3}$ or $\frac{2}{3}$	**f** $\frac{2}{3}$ or $\frac{1}{3}$	**g** $\frac{5}{8}$ or $\frac{7}{8}$	**h** $\frac{3}{8}$ or $\frac{7}{8}$

2. Which fraction is less?

a $\frac{1}{4}$ or $\frac{1}{3}$	**b** $\frac{2}{3}$ or $\frac{2}{8}$	**c** $\frac{3}{6}$ or $\frac{3}{8}$	**d** $\frac{2}{4}$ or $\frac{2}{6}$
e $\frac{2}{4}$ or $\frac{3}{4}$	**f** $\frac{1}{6}$ or $\frac{2}{6}$	**g** $\frac{4}{8}$ or $\frac{5}{8}$	**h** $\frac{5}{6}$ or $\frac{4}{6}$

3. Complete to make each sentence true.

a $\frac{1}{2} = \frac{?}{4}$ 2	**b** $\frac{2}{3} = \frac{?}{6}$ 4	**c** $\frac{1}{2} = \frac{?}{8}$ 4	**d** $\frac{4}{4} = \frac{?}{8}$ 8
e $\frac{6}{8} = \frac{?}{4}$ 3	**f** $\frac{1}{4} = \frac{?}{8}$ 2	**g** $\frac{1}{3} = \frac{?}{6}$ 2	**h** $\frac{3}{4} = \frac{?}{8}$ 6
i $\frac{2}{8} = \frac{?}{4}$ 1	**j** $\frac{3}{3} = \frac{?}{6}$ 6	**k** $\frac{2}{4} = \frac{?}{8}$ 4	**l** $\frac{4}{6} = \frac{?}{3}$ 2

241

goal Comparison of fractions, using a model

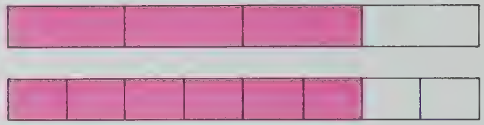
page 241 The learners should be able to work independently with the aid of the number strips. You will want to check that they know how to use the strips and aren't simply guessing the answers.

goal Development of the concept of renaming fractions

page 242 What's in a name? Discuss and explore! Could region **A** be changed to look like region **B**? The discussion can turn into a simple paper-folding activity for each student. Shade half a piece of paper. Make the folds indicated in the text—change from one region to the next by folding. *What's a new name for the shaded part? As more folds were made, what happened to the size of the parts?*

Some of Robert's friends call him Robert. Others call him Bob. Still others call him Rob. His mother calls him Bobby. Is Robert still the same person even though different people call him different names? Is $\frac{1}{2}$ still the same number Yes even though at different times it might be called $\frac{2}{4}$, $\frac{4}{8}$, or $\frac{8}{16}$? Yes

1. A



B

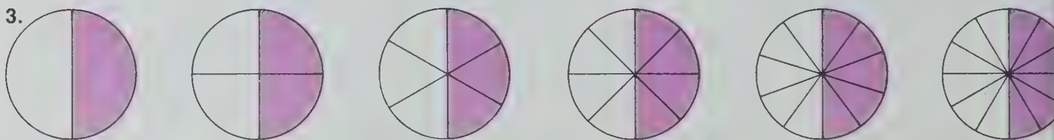
What fraction represents the shaded part of region A? of region B? Do these fractions $\frac{3}{4}$ $\frac{6}{8}$ name the same fractional number? Yes

2. C

D

What fraction represents the shaded part of region C? of region D? Do these fractions name the same fractional number? No

3.



A $\frac{1}{2}$ B $\frac{2}{4}$ C $\frac{3}{6}$ D $\frac{4}{8}$ E $\frac{5}{10}$ F $\frac{6}{12}$

What fraction names the amount shaded in each of these regions? Do these fractions all See above. name the same fractional number? Yes

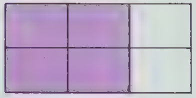
This type of renaming is often needed.

For example: $\frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4}$

Is there another name for $\frac{2}{4}$? What is it? $\frac{1}{2}$ Yes

242

y these on your own

A  B  C 

What part of region A is shaded? What part isn't shaded? $\frac{2}{3}$ $\frac{1}{3}$

What part of region B is shaded? What part isn't? $\frac{4}{6}$ $\frac{2}{6}$

What part of region C is shaded? What part isn't? $\frac{6}{9}$ $\frac{3}{9}$

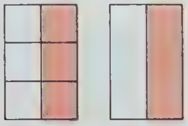
Is the same amount shaded in each region? Yes
Is the same amount unshaded? Yes

Are $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{6}{9}$ all names for the same number? Yes

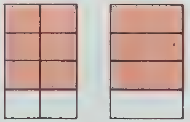
Does $\frac{2}{3} = \frac{4}{6}$? Yes Does $\frac{2}{3} = \frac{6}{9}$? Yes Does $\frac{4}{6} = \frac{6}{9}$? Yes Does $\frac{1}{3} = \frac{2}{6}$? Yes Does $\frac{1}{3} = \frac{6}{9}$? No

$\frac{2}{6} + \frac{2}{6} = \frac{4}{6}$ Is there another name for $\frac{4}{6}$? Yes
What is the simpler name? $\frac{2}{3}$

Now look at pairs of regions. You will be finding other names for fractions.



a $\frac{3}{6} = \frac{?}{2}$ 1



b $\frac{6}{8} = \frac{?}{4}$ 3

Look at the unshaded parts.
$\frac{2}{8} = \frac{?}{4}$ 1

$\frac{1}{6} + \frac{2}{6} = \frac{1+2}{6} = \frac{3}{6}$ What is a simpler name for $\frac{3}{6}$? $\frac{1}{2}$

$\frac{3}{8} + \frac{3}{8} = \frac{3+3}{8} = \frac{6}{8}$ What is a simpler name for $\frac{6}{8}$? $\frac{3}{4}$

goal Practice in renaming fractions

things for each pupil:
2 rectangular pieces of paper

page 243 Follow up the youngsters' work with another paper-folding activity. Have them take one piece of paper and make a model of region **A.** *Now with one fold, change region* **A** *into region* **B.** Take the other piece of paper. This time change **A** to **C.**

Focus on the term SIMPLER NAME. *Why do you suppose that expression is used?* One name isn't any easier to say than the other.

goal Practice in renaming fractions; **Progress Check** – renaming fractions, using a model

things small cards

page 244 Everyone on his own. Check youngsters who are having problems – when given a model, are they able to name the numerator correctly? the denominator?

Here's a way to involve everyone and provide additional practice. Have your pupils jot down sets of 4 equivalent fractions. Use these sets to prepare a spirit master as shown.

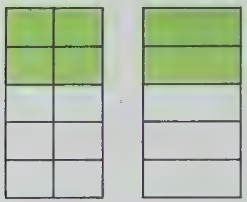
Connect the equivalent fractions.

$\frac{6}{8}$ $\frac{10}{15}$ $\frac{1}{5}$ $\frac{9}{12}$

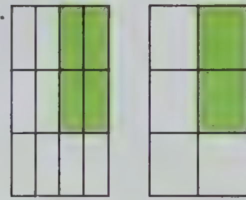
$\frac{9}{45}$ $\frac{3}{4}$ $\frac{15}{20}$ $\frac{2}{10}$

$\frac{8}{12}$ $\frac{5}{25}$ $\frac{2}{3}$ $\frac{4}{6}$

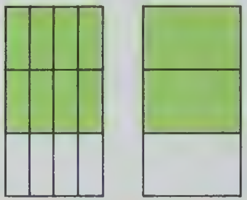
Look at more pairs of regions. First look at the shaded parts. Then look at the unshaded parts.

1. 

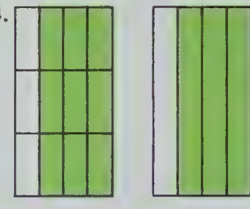
a $\frac{4}{10} = \frac{?}{5}$ 2
shaded parts
and
b $\frac{6}{10} = \frac{?}{5}$ 3
unshaded parts

2. 

a $\frac{?}{12} = \frac{2}{6}$ 4
shaded parts
and
b $\frac{8}{12} = \frac{?}{6}$ 4
unshaded parts

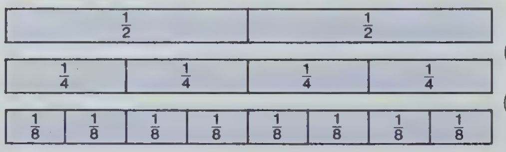
3. 

a $\frac{8}{12} = \frac{?}{3}$ 2
shaded parts
and
b $\frac{?}{12} = \frac{1}{3}$ 4
unshaded parts

4. 

What pairs of equal fractions name the shaded parts? $\frac{9}{12}$
the unshaded parts?

Use these number strips to complete each pair of fractions. Skill: Renaming fractions



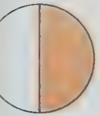
1. $\frac{1}{2} = \frac{?}{4}$ 2
2. $\frac{1}{2} = \frac{?}{8}$ 4
3. $\frac{4}{4} = \frac{?}{8}$ [illegible]
4. $\frac{6}{8} = \frac{?}{4}$
5. $\frac{1}{4} = \frac{?}{8}$ 2
6. $\frac{3}{4} = \frac{?}{8}$ 6
7. $\frac{2}{8} = \frac{?}{4}$ 1
8. $\frac{2}{4} = \frac{?}{8}$

244

See activity 1, page 258a.

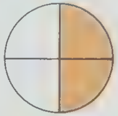
See activity 2, page 258a.

The shaded parts in each of the regions above show the same fraction. **Why?**

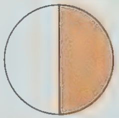


Shaded parts → $\frac{1}{2}$ ← Parts in all

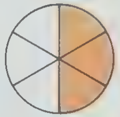
Double these and what happens?



The number of shaded parts doubled also.
$\frac{1}{2}$ is the same as $\frac{1 \times 2}{2 \times 2}$ or $\frac{2}{4}$.



Make three times as many parts.



$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$

When you multiply the numerator and denominator by the same number, you can find another name for a fraction.

Are these names for the same fraction? $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}$ Yes

Think! What will result from multiplying the numerator and the denominator by the same number?

Start with the simplest name, $\frac{1}{3}$. $\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$ $\frac{1}{3}$ does equal $\frac{2}{6}$!

$\frac{1 \times 3}{3 \times 3} = \frac{3}{9}$ $\frac{1}{3}$ does equal $\frac{3}{9}$! $\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$ $\frac{1}{3}$ does equal $\frac{4}{12}$!

Go back to the regions at the top of the page.
What number times $\frac{1}{2}$ has been used to get the fraction $\frac{4}{8}$? $\frac{5}{10}$? $\frac{6}{12}$? 6
4 5

See activity 3, page 258b.

lesson Pages 245, 246, 247, 248, 249

goal Introducing a way to rename fractions

memo A new idea is being introduced. It's important to get the correct start. Pages 245 through 248 will require your guidance.

page 245 No writing necessary—just **thinking** and discussion. Ask lots of questions. Encourage questions.

goal Introducing a way to find the SIMPLEST NAME for a fraction

page 246 What goes up must come down. Right? But how? That's what the page is all about. Nice and easy. Don't rush.

The expression COMMON FACTOR may cause confusion. What solutions would make these math sentences true?

$t \times q = 6$	$r \times s = 8$
(1×6)	(1×8)
(2×3)	(2×4)

Which factors are common factors of both 6 and 8? (1, 2) Try dividing both the numerator and the denominator of $\frac{6}{8}$ by 1. What happens? Now try dividing each by 2.

Be ready to help find those common factors for problems 1 through 10.

Is $\frac{2}{6}$ the simplest name for a fraction?

Look at the model. Does $\frac{2}{6}$ also name $\frac{1}{3}$?

WHAT'S HAPPENING NOW?

THINK

I can multiply by the same number and get a larger numerator and denominator.

WHY NOT divide by the same number to get a smaller numerator and denominator?

The number you use to divide is called a common facto

$\frac{2 \div ?}{6 \div ?}$ What is a common factor? Will 2 work? $\frac{2 \div 2}{6 \div 2}$

2 is a common factor.

AND $\frac{2}{6}$ does equal $\frac{1}{3}$.

LOOK AT ANOTHER

Is $\frac{6}{8}$ the simplest name for a fraction?

Do $\frac{6}{8}$ have a common factor? Try 2 again. $\frac{6 \div 2}{8 \div 2} =$

Is $\frac{3}{4}$ the simplest name? Do $\frac{3}{4}$ have a common fact

$\cancel{\frac{3 \div 3}{4 \div 3}}$ $\cancel{\frac{3 \div 2}{4 \div 2}}$ No! $\frac{3}{4}$ is the simplest name.

YOUR TURN

Rename each fraction with its simplest name.

1. $\frac{9}{12}$ **Think** $\frac{9 \div ?}{12 \div ?}$ What is a common factor?
2. $\frac{12}{16}$ **Think** What is a common factor? 4
3. $\frac{2}{6}$ $\frac{1}{3}$
4. $\frac{3}{9}$ $\frac{1}{3}$
5. $\frac{4}{12}$ $\frac{1}{3}$
6. $\frac{2}{12}$ $\frac{1}{6}$
7. $\frac{3}{18}$ $\frac{1}{6}$
8. $\frac{4}{24}$ $\frac{1}{6}$
9. $\frac{5}{10}$ $\frac{1}{2}$
10. $\frac{8}{10}$ $\frac{4}{5}$

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See activity 4, page 258b.

ake sure you can always find the simplest name for a fraction if
ou divide numerator and denominator by a common factor.

XPLORE! EXPERIMENT! TRY IT!

Try $\frac{3}{6}$. Is there a common factor of the numerator and denominator?

Try 3. Divide. $\frac{3 \div 3}{6 \div 3} = \frac{1}{2}$ It worked again!

y some more.
vide numerator and denominator by a common factor.

1. $\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{?}{?}$ $\frac{2}{3}$ 2. $\frac{3}{12} = \frac{3 \div 3}{12 \div 3} = \frac{?}{?}$ $\frac{1}{4}$ 3. $\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{?}{?}$ $\frac{1}{4}$

IME OUT TO LOOK T A CURVE BALL

$\frac{4}{12}$ It's true that 2 is a common factor.

$\frac{4 \div 2}{12 \div 2} = \frac{2}{6}$ But that is not the simplest name. You can see 2 is another common factor. Don't be discouraged. Divide again.

$\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$ That's it!

You could have saved the extra step had you divided by the common factor 4. $\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$ Same answer.

The common factor 2 works. The greater common factor 4 works faster. Sooner or later you come out with the simplest name.

some more. Find the simplest name.

4. $\frac{2}{12}$ $\frac{1}{6}$ 5. $\frac{3}{12}$ $\frac{1}{4}$ 6. $\frac{6}{12}$ $\frac{1}{2}$ 7. $\frac{3}{6}$ $\frac{1}{2}$ 8. $\frac{4}{6}$ $\frac{2}{3}$

See activity 5, page 258b.

goal Practice in finding the simplest name for a fraction

page 247 Some youngsters are able to name the GREATEST COMMON FACTOR for two numbers right off the top of their heads. Others draw a complete blank; furthermore, they become confused by lengthy factoring procedures. The development on the page accommodates everyone. Skills will improve with practice.

Question: *How do I know when I have found the simplest name for a fraction?* When the only common factor of both the numerator and the denominator is 1.

goal Practice in renaming addition sums with their simplest name

page 248 We're combining an old skill with a new skill. Discuss the development together. Rows 1 through 4 should be independent work.

What is the simplest name for $\frac{8}{12}$?

Think Is there a common factor of the numerator and denominator?
2? 3? 4? 6? You can pick 2 as the common factor.
Or you could pick 4.

Here is what happens when you pick 2.

$$\frac{8}{12} = \frac{8 \div 2}{12 \div 2} = \frac{4}{6}$$

You have to divide again.

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

It took two steps, but you could find the simplest name. Now look what happens when you pick 4.

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

It took only one step to find the simplest name.

Put your renaming skill together with your addition skill. Remember how to add fractions?

$\frac{1}{10} + \frac{7}{10} = \frac{1+7}{10} = \frac{8}{10}$ Is $\frac{8}{10}$ the simplest name? No

Is there a common factor of the numerator and denominator? Yes, 2

$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}$ This is the simplest name.

You don't have to write down all these steps. Write only what you have to in order to get the right answer. Try these.

ADD

Make sure the sum has its simplest name.

	a	b	c
1.	$\frac{1}{4} + \frac{1}{4}$ $(\frac{2}{4})$ $\frac{1}{2}$	$\frac{7}{12} + \frac{1}{12}$ $(\frac{8}{12})$ $\frac{2}{3}$	$\frac{1}{6} + \frac{1}{6}$ $(\frac{2}{6})$ $\frac{1}{3}$
2.	$\frac{3}{8} + \frac{1}{8}$ $(\frac{4}{8})$ $\frac{1}{2}$	$\frac{1}{9} + \frac{2}{9}$ $(\frac{3}{9})$ $\frac{1}{3}$	$\frac{5}{12} + \frac{1}{12}$ $(\frac{6}{12})$ $\frac{1}{2}$
3.	$\frac{5}{9} + \frac{1}{9}$ $(\frac{6}{9})$ $\frac{2}{3}$	$\frac{1}{10} + \frac{1}{10}$ $(\frac{2}{10})$ $\frac{1}{5}$	$\frac{3}{16} + \frac{5}{16}$ $(\frac{8}{16})$ $\frac{1}{2}$
4.	$\frac{3}{10} + \frac{3}{10}$ $(\frac{6}{10})$ $\frac{3}{5}$	$\frac{4}{7} + \frac{3}{7}$ $(\frac{7}{7})$ 1	$\frac{5}{10} + \frac{3}{10}$ $(\frac{8}{10})$ $\frac{4}{5}$

ometimes you have to ind the simplest name for he answer to a subtraction roblem, too. Remember how o subtract.

$\frac{7}{12} - \frac{3}{12} = \frac{7-3}{12} = \frac{4}{12}$ Rename. Is there a common factor? Yes, 4

$\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$ That's the simplest name.

YOUR TURN

ubtract and rename he answers. (These problems re rigged. Every nswer needs renaming.)

	a	b	c	d
1.	$\frac{5}{6} - \frac{1}{6}$ $(\frac{4}{6})$ $\frac{2}{3}$	$\frac{7}{10} - \frac{3}{10}$ $(\frac{4}{10})$ $\frac{2}{5}$	$\frac{7}{8} - \frac{1}{8}$ $(\frac{6}{8})$ $\frac{3}{4}$	$\frac{11}{12} - \frac{7}{12}$ $(\frac{4}{12})$ $\frac{1}{3}$
2.	$\frac{7}{9} - \frac{4}{9}$ $(\frac{3}{9})$ $\frac{1}{3}$	$\frac{3}{4} - \frac{1}{4}$ $(\frac{2}{4})$ $\frac{1}{2}$	$\frac{5}{8} - \frac{3}{8}$ $(\frac{2}{8})$ $\frac{1}{4}$	$\frac{9}{10} - \frac{7}{10}$ $(\frac{2}{10})$ $\frac{1}{5}$

PROGRESS CHECK

Skill: Addition and subtraction of fractions; renaming answers

Are you feeling brave? You will find both addition and subtraction problems. Not every answer will have to be renamed.

1. $\frac{1}{12} + \frac{1}{12}$ $(\frac{2}{12})$ $\frac{1}{6}$	2. $\frac{6}{7} - \frac{3}{7}$ $\frac{3}{7}$	3. $\frac{3}{10} + \frac{7}{10}$ $(\frac{10}{10})$ 1	4. $\frac{9}{10} - \frac{1}{10}$ $(\frac{8}{10})$ $\frac{4}{5}$
5. $\frac{2}{3} + \frac{1}{3}$ $(\frac{3}{3})$ 1	6. $\frac{2}{3} - \frac{1}{3}$ $\frac{1}{3}$	7. $\frac{2}{5} + \frac{2}{5}$ $\frac{4}{5}$	8. $\frac{7}{8} - \frac{5}{8}$ $(\frac{2}{8})$ $\frac{1}{4}$
9. $\frac{1}{9} + \frac{2}{9}$ $(\frac{3}{9})$ $\frac{1}{3}$	10. $\frac{6}{7} - \frac{5}{7}$ $\frac{1}{7}$	11. $\frac{1}{10} + \frac{7}{10}$ $(\frac{8}{10})$ $\frac{4}{5}$	12. $\frac{4}{5} - \frac{4}{5}$ $(\frac{0}{5})$ 0
13. $\frac{9}{16} - \frac{1}{16}$ $(\frac{8}{16})$ $\frac{1}{2}$	14. $\frac{16}{16} - \frac{1}{16}$ $\frac{15}{16}$	15. $\frac{13}{16} - \frac{7}{16}$ $(\frac{6}{16})$ $\frac{3}{8}$	16. $\frac{7}{16} + \frac{5}{16}$ $(\frac{12}{16})$ $\frac{3}{4}$

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STOP See activity 3, variation A, page 258b.

See activity 6, page 258b.

goal Learning to rename answers to subtraction problems with their simplest name; **Progress Check** – adding and subtracting fractions with like denominators and renaming answers with their simplest name

memo Consider taking two days to complete this page – one to practice subtraction and renaming, the other for the Progress Check.

page 249 Each problem of the Progress Check requires three distinct skills:
- Computing correctly
- Recognizing when the answer can be renamed
- Renaming correctly where possible

Watch for careless reading of operation signs – both addition and subtraction problems are mixed in each row.

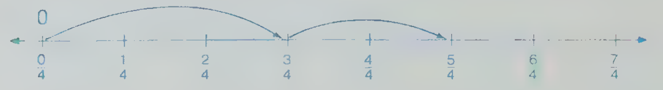
Give praise for computing correctly even when renaming errors are made or where the youngster neglects to rename. Those who perform both computations correctly are Supercomputers.

lesson Pages 250, 251, 252, 253

goal Examining the renaming of addition sums greater than 1

page 250 Nothing new. Pupils met fractions greater than one in a previous chapter. Adding fractions with like denominators should be no problem. Independent learners can combine these skills by themselves. The page gives sufficient guidance. You decide how best to handle this page with the others.

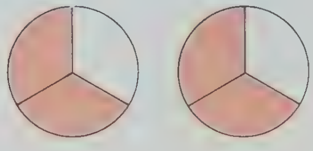
1. Dan is on the track team. He practices every day. Today he ran $\frac{3}{4}$ of a mile. Then he rested. Then he ran another $\frac{2}{4}$ of a mile. Did he run more than 1 mile? How far in all? Yes $\frac{5}{4}$ mile



$\frac{3}{4} + \frac{2}{4} = \frac{?}{4}$ 5

This problem signals the next step in operations with fractions. Take a good look at your answer. What looks different about that fraction? The numerator is greater than the denominator.

2. Here's another example to study.



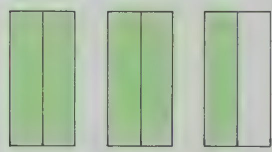
How many thirds are shaded?

$\frac{2}{3} + \frac{2}{3} = \frac{?}{3}$ 4

Hey! The numerator is greater than the denominator again.

You have named a fraction greater than 1. $\frac{4}{3}$ is another name for $1\frac{1}{3}$

3. How many halves are shaded? 5
How many whole regions are shaded? 2
   a How many more halves than two whole regions are shaded? 1
   b Is the sentence $\frac{5}{2} = 2\frac{1}{2}$ true? Yes



Do you remember? $1\frac{1}{4}$ is called a *mixed number*. So are $1\frac{1}{3}$ and $2\frac{1}{2}$.

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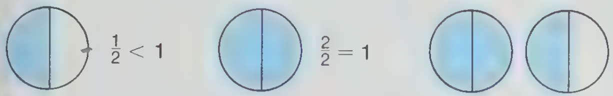
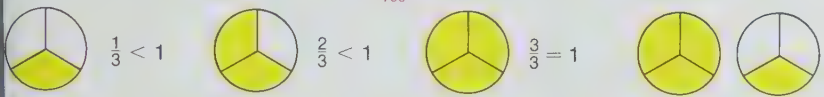


$\frac{1}{2} < 1$ $\frac{2}{2} = 1$ $\frac{3}{2} > 1$

Compare the numerator with the denominator.
Are these statements true?

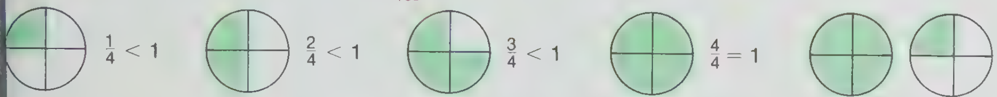
1. When the numerator is smaller than the denominator, the fraction is less than 1. Yes
2. When the numerator is the same number as the denominator, the fraction is equal to 1. Yes
3. When the numerator is greater than the denominator, the fraction is greater than 1. Yes

Are the statements true for thirds? Yes



$\frac{1}{3} < 1$ $\frac{2}{3} < 1$ $\frac{3}{3} = 1$ $\frac{4}{3} > 1$

Are the statements true for fourths? Yes



$\frac{1}{4} < 1$ $\frac{2}{4} < 1$ $\frac{3}{4} < 1$ $\frac{4}{4} = 1$ $\frac{5}{4} > 1$

Try to find a time when the statements are not true. There isn't one.

Use >, <, or = to replace the ?. Draw a picture if you need to.

	a	b	c	d	e	f	g
.	$\frac{5}{6}$? 1 (<)	$\frac{5}{2}$? 1 (>)	$\frac{6}{6}$? 1 (=)	$\frac{2}{5}$? 1	$\frac{1}{9}$? 1 (<)	$\frac{6}{4}$? 1 (>)	$\frac{9}{3}$? 1 (>)
.	$\frac{0}{4}$? 1 (<)	$\frac{5}{5}$? 1 (=)	$\frac{12}{1}$? 1	$\frac{1}{12}$? 1	$\frac{12}{12}$? 1 (=)	$\frac{7}{6}$? 1 (>)	$\frac{8}{8}$? 1 (=)

goal Comparison of fractions to one whole

page 251 The page itself provides the necessary guidance. You may need to discuss the generalizations with your strugglers.

Careful! There's one curve ball in the second row.

Think

12 ← shaded parts
1 ← parts in all

? Why that's another name for 12!

goal Recognition of fractions that name numbers greater than 1

page 252 Examine the number line closely. Does every fraction whose numerator is greater than its denominator name a mixed number? What other kind of numbers can such a fraction name? (Whole numbers) When does such a fraction name a whole number? (Numerator a multiple of the denominator) a mixed number? (Numerator not a multiple of the denominator)

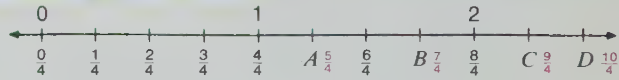
1. *How many fourths are shaded?*

$\frac{2}{4} + \frac{3}{4} = \frac{?}{4}$ 5

Does this fraction name a number greater than 1? Yes

What is another name for the sum? $1\frac{1}{4}$

2. *What are the missing fractions?*



a Is $\frac{4}{4}$ another name for 1? Yes

b Is $1\frac{1}{4}$ more than 1? Yes How much more? $\frac{1}{4}$ Is $1\frac{1}{4}$ a mixed number? Yes

c What is another name for $\frac{8}{4}$? 2 Does $\frac{8}{4}$ name a mixed number? No. It names a whole number.

3. *Don't rename. Just tell which fractions*

can be renamed as mixed numbers. Draw diagrams if you need help.

a $\underline{\frac{3}{2}}$ **b** $\underline{\frac{5}{4}}$ **c** $\underline{\frac{7}{4}}$ **d** $\frac{3}{4}$ **e** $\frac{12}{15}$ **f** $\underline{\frac{7}{3}}$

g How did you decide which numbers were mixed numbers?
Those whose numerator was greater than their denominator.

*4. *Copy and complete the chart.*

 ate	 ate	Did they eat more than 1 whole thing?
$\frac{1}{4}$ of a melon	$\frac{1}{2}$ of a melon	yes **?** <u>no</u>
$\frac{1}{4}$ of a cake	$\frac{1}{3}$ of a cake	yes **?** <u>no</u>
$\frac{1}{4}$ of a pie	$\frac{7}{8}$ of a pie	<u>yes</u> **?** no

Who had a bigger appetite?

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$\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \frac{7}{5}$  $\frac{5}{5}$ names 1 and there is another $\frac{2}{5}$.

So $\frac{7}{5} = 1\frac{2}{5}$

$\frac{2}{3} + \frac{2}{3} = \frac{2+2}{3} = \frac{4}{3}$  $\frac{3}{3}$ names 1 and there is another $\frac{1}{3}$.

So $\frac{4}{3} = 1\frac{1}{3}$

$\frac{5}{7} + \frac{3}{7} = \frac{5+3}{7} = \frac{8}{7}$ **Think** $\frac{7}{7}$ names 1. There is another $\frac{1}{7}$.

So $\frac{8}{7} = \underline{?}$ $1\frac{1}{7}$

Look out! $\frac{5}{9} + \frac{4}{9} = \frac{5+4}{9} = \underline{?}$ $\frac{9}{9}$ What is another name? 1

Is it a mixed number? No. It's a whole number.

Keep on your toes! $\frac{5}{8} + \frac{1}{8} = \frac{5+1}{8} = \underline{?}$ $\frac{6}{8}$ What is another name? Is it a mixed number? $\frac{3}{4}$ No

$\frac{4}{7} + \frac{1}{7} = \frac{4+1}{7} = \underline{?}$ $\frac{5}{7}$ Is there another name? No

Remember please

Fractions greater than 1 can be renamed.
Fractions that are equal to 1 can be renamed.
Some fractions less than 1 can be renamed.

All these fractions can be renamed. Do a good job.
Skill: Renaming fractions with simplest name

1. $\frac{6}{5}$ $1\frac{1}{5}$ 2. $\frac{7}{4}$ $1\frac{3}{4}$ 3. $\frac{5}{3}$ $1\frac{2}{3}$ 4. $\frac{3}{2}$ $1\frac{1}{2}$ 5. $\frac{6}{6}$ 1

6. $\frac{9}{7}$ $1\frac{2}{7}$ 7. $\frac{11}{6}$ $1\frac{5}{6}$ 8. $\frac{5}{4}$ $1\frac{1}{4}$ *9. $\frac{9}{3}$ 3 *10. $\frac{4}{2}$ 2

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See activity 3, variation B, page 258b.

See activity 7, page 258b.

goal Review of three possible types of renaming; **Progress Check**—renaming fractions

page 253 At this point in time, the pupils are renaming fractions greater than 1 by thinking ones and how much more. For example: $\frac{8}{5}$ can be renamed as $\frac{5}{5}$ and $\frac{3}{5}$ more. Therefore, $\frac{8}{5} = 1\frac{3}{5}$ This technique should carry the learner through renaming $\frac{9}{3}$ as $\frac{3}{3}$ and $\frac{6}{3}$ more. $\frac{6}{3}$ can then be renamed as $\frac{3}{3}$ and $\frac{3}{3}$. How many $\frac{3}{3}$s are there? Three. So, $\frac{9}{3} = 3$. Renaming fractions of this type has not been practiced; therefore, problems 9 and 10 on the Progress Check have been starred.

lesson Pages 254, 255, 256, 257

goal Practice in renaming sums that are 1 or greater than 1

page 254 Better talk about this one. Two renaming skills are being used on one sum—eliminating common factors first and then changing to a mixed number. Of course the order can be reversed. We found in field testing that many youngsters tend to leave $1\frac{4}{6}$, believing that they have completed the renaming once they have written it as a mixed number.

The computation practice is independent work. Watch those directions! Certainly wouldn't want to do more work than necessary.

Now that you have reviewed what a mixed number is, find out when you might run into them.

$\frac{3}{4} + \frac{3}{4} = \frac{3+3}{4} = \frac{6}{4}$ Aha! There is a mixed number!

Ask first, do the numerator and denominator have a common factor? $\frac{6 \div 2}{4 \div 2} = \frac{3}{2}$
Now rename. What mixed number does $\frac{3}{2}$ name? $1\frac{1}{2}$

Look at these three examples.
The idea of common factor will help you rename.

$\frac{5}{6} + \frac{5}{6} = \frac{5+5}{6} = \frac{10}{6} \rightarrow$ Common factor? Yes $\frac{10 \div 2}{6 \div 2} = \frac{5}{3}$
Now rename so that you have a mixed number. $1\frac{2}{3}$

$\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \frac{7}{5} \rightarrow$ Common factor? No
Rename as a mixed number. $1\frac{2}{5}$

$\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2} = \frac{2}{2} \rightarrow$ Common factor? You don't have to Yes write anything. You know the number that kind of fraction names. 1

You are ready to practice.
Add. Rename the answer so that it has its simplest name.

Boys do these.

1. $\frac{3}{4} + \frac{1}{4}$ $(\frac{4}{4})$ 1
2. $\frac{4}{7} + \frac{6}{7}$ $(\frac{10}{7})$ $1\frac{3}{7}$
3. $\frac{4}{5} + \frac{2}{5}$ $(\frac{6}{5})$ $1\frac{1}{5}$
4. $\frac{9}{10} + \frac{1}{10}$ $(\frac{10}{10})$ 1
5. $\frac{1}{6} + \frac{5}{6}$ $(\frac{6}{6})$ 1
6. $\frac{5}{8} + \frac{6}{8}$ $(\frac{11}{8})$ $1\frac{3}{8}$
7. $\frac{3}{8} + \frac{5}{8}$ $(\frac{8}{8})$ 1
8. $\frac{4}{8} + \frac{7}{8}$ $(\frac{11}{8})$ $1\frac{3}{8}$

Girls do these.

1. $\frac{4}{5} + \frac{4}{5}$ $(\frac{8}{5})$ $1\frac{3}{5}$
2. $\frac{2}{4} + \frac{3}{4}$ $(\frac{5}{4})$ $1\frac{1}{4}$
3. $\frac{7}{8} + \frac{5}{8}$ $(\frac{12}{8})$ $1\frac{1}{2}$
4. $\frac{3}{7} + \frac{6}{7}$ $(\frac{9}{7})$ $1\frac{2}{7}$
5. $\frac{1}{6} + \frac{5}{6}$ $(\frac{6}{6})$ 1
6. $\frac{1}{10} + \frac{9}{10}$ $(\frac{10}{10})$ 1
7. $\frac{5}{8} + \frac{4}{8}$ $(\frac{9}{8})$ $1\frac{1}{8}$
8. $\frac{7}{9} + \frac{4}{9}$ $(\frac{11}{9})$ $1\frac{2}{9}$

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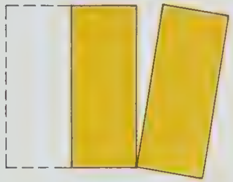
Time out to review subtraction

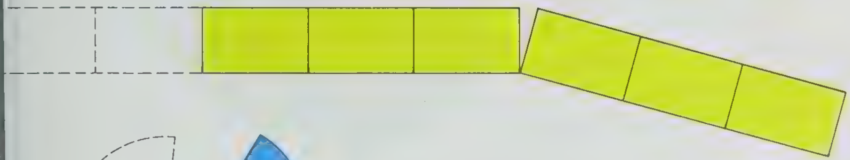


There were $\frac{3}{3}$.

Now there are $\frac{2}{3}$.

Take $\frac{1}{3}$ from $\frac{2}{3}$.

$\frac{2}{3} - \frac{1}{3} = ?$ $\frac{1}{3}$

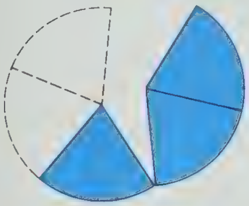


There were $\frac{8}{8}$.

Now there are $\frac{6}{8}$.

Take $\frac{3}{8}$ from $\frac{6}{8}$.

$\frac{6}{8} - \frac{3}{8} = ?$ $\frac{3}{8}$



There were $\frac{5}{5}$.

Now there are $\frac{3}{5}$.

Take $\frac{2}{5}$ from $\frac{3}{5}$.

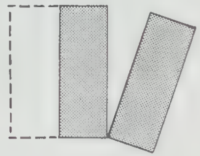
$\frac{3}{5} - \frac{2}{5} = ?$ $\frac{1}{5}$

Practice subtraction. Rename when you can. (Only renamed answers given.)

	a	b	c	d
1.	$\frac{3}{4} - \frac{1}{4} = ?$ $\frac{1}{2}$	$\frac{7}{8} - \frac{3}{8} = ?$ $\frac{1}{2}$	$\frac{4}{9} - \frac{2}{9} = ?$ 2/[illegible]	$\frac{7}{12} - \frac{3}{12} = ?$ $\frac{1}{3}$
2.	$\frac{5}{6} - \frac{1}{6} = ?$ $\frac{2}{3}$	$\frac{6}{7} - \frac{5}{7} = ?$ $\frac{1}{7}$	$\frac{7}{9} - \frac{3}{9} = ?$ $\frac{4}{9}$	$\frac{5}{12} - \frac{3}{12} = ?$ $\frac{1}{6}$
3.	$\frac{3}{4} - \frac{1}{4} = ?$ $\frac{1}{2}$	$\frac{4}{5} - \frac{2}{5} = ?$ $\frac{2}{5}$	$\frac{4}{5} - \frac{1}{5} = ?$ $\frac{3}{5}$	$\frac{5}{8} - \frac{3}{8} = ?$ $\frac{1}{4}$

goal Practice in subtracting fractions with like denominators and renaming the answer

page 255 The breakaway model for subtraction of fractions is superior to the number line, but it is tricky to handle. Any fractional part of a region by itself is visually a whole region in and of itself. It is necessary to establish what the whole region is before looking at an operation on part of the region.



The class can pretend this represents a cake pan. The cake pan was full. Then the cake was cut into thirds. There were $\frac{3}{3}$ of the cake.

The shaded parts show that now there are $\frac{2}{3}$ of the cake. And $\frac{1}{3}$ is being taken away.

Let the second model represent what was an 8-inch strip of something. The 8 inches had been divided into 8 equal parts. 6 parts ($\frac{6}{8}$) remain, according to the shading. And 3 parts ($\frac{3}{8}$) are being taken away from the $\frac{6}{8}$.

Our friend the fraction pie will do nicely for the third circular region.

The subtraction computation is not new. Requiring that the answers be renamed is new. Assign only as many problems as you think are necessary.

goal Practice in renaming answers for subtraction of fractions with like denominators

page 256 The model becomes more abstract. The double shading now represents the part to be taken away. If any child is confused by this, give him a chance to work with manipulatives and go back to the breakaway model (page 255).

These are very easy problems. The emphasis is on renaming the answers where possible. Why not let the pupils select any three rows to complete? Save the other problems for a later time for the insecure learners who need more practice.

This kind of picture can show subtraction too.

$\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

$\frac{5}{6} - \frac{3}{6} = \frac{2}{6}$

Subtract. Rename when you can. (Only renamed answers given)

1. $\frac{3}{3} - \frac{2}{3}$ $\frac{1}{3}$
2. $\frac{4}{5} - \frac{1}{5}$ $\frac{3}{5}$
3. $\frac{2}{2} - \frac{1}{2}$ $\frac{1}{2}$
4. $\frac{3}{6} - \frac{2}{6}$ $\frac{1}{6}$
5. $\frac{4}{7} - \frac{2}{7}$ $\frac{2}{7}$
6. $\frac{6}{8} - \frac{3}{8}$ $\frac{3}{8}$
7. $\frac{1}{4} - \frac{1}{4}$ 0
8. $\frac{3}{9} - \frac{1}{9}$ $\frac{2}{9}$
9. $\frac{3}{9} - \frac{2}{9}$ $\frac{1}{9}$
10. $\frac{7}{11} - \frac{4}{11}$ $\frac{3}{11}$
11. $\frac{2}{3} - \frac{1}{3}$ $\frac{1}{3}$
12. $\frac{2}{4} - \frac{1}{4}$ $\frac{1}{4}$
13. $\frac{4}{5} - \frac{3}{5}$ $\frac{1}{5}$
14. $\frac{7}{7} - \frac{7}{7}$ 0
15. $\frac{1}{2} - \frac{0}{2}$ $\frac{1}{2}$
16. $\frac{4}{6} - \frac{3}{6}$ $\frac{1}{6}$
17. $\frac{7}{9} - \frac{4}{9}$ $\frac{1}{3}$
18. $\frac{7}{9} - \frac{3}{9}$ $\frac{4}{9}$

Practice your renaming skills in the subtraction problems.
Rename the answers so that each has its simplest name. (Only renamed answers given)

	a		b		c		d	
1.	$\frac{5}{6} - \frac{1}{6}$	$\frac{2}{3}$	$\frac{7}{8} - \frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4} - \frac{1}{4}$	$\frac{1}{2}$	$\frac{7}{10} - \frac{5}{10}$	$\frac{1}{5}$
2.	$\frac{5}{8} - \frac{3}{8}$	$\frac{1}{4}$	$\frac{11}{12} - \frac{5}{12}$	$\frac{1}{2}$	$\frac{7}{9} - \frac{1}{9}$	$\frac{2}{3}$	$\frac{7}{8} - \frac{1}{8}$	$\frac{3}{4}$

Now try it with addition.
Rename the answers so that each has its simplest name.
You might name some mixed numbers, too. (Only renamed answers given)

	a		b		c		d	
3.	$\frac{1}{4} + \frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8} + \frac{7}{8}$	1	$\frac{1}{6} + \frac{5}{6}$	1	$\frac{1}{8} + \frac{5}{8}$	$\frac{3}{4}$

Look out. These are only for the very brave. (Only renamed answers given)

	a		b		c		d	
*4.	$\frac{7}{10} + \frac{9}{10}$	$1\frac{3}{5}$	$\frac{5}{6} + \frac{5}{6}$	$1\frac{2}{3}$	$\frac{7}{8} + \frac{5}{8}$	$1\frac{1}{2}$	$\frac{3}{4} + \frac{3}{4}$	$1\frac{1}{2}$

Answer these with a fraction. The simplest names, please.

5. There were 12 hotdogs.
   You ate 4. I ate 2.
   What part of the 12 hotdogs did you eat? $\frac{1}{3}$
   What part of the 12 hotdogs did I eat? $\frac{1}{6}$
   What part of the 12 hotdogs are left? $\frac{1}{2}$

6. There were 6 candy bars.
   Ron ate 2. Don ate 3.
   What part of the 6 candy bars did Ron eat? $\frac{1}{3}$
   What part of the 6 candy bars did Don eat? $\frac{1}{2}$
   What part of the 6 candy bars are left? $\frac{1}{6}$

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goal Practice in renaming answers for subtraction and addition

page 257 Here is the first chance you have to identify those pupils to whom you can turn for help when you need peer tutors. Please notice that the answers to row 4 are mixed numbers. Anyone in trouble in row 1, 2, or 3 should not tackle the remainder of the page. These youngsters need more practice in renaming. Assign peer tutors and have a supply of region models handy so that learners in trouble can continue to have manipulative experiences (or visual support for the work to come).

goal **Checkout**—concept of fractions; comparing, adding, and subtracting fractions with like denominators; renaming answers

page 258 The page requires a great deal from the learner. Problem sets 1, 2, and 3 should be completed by everyone. Use your own judgment in assigning other parts of the Checkout.

Specific skills are identified on the answer key to help you identify the pupil's weaknesses.

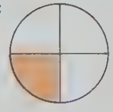
CHECKOUT

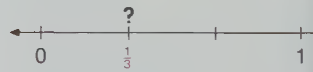
Skill: Naming fraction shown by model

1. Which fraction names each picture?

a  $\frac{5}{6}$

b  $\frac{3}{8}$

c  $\frac{1}{4}$

d 

Skill: Drawing fraction models

2. Draw a picture to show $\frac{1}{2}$. Draw one to show $\frac{2}{3}$. Draw another one to show $\frac{5}{8}$.

Skill: Comparing fractions

3. Which is greater?

a $\frac{1}{3}$ or $\frac{2}{3}$ b $\frac{3}{4}$ or $\frac{2}{4}$ c $\frac{5}{6}$ or $\frac{1}{6}$ d $\frac{7}{9}$ or

Skill: Addition of fractions with like denominators (renaming sums)

4. Add. Rename when you can.

a $\frac{3}{5}+\frac{1}{5}$ $\frac{4}{5}$ b $\frac{2}{6}+\frac{1}{6}$ $(\frac{3}{6})$ $\frac{1}{2}$ c $\frac{2}{3}+\frac{1}{3}$ $(\frac{3}{3})$ 1 d $\frac{5}{6}+\frac{1}{6}$

*e $\frac{3}{8}+\frac{7}{8}$ $(\frac{10}{8})$ $1\frac{1}{4}$ *f $\frac{7}{12}+\frac{9}{12}$ $(\frac{16}{12})$ $1\frac{1}{3}$ *g $\frac{3}{10}+\frac{2}{10}$ $(\frac{5}{10})$ $\frac{1}{2}$ *h $\frac{7}{8}+\frac{5}{8}$ $(\frac{12}{8})$ 1

5. Subtract. Rename when you can.

Skill: Subtraction of fractions w like denominators (renaming diff

a $\frac{2}{3}-\frac{1}{3}$ $\frac{1}{3}$ b $\frac{4}{5}-\frac{1}{5}$ $\frac{3}{5}$ c $\frac{4}{6}-\frac{3}{6}$ $\frac{1}{6}$ d $\frac{5}{8}-\frac{3}{8}$ (

e $\frac{9}{10}-\frac{3}{10}$ $(\frac{6}{10})$ $\frac{3}{5}$ f $\frac{3}{4}-\frac{1}{4}$ $(\frac{2}{4})$ $\frac{1}{2}$ g $\frac{7}{8}-\frac{1}{8}$ $(\frac{6}{8})$ $\frac{3}{4}$ h $\frac{8}{9}-\frac{2}{9}$ (

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See activity 8, page 258c.

See activity 9, page 258c.

RESOURCES

another form of evaluation

for Progress Check—page 244

Use these number strips to complete each pair of fractions.

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

1. $\frac{1}{3} = \frac{?}{6}$ 2
2. $\frac{3}{6} = \frac{?}{12}$ 6
3. $\frac{4}{12} = \frac{?}{3}$ 1
4. $\frac{8}{12} = \frac{?}{6}$ 4
5. $\frac{2}{3} = \frac{?}{12}$ 8
6. $\frac{4}{6} = \frac{?}{3}$ 2
7. $\frac{2}{6} = \frac{?}{12}$ 4
8. $\frac{3}{3} = \frac{?}{12}$ 12

for Progress Check—page 249

Compute. Watch the operation signs. Not every answer will have to be renamed.

1. $\frac{2}{4} + \frac{1}{4}$ $\frac{3}{4}$
2. $\frac{7}{9} - \frac{4}{9}$ $\frac{1}{3}$
3. $\frac{1}{2} + \frac{1}{2}$ 1
4. $\frac{7}{8} - \frac{1}{8}$ $\frac{3}{4}$
5. $\frac{9}{10} - \frac{7}{10}$ $\frac{1}{5}$
6. $\frac{3}{7} + \frac{2}{7}$ $\frac{5}{7}$
7. $\frac{11}{12} - \frac{5}{12}$ $\frac{1}{2}$
8. $\frac{2}{5} + \frac{1}{5}$ $\frac{3}{5}$
9. $\frac{1}{8} + \frac{3}{8}$ $\frac{1}{2}$
10. $\frac{3}{3} - \frac{1}{3}$ $\frac{2}{3}$
11. $\frac{5}{8} - \frac{3}{8}$ $\frac{1}{4}$
12. $\frac{5}{6} + \frac{0}{6}$ $\frac{5}{6}$
13. $\frac{5}{6} - \frac{1}{6}$ $\frac{2}{3}$
14. $\frac{5}{16} + \frac{7}{16}$ $\frac{3}{4}$
15. $\frac{2}{9} + \frac{4}{9}$ $\frac{2}{3}$
16. $\frac{11}{16} - \frac{7}{16}$ $\frac{1}{4}$

for Progress Check—page 253

Rename these fractions.

1. $\frac{4}{3}$ $1\frac{1}{3}$
2. $\frac{7}{5}$ $1\frac{2}{5}$
3. $\frac{2}{2}$ 1
4. $\frac{7}{6}$ $1\frac{1}{6}$
5. $\frac{9}{5}$ $1\frac{4}{5}$
6. $\frac{11}{9}$ $1\frac{2}{9}$
7. $\frac{0}{3}$ 0
8. $\frac{10}{7}$ $1\frac{3}{7}$

*9. $\frac{8}{4}$ 2

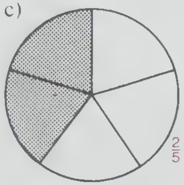
*10. $\frac{8}{2}$ 4

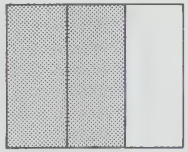
for Checkout—page 258

1. Which fraction names each picture?

a) 

b) 

c) 

d) 

2. Draw a picture to show $\frac{1}{3}$. Draw one to show $\frac{4}{4}$. Draw one to show $\frac{3}{5}$. Pictures may vary.

Examples: 

3. Which is greater?

a) $\frac{2}{2}$ or $\frac{1}{2}$ b) $\frac{2}{5}$ or $\frac{4}{5}$ c) $\frac{2}{6}$ or $\frac{5}{6}$

d) $\frac{7}{8}$ or $\frac{3}{8}$

4. Add. Rename when you can.

a) $\frac{2}{6} + \frac{3}{6}$ $\frac{5}{6}$ b) $\frac{1}{8} + \frac{5}{8}$ $\frac{3}{4}$ c) $\frac{4}{5} + \frac{1}{5}$ 1

d) $\frac{4}{9} + \frac{2}{9}$ $\frac{2}{3}$ *e) $\frac{3}{4} + \frac{2}{4}$ $1\frac{1}{4}$ *f) $\frac{4}{5} + \frac{3}{5}$ $1\frac{2}{5}$

*g) $\frac{7}{9} + \frac{5}{9}$ $1\frac{1}{3}$ *h) $\frac{11}{12} + \frac{5}{12}$ $1\frac{1}{3}$

5. Subtract. Rename when you can.

a) $\frac{5}{6} - \frac{1}{6}$ $\frac{2}{3}$ b) $\frac{6}{7} - \frac{3}{7}$ $\frac{3}{7}$ c) $\frac{4}{5} - \frac{2}{5}$ $\frac{2}{5}$

d) $\frac{7}{9} - \frac{4}{9}$ $\frac{1}{3}$ e) $\frac{7}{10} - \frac{5}{10}$ $\frac{1}{5}$ f) $\frac{3}{3} - \frac{2}{3}$ $\frac{1}{3}$

g) $\frac{11}{12} - \frac{3}{12}$ $\frac{2}{3}$ h) $\frac{7}{8} - \frac{3}{8}$ $\frac{1}{2}$

activities

1. **things** squares of poster board; felt pen; clear acetate; washable crayon

Make an appropriate board, as shown, for each multiplication factor—2 through 9. Laminate the board or cover it with clear acetate. With washable crayon, the youngster traces a path of multiples from the star to the triangle. This path can be erased with tissue or cloth and the board reused.

Here is an example for the multiples of 4:

* 4	8	10	14	18
6	12	26	28	32
10	16	20	24	36
55	50	48	44	40
54	56	52	46	42

2. **things** small cards

Have the pupils jot down sets of 3 equivalent fractions. These fractions are then written on the small cards, 1 per card, to form a deck of playing cards. The number of cards will depend on the size of the group of players.

The deck is shuffled. Players draw for dealer—greatest number indicates the dealer. Each player is dealt 8 cards. The remaining cards are placed facedown in a stack. The top card is turned faceup to form a discard stack. Play begins with the first player to the left of the dealer. He draws a card from either stack. If he has 3 cards that show equivalent fractions, he lays these down in front of him. To complete his turn, he discards a card on the faceup stack. The game continues with the next player on the left. The game ends when a player has laid down all his cards. Player with the most sets of 3 equivalent fractions wins.

3. things game board; 2 sets of markers

Prepare a game board as shown—all fractions written in their simplest form.

$\frac{3}{5}$	$\frac{2}{9}$	$\frac{5}{6}$
$\frac{7}{8}$	$\frac{5}{9}$	$\frac{2}{3}$
$\frac{3}{10}$	$\frac{1}{2}$	$\frac{3}{4}$

To cover a fraction on the game board with his marker, the player must rename the fraction correctly. Players take turns. The first person to place 3 markers in a row, column, or diagonal wins.

Variations:

A. Rename fractions in simplest form.

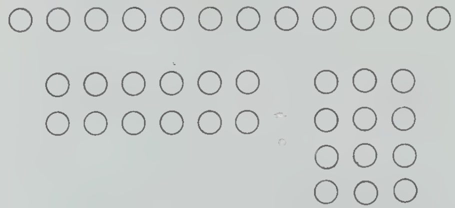
$\frac{6}{18}$	$\frac{10}{15}$	$\frac{9}{18}$
$\frac{18}{24}$	$\frac{4}{6}$	$\frac{2}{10}$
$\frac{4}{8}$	$\frac{20}{25}$	$\frac{6}{12}$

B. Rename fractions as whole or mixed numbers.

$\frac{15}{5}$	$\frac{7}{4}$	$\frac{9}{9}$
$\frac{0}{7}$	$\frac{9}{3}$	$\frac{3}{2}$
$\frac{9}{8}$	$\frac{5}{5}$	$\frac{8}{4}$

4. things counters

Have the pupil take out 12 counters and form an array. Three different arrays are possible, and each array can be turned—even turned consider these to be the same array.



A multiplication sentence can be written for each array.

number of rows $\times$ number in each row $=$ product

Challenge the youngster to find as many arrays and multiplication sentences for a given number as he can. Interesting numbers are 12, 16, 18, 24, 30, 36, 64.

For some numbers only one array is possible. Challenge your sharpies to find some of these. (2, 3, 5, 7, 11, 13, 17, 19, 23) You're right—these are the prime numbers. No need to introduce the vocabulary now. That will happen at the next level.

5. Have the pupil fold a piece of paper into sections. He is to select a number less than 90 for each section. His job: to find pairs of factors which when multiplied yield that number for the product.

16	24	8	30
1×16	1×24	1×8	1×30
2×8	2×12	2×4	2×15
4×4	3×8		3×10
	4×6		5×6

The youngster who has difficulty will benefit from first forming arrays (see activity 4).

6. things deck of playing cards

Remove all face cards. Change the aces to ones. Four cards are dealt to each player. The remaining cards are placed facedown in a stack. The goal is to form a pair of equal fractions.

The first player draws a card from the stack and discards 1 card from his hand to form a discard stack. Players that follow may draw 1 card from either stack. A player may have no more than 4 cards in his hand at a time. The first player to arrange a pair of equal fractions with the 4 cards in his hand is the winner. Some sets of cards can be arranged in several ways:

2	2		3	2		3	3
3	3		3	2		2	2

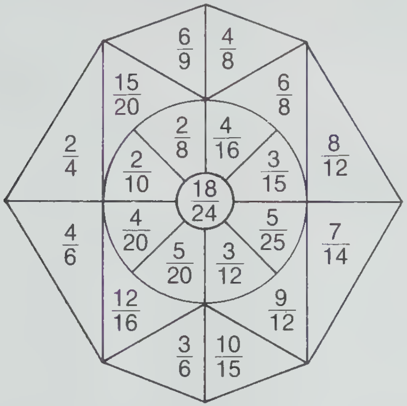
7. things game board; 24 small cards

Prepare a 4-by-6 array game board of 2-inch squares. Make pairs of cards, writing a fraction name on 1 card and the equivalent whole or mixed-number name on the other.

The cards are shuffled and placed facedown on the squares of the game board. To begin, a player turns over 2 cards. If the cards match (name the same number), he removes the cards from the board. If the cards do not match, they are again turned facedown and play continues. Play continues until all the cards are matched. The player with the most cards wins.

8. things spirit master; crayons

Prepare a spirit master as shown.



Color each name for $\frac{2}{3}$ orange.
Color each name for $\frac{1}{4}$ blue.
Color each name for $\frac{1}{2}$ yellow.
Color each name for $\frac{1}{5}$ green.
Color each name for $\frac{3}{4}$ red.

9. things 6 wood cubes

Number the faces of the cubes as follows:

- 2 cubes – 1 through 6
- 2 cubes – 2, 4, 6, 8, 10, 12
- 1 cube – 3, 6, 9, 12, 15, 18
- 1 cube – 4, 8, 12, 16, 20, 24

The pupil rolls all 6 cubes at once and tries to form a pair of equal fractions, using the numbers that land faceup. One point is earned for each pair formed. An additional point is awarded if the player identifies a fraction in simplest form.

Players predetermine rules for the following:

- The number of points needed to win
- Challenging incorrect pairs of equal fractions
- Overlooking pairs of equal fractions

additional learning aids

notation – chapter objectives 1, 2, 3

SRA products

Computapes, SRA (1972)
Module 5, Lessons: FR 5, 8
Mathematics Involvement Program, SRA (1971)
Cards: 224, 234, 265, 166, 167
Skill Modes in Mathematics, SRA (1974)
Level 1, Molecule: F
Visual Approach to Mathematics, level 3, SRA (1967)
Visuals: 23, 24, 26
Visual Approach to Mathematics, Rational Numbers, SRA (1967)
Visuals: 1, 2, 3, 4, 5, 6

other learning aids (described on page 288g)
Experiments in Fractions, Fraction Dominoes

12 MEASUREMENT MONEY AND MASS

before this chapter the learner has—

1. Read and written money notation (dollar sign, cent point)
2. Mastered finding the sum or difference of two 3-digit numbers
3. Experienced working with basic metric units of measure: grams, kilograms
4. Experienced researching for specific information.

in chapter 12 the learner is—

1. Mastering writing the value of a set of coins by using a dollar sign and a decimal point
2. Writing the fractional part of a dollar that a set of coins represents
3. Adding and subtracting with money notation
4. Counting change in problem situations
5. Computing tax in problem situations
6. Determining which is the better buy in real-life problem situations involving unit prices
7. Analyzing various factors that could affect the price of an item
8. Determining the relationship of grams to kilograms and identifying which is the greater measure of mass
9. Becoming familiar with grams and kilograms in relation to consumer products and prices

in later levels the learner will—

Find the sum or difference of two decimal fractions that are less than 100 and expressed in tenths or hundredths

Notes & Things

The major thrust of this chapter is to help the pupil gain confidence and power in applying to the consumer's world the mathematical knowledge that has already been learned. This is an exploratory chapter that will take the study of mathematics far beyond traditional bounds. Children of every ability level will be able to make contributions to each other's learning.

Children are generally much more capable in computing money than we give them credit for. Too many times problems with money are thought to be a set of skills that can't be handled until the child knows all about decimals and their place value. Money does involve decimals, but most children have been computing with money since grandpa put that first dollar bill in a birthday card or since that first toy that cost only $2.98 was advertised on TV. Place value and $2.98 just don't go together for a youngster. This chapter lets the child use his knowledge of money to review some fraction concepts, as a means for some computation practice, and as a way to look at measurement of mass. The intent of the chapter is to make the child more sophisticated about the money he spends. Consumer education can start with young children.

There is great emphasis here on economic judgments. Applications of estimating are made dramatically obvious for the first time in this chapter. Basic economic concepts that are a part of our society are featured in problem-solving situations. These situations are the basis for rich discussions that will foster tolerance of other people's ideas and values.

Money is related primarily to things that are sold by mass. But don't expect the typical study of measurement of mass.

The pupil is *not* required to make numerous conversions from one unit to another. Rather the approach is from the pupil's world. He is in charge. He must make value judgments and then look at the consequences. He is asked to examine *cause and effect* and learn by *trial and error.* He is required to use what he knows about number as it is used in the world, rather than to perform countless computations.

Very capable pupils will need to do little or no computation. Exact answers are not always necessary. Rounding and estimation often yield the necessary information. Less able pupils may be more dependent on computation.

Fair warning to you. There are a few traditional economic notions that aren't holding up in our fast-changing world. Possible problems will be signaled on specific lesson pages by comments.

things

food labels showing SI measurements.

goal Think about and explore ideas through a picture clue

page 259 The nature of this chapter makes the photograph one that you will not spend much time with. The entire chapter is full of research. You don't need any more. But the photograph can be used to get the youngsters started thinking about their purchases. Do you always have as much money as you would like to have? What coins do you most often have in your pocket? What coins do you think are used the most? Why? Does the cost of something depend on the size of the object? What does the cost depend on?

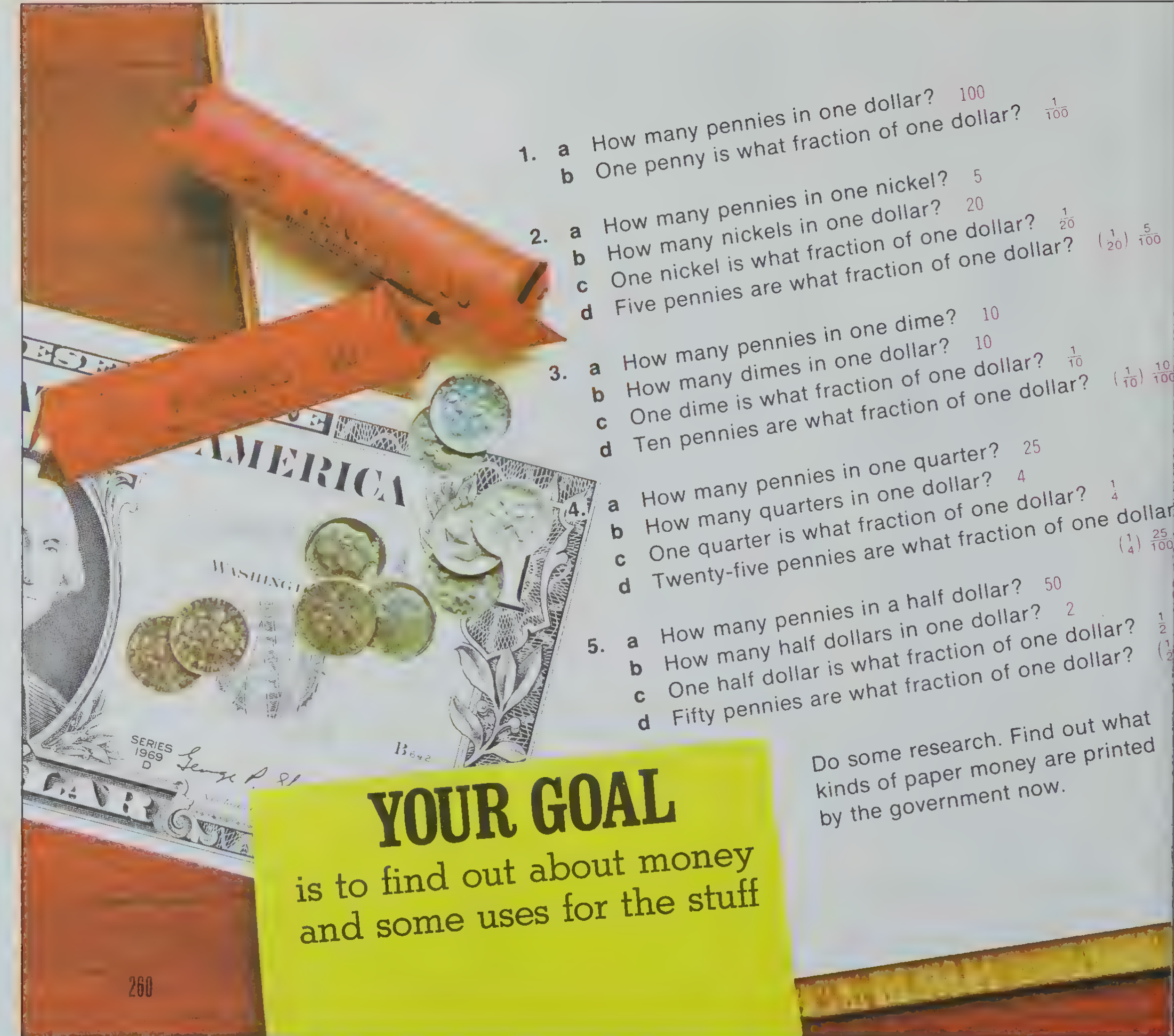
Now they are thinking along the right track. Turn the page and get started on a very exciting chapter.

lesson Pages 260, 261, 262

goal **Survey**—naming equivalent sets of coins, naming fractional parts of a dollar

page 260 Many youngsters at this age are great money handlers. They are familiar with equivalent sets of coins, yet have little or no concept of the fractional part of a dollar that a particular set of coins represents. The questions on the page will help you identify the learner's skill in each of these two areas.

Note the research challenge. The job can be extended to finding the denominations that are no longer printed. The almanac and the encyclopedia usually have this information. Anyone know a collector of old coins and bills? Would this person be interested in talking to the class?


1. a How many pennies in one dollar? 100
   b One penny is what fraction of one dollar? $\frac{1}{100}$

2. a How many pennies in one nickel? 5
   b How many nickels in one dollar? 20
   c One nickel is what fraction of one dollar? $\frac{1}{20}$
   d Five pennies are what fraction of one dollar? $(\frac{1}{20})$ $\frac{5}{100}$

3. a How many pennies in one dime? 10
   b How many dimes in one dollar? 10
   c One dime is what fraction of one dollar? $\frac{1}{10}$
   d Ten pennies are what fraction of one dollar? $(\frac{1}{10})$ $\frac{10}{100}$

4. a How many pennies in one quarter? 25
   b How many quarters in one dollar? 4
   c One quarter is what fraction of one dollar? $\frac{1}{4}$
   d Twenty-five pennies are what fraction of one dollar? $(\frac{1}{4})$ $\frac{25}{100}$

5. a How many pennies in a half dollar? 50
   b How many half dollars in one dollar? 2
   c One half dollar is what fraction of one dollar? $\frac{1}{2}$
   d Fifty pennies are what fraction of one dollar? $(\frac{1}{2}$

Do some research. Find out what kinds of paper money are printed by the government now.

YOUR GOAL
is to find out about money and some uses for the stuff

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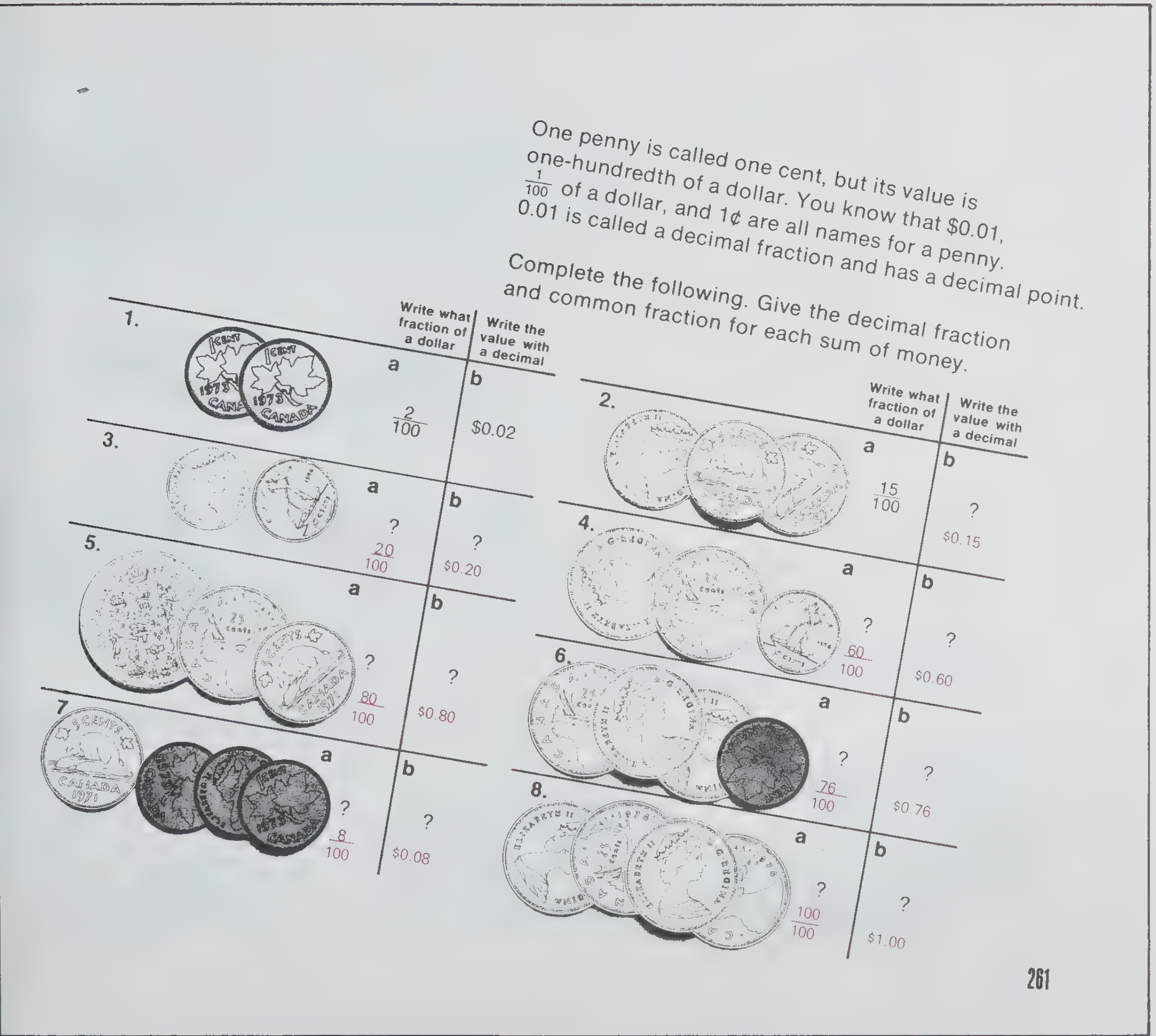

One penny is called one cent, but its value is one-hundredth of a dollar. You know that \$0.01, $\frac{1}{100}$ of a dollar, and 1¢ are all names for a penny. 0.01 is called a decimal fraction and has a decimal point.

Complete the following. Give the decimal fraction and common fraction for each sum of money.

	a Write what fraction of a dollar	b Write the value with a decimal
1.	$\frac{2}{100}$	\$0.02
2.	$\frac{15}{100}$	? \$0.15
3.	? $\frac{20}{100}$	? \$0.20
4.	? $\frac{60}{100}$	? \$0.60
5.	? $\frac{80}{100}$	? \$0.80
6.	? $\frac{76}{100}$	? \$0.76
7	? $\frac{8}{100}$	? \$0.08
8.	? $\frac{100}{100}$	? \$1.00

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goal Writing the DECIMAL value and the fractional part of a dollar for a set of coins

memo The focus here is not on introducing DECIMAL FRACTIONS. Readiness is being developed, however, for later chapters in the program dealing with decimals. The youngsters are familiar with the decimal point used to mark cents. Here it is given its technical name, DECIMAL POINT. And indeed the youngsters are writing decimal fractions in terms of hundredths (cents) and equivalent common fractions with the denominator 100. They will not know the decimal place-value positions of tenths and hundredths. There is no need to introduce these concepts. The emphasis is on writing decimal values for a set of coins before actually computing with dollars and cents.

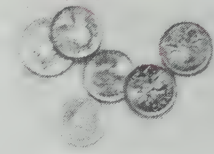
page 261 Some discussion will be necessary to clarify language and the type of recording expected.

goal Practice in writing the decimal value for a set of coins

page 262 There should be no problems after page 261. Writing dollars in decimal notation in problem 7 may need some special attention, but don't ask that the zeros be annexed. \$3 tells the value just as well as \$3.00. It is a good idea to put the decimal point after the \$3. to let everyone know that you haven't forgotten anything.

Here are some sets of coins. Write the value of each set, using a decimal.

1. \$0.19

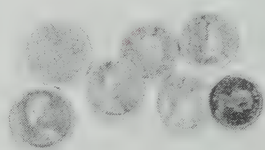


2. \$0.09

3. \$0.03

4. \$0.30

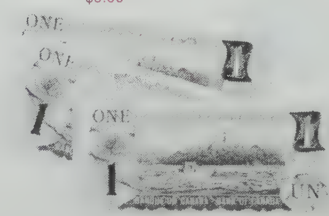
5. \$0.71



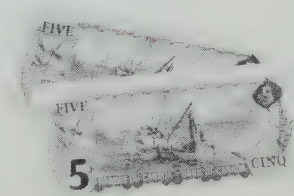
6. \$0.95

How good are you at writing how many dollars? Prove it. Use the symbol that stands for dollar.

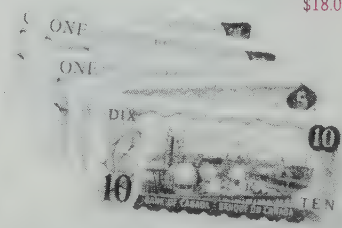
7. \$3.00



8. \$10.00



9. \$18.00



262

It is probably silly to take too much time to practice adding or subtracting money. You have been doing that for years. But take time out for a short review.

$3.98
- 8 ← That is the number of pennies.
- 9 — The number of dimes
- . — The decimal point separates the dollars from cents
- 3 — The number of dollars

Bill started to add some money this way.

What's wrong?

```
 $1.56
+  1.97
```

He is not adding dollars to dollars, dimes to dimes, or cents to cents

Cash started to subtract this way.

What's wrong?

```
 $7.27
-    5
```

We don't know what the 5 means

Don't make these mistakes when you compute. Watch the signs. There are both addition and subtraction problems.

	a	b	c	d	e	f	g
1.	$4.43 + $1.26 = $5.69	$5.75 − $3.20 = $2.55	$12.56 + $ 6.11 = $18.67	$8.12 + $ .76 = $8.88	$9.56 − $ .25 = $9.31	$7.10 − $1.05 = $6.05	$3.75 + $2.25 = $6.00

Did you mark your answer so that it has a dollar symbol and a decimal point?

	a	b	c	d	e	f	g
2.	$5.25 + $4.38 = $9.63	$3.62 + $4.57 = $8.19	$7.50 − $5.25 = $2.25	$8.25 − $2.50 = $5.75	$8.67 + $1.75 = $10.42	$6.65 + $3.35 = $10.00	$2.25 − $1.30 = $.95

Work carefully on the next ones. Make sure your answers are reasonable.

	a	b	c	d	e	f	g
3.	$5.00 + $5.00 = $10.00	$5.00 − $3.50 = $1.50	$3.69 + $2.31 = $6.00	$7.85 + $2.15 = $10.00	$9.00 − $7.95 = $1.05	$10.00 − $ 9.98 = $.02	$10.00 − $ 5.01 = $4.99

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lesson Pages 263, 264, 265

goal Practice in adding and subtracting money

page 263 You'll want to point out the two common errors shown on the page. The computation itself should cause no problems—unless someone is careless. If aligning place-value positions is a problem for anyone, have him turn lined paper sideways.

goal Counting change for problem situations

memo Studies show that young people are often mistreated as consumers in many ways. Giving the incorrect amount of change is altogether too frequent. Most of the time the mistake is an "honest error." Imagine the courage it would take for a 10-year-old to say to an adult cashier, "I think you have given me the wrong amount of change." Yet that child must do it on the spot if an error has been made and is to be corrected. Giving the youngsters the skill to detect an error and then the confidence to ask that it be corrected is a big job. The customer must always make sure the change is correct whether it is handed to him or dispensed by machine. And machines make errors too.

page 264 You decide how best to use this page with your pupils. Focus on verifying the change received by counting **from** the amount spent **to** the amount given to the clerk.

This is a great page for enactment. Challenge your sharpies to figure out all combinations of coins you could use as change for any or all of the problems.

Do you count your change?
Pretend you are in a store. You are buying lots of things. Would you be satisfied if the following happened?

1. Something cost 29¢ in all.
You gave the clerk two quarters.
You get back 11¢ in change. O.K.? No

What *coins* should you get back?
1 penny and 2 dimes or any combination making 21¢

2. Something cost $1.26 in all.
You gave the clerk a five-dollar bill.
You get back $2.74. O.K.? No

What *bills* and *coins* should you get back?
4 pennies, 2 dimes, 2 quarters, and 3 dollars or any combination making $3.74

3. Something cost $3.89 in all.
You gave the clerk a five-dollar bill.
You get back $1.11. O.K.? Yes

What *bills* and *coins* should you get back?
1 penny, 1 dime, and 1 dollar or any combination making $1.11

4. Something cost $1.61 in all.
You gave the clerk two one-dollar bills and one penny. You get back $.39. O.K.? No

What *coins* should you get back?
1 dime, 1 nickel, and 1 quarter or any combination making $.40

Counting *from* the amount spent up *to* the amount of money received is a good way to count change. Try it.

264

SALES TAX

On sales over one dollar, the tax shall be computed at the straight rate of 3% on even dollar amounts and the bracket system shall be used to compute the tax on amounts between even dollars.

Amount of Sale			Tax Due
$0.01	to	$0.14	None
0.15	to	0.44	$0.01
0.45	to	0.74	0.02
0.75	to	1.14	0.03
1.15	to	1.44	0.04
1.45	to	1.74	0.05
1.75	to	2.14	0.06
2.15	to	2.44	0.07
2.45	to	2.74	0.08
2.75	to	3.14	0.09
3.15	to	3.44	0.10
3.45	to	3.74	0.11
3.75	to	4.14	0.12
4.15	to	4.44	0.13
4.45	to	4.74	0.14
4.75	to	5.14	0.15
5.15	to	5.44	0.16
5.45	to	5.74	0.17
5.75	to	6.14	0.18
6.15	to	6.44	0.19
6.45	to	6.74	0.20
6.75	to	7.14	0.21
7.15	to	7.44	0.22
7.45	to	7.74	0.23
7.75	to	8.14	0.24
8.15	to	8.44	0.25
8.45	to	8.74	0.26
8.75	to	9.14	0.27
9.15	to	9.44	0.28
9.45	to	9.74	0.29
9.75	to	10.14	0.30

A tax is added to the price of things you buy in some parts of the country. Here is a table that tells how much sales tax is charged in one place.

The print is very small. Name one reason it might be so small. Who might use a table like this everyday? Salespeople

How much tax would you pay on each of the following purchases?

1. $.75 $.03	**2.** $8.45 $.26
3. $3.44 $.10	**4.** $6.15 $.19
5. $.14 None	**6.** $2.98 $.09
7. $8.20 $.25	**8.** $9.99 $.30
9. $1.00 $.03	**10.** $4.50 $.14
11. $7.10 $.21	**12.** $2.25 $.07
13. $5.60 $.17	**14.** $.89 $.03
15. $.10 None	**16.** $.78 $.03

Sometimes tax is already figured into the price you pay. If this is true, you will see the words "tax included."

Do some research

What are some of the things tax money is used for?

goal Computing sales tax on a purchase

page 265 Taxes are a very real part of all our lives. Have you ever wanted to buy something, had enough money in your pocket for the whatever, **but** not enough for the tax? This is a common experience for many youngsters—especially in the candy or toy shop.

Try to obtain a local tax table and provide additional practice, using the local rate. These charts are not always easy to obtain. Perhaps the youngsters can help.

That research problem is a good one, but the question is broad. You may want to limit it to sales tax rather than open it up to include gas tax, state and federal income tax, and so forth.

lesson Pages 266, 267, 268

goal Survey - knowledge of metric units of measure

memo You may wish to prepare a spirit master of the chart on this page to help pupils in recording their estimates. You may also want to assign two or three independent workers to prepare the 50 g, 100 g, 200 g, 300 g, and 500 g masses of sand or whatever ahead of time.

things bathroom scales
balance scale and
metric masses
plasticine, sand, or stones

page 266 This is mainly an activity page. Start with discussion of the pupils' mass in kilograms and in grams. *Why do we usually record a person's mass in kilograms, not grams?*

Then allow pupils plenty of time to become familiar with the various gram masses, estimating and keeping records of their estimates. Do their estimates become more accurate from day to day?

266

1. What is your mass in kilograms? Check it when you get a chance to use a scale.

2. What is your mass in grams? (Remember: 1 kg = 1000 g)

3. Practise estimating in grams. Have someone in the class make up at least two of each of these masses and put them in paper bags. (They can be made up of plasticine or small jars of sand or stones.)
   50 g   100 g   200 g   300 g   500 g
   The mass can be written on the bottom of the bag so that it is not too easy to see.

Over the next few days, when you get a chance, practise estimating these masses. Without lookin at what is written on the bag, hold the bag and estimate which mass it is. Write each one down in a column like this. Beside each one put a check o an X each time you get it right or wrong.

	Estimates	Right or Wrong
50 g		
100 g		
200 g		
300 g		
500 g		

4. How many different ways can you make up a kilogram with these masses if you have two o each?
   a. 500 + 500
   b. 500 + 300 + 200
   c. 300 + 200 + 300 + 200
   d. 500 + 200 + 200 + 100
   e. 300 + 300 + 200 + 100 + 50 + 50
   and many more

Here are some things you might buy in a store. The mass of each one is about what you would find in the store.

1. Would 400 g of bread always be the same shape as the bread in the picture? No

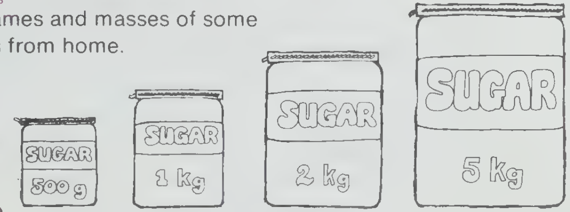
2. If you bought a small box of tea bags with a mass of 100 g, how many tea bags do you think would be in it? 30

3. The jar of coffee has 100 g on the label. Does that mean:
   a the coffee and the jar have a mass of 100 g altogether? No
   b the coffee, all by itself, has a mass of 100 g? Yes

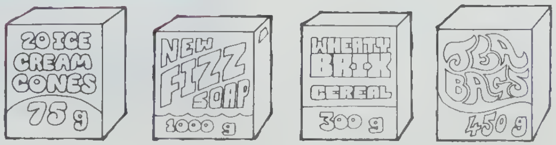
4. What do you think would be the usual sizes of these packages in a store near you? (They come in two or three sizes. Estimate what you think one size may be.)
   a butter b salt c flour d canned peaches
   Answers will vary. Discuss all answers

5. Next time you are in a store, copy down the names and masses of some of these things from the labels. Or bring labels from home.

6. Look at these bags of sugar.
   How many of the 500 g bags would you need to have the same amount as the 1 kg bag? 2 the 2 kg bag? 4 the 5 kg bag? 10



7. Look at these packages. They are all about the same size. Why are the masses not about the same? Some goods are heavier than others



goal Examining metric measurement in relation to consumer products

things food labels showing metric measurements

page 267 Use this page as the basis for discussion. It can serve to reinforce understanding of metric measures and of packaging of consumer goods. Everyone should do Question 6 independently.

Question 5 is a research project. Research is an integral part of this chapter. Encourage students to investigate labels. The activity is designed to help relate the work in school to the real world. Show them a sample—they should know what they are looking for.

goal Examining prices of products in relation to metric units of measure

memo Do not expect the pupils to have knowledge of operations on decimals or multiplication of fractions. They have been consumers for years. They know a surprising amount about money. Let them operate intuitively and answer without writing down any more of the problem than they need to.

page 268 This is the real world! Often the greater the quantity purchased, the lower the price. In order to take advantage of a lower price, however, the large quantity must be purchased. Discuss problem 1. A quantity less than 400 g of peanuts will have to be computed at $0.75 for 100 g. Is this true for any other examples?

How much can you buy? may be interpreted as *How much do I get for my money?* This question may be viewed as cost per unit or as a value judgment. Is this a fair price? What conditions determine price? That is the focus of the remainder of this chapter. Watch for broadened viewpoints as the youngsters progress through this short unit in economics.

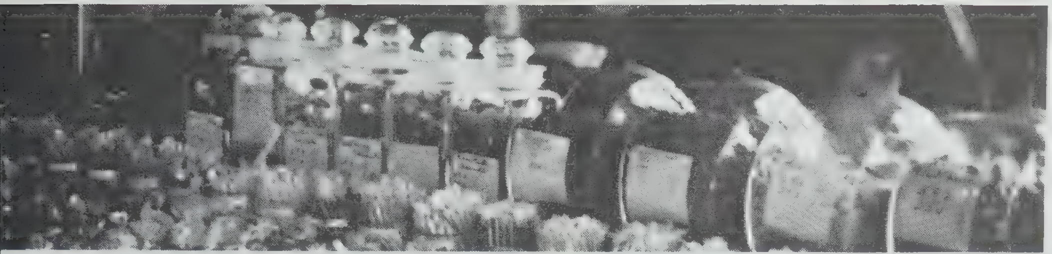
In your neighborhood there is a wonderful shop. It is a small shop. It is packed full of all sorts of good things to eat.

Sometimes you want to buy everything in the shop. Sometimes you want to buy just a little of your favorite thing. And sometimes you are very, very hungry and want to buy as much as you can for your money.

The prices in the table are for items sold in the shop.

Peanuts	Mints	French almonds	Toffee	
400 g $2.00	1 kg $8.00	1 kg $5.00	100 g	$0.40
			Chocolates	
100 g $0.75	100 g $1.00	500 g $3.00	1 kg	$3.50
			500 g	$2.00

1. How much does each of the following cost?
   - a 200 g peanuts $1.00
   - b 300 g mints $3.00
   - c 2 kg French almonds $10.00
   - d 500 g mints $5.00
   - e 200 g toffee $0.80
2. How much does each of these cost?
   - a 2 kg mints $16.00
   - b 800 g peanuts $ 4.00
   - c 300 g toffee $ 1.20



Today is your lucky day. Pretend you won the door prize at the candy shop. Your prize was ten dollars. You are very hungry and decide to spend it all in the shop. You want to buy different kinds of candy and fruit. Of course you want to buy things you like. Go ahead, enjoy yourself!

Name	Package	Cost
Chocolate crunch bar	30 g	10¢
Almond bar	100 g	15¢
Chocolate bar	100 g 500 g	35¢ $1.25
Nut bar	200 g	40¢
Lemon drops	200 g bag 1000 g bag	50¢ $2.10
Toffee	500 g	40¢

1. Cost of 300 g box? $1.00
2. Cost of 1 kg? $1.50
3. Cost of 1 kg? $2.50
4. Cost of 1 kg? $2.00
5. Cost of 1 kg? $2.10
6. Cost of 1 kg? $0.80

269

lesson Pages 269, 270

goal Practice in arithmetic skills involving money and measurement; introduction to comparison shopping

memo Pages 269 and 270 are related. Use them together please.

page 269 Pupils should decide on their purchases independently. This problem is given an interesting twist in the Supersleuth assignment on page 270. Here the pupil buys for himself. On page 270, he is asked to buy for 100 people. The pupil may choose favorites or choose items that will yield the greatest amount. But when buying for 100, quantity is most important. Everyone wants a share.

Some pupils may require help in determining how to compute the cost per kilogram for each item.

goal Examining quantity in relation to price; selecting the greatest value for a given amount of money

page 270 Be a good shopper—examine the prices. What is special about certain prices? (Some based on mass, some on number of pieces).

Capable people will probably perform all of the computations mentally. Great! Have them write only if necessary. Watch for youngsters who answer so quickly that others don't have a chance. There is a lot of comparative work to be done. You might want to split into teams.

The Supersleuth assignment should be very revealing. What motivated the pupil to make specific choices when buying for himself? when buying for 100?
Look for the lone pupil who chooses only one specific item consistently.

As an extension, the number of pupils who selected each item can be tallied and recorded in graph form. A comparison of the items selected when doing personal shopping with the items selected when buying for 100 should be

Interesting

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Peanut Clusters	Peppermint Sticks	Fudge
1 kg $3.00	1 for 15¢	1 kg $2.50
500 g $1.70	5 for 50¢	250 g $0.75

1. Why do lemon-drop makers give you more lemon drops for your money if you buy a bigger package?
To get you to buy the larger package—producer saves in packaging and h
2. If you and a friend each want 500 g of peanut clusters, how should you buy them to save money? What will you pay? How much will you save? Buy 1 kg and split it. $3.00 $0.40
3. How many peppermint sticks can you buy for $1.00? for 45¢? 10 3
4. If four friends each bought 250 g of fudge, how much would they pay all together? $3.00
What does this tell you? It's cheaper by the kilogram.
5. Which is cheaper — 1 kg of lemon drops or five 200 g bags? How much cheaper? 1 kg $0.40 chea
Which is the better buy? 1 kg
6. How much does a 2 kg box of fudge cost *per* kg? a 5 kg box? $2.50/kg $2.50/kg

SUPER SLEUTH

You still have $10, but this time you are buying things to feed 100 people. You won't be eating any of it. Now what will you buy?

FRUIT	
Oranges 1 for 15¢ 4 for 50¢	**Watermelon** Whole 10¢ a lb. Half 15¢ a lb.
Apples 1 pound 30¢ 2 pounds 60¢	**Bananas** 1 pound 10¢ 2 pounds 15¢

(Please do this in a discussion group. There are lots of right answers.)

1. Four oranges cost less *per* orange than one orange. Right? Why are oranges priced this way?
   Yes To get you to buy more oranges; to save you money; to move stock
2. Why is a whole watermelon cheaper per pound than a cut-up one? Less risk; whole melons will keep better than cut-up ones.
3. Why should you and your friend buy candy or fruit together and divide it up later?
   Saves you money to buy larger amounts.
4. Are there other things in the shop that are cheaper per pound if you buy more? Name some.
   Yes Bananas, lemon drops
5. If the shopkeeper had only one of some item left, what could he do to its price?
   He may or may not change it. He might charge less to get rid of it.
6. If something didn't sell, what could the shopkeeper do to get more people to buy it?
   Have a sale
7. Lots of people want a product, and the store is running out of it. What could happen to its price?
   It could go up.
8. No one has bought a product, and the store has lots of it. What could happen to its price?
   It could go down.

lesson Pages 271, 272, 273, 274, 275

goal Revealing the importance of arithmetic skills in shopping; examining some factors that affect price

memo These pages are self-explanatory. They are meant for discussion and a sharing of ideas. They can help make pupils aware of some of the factors that determine the price of an item. (Number and arithmetic **are** often hidden factors.) You are the best judge of how to handle these exploration pages with your pupils. Do not let a reading difficulty hinder any pupil from gaining experience in making these real-life judgments.

page 271 The discussion of these questions should be very revealing. Sharp shoppers will be quickly identified.

Discussion of question 5 might bring out that the shopkeeper could give away the last item, keep it for himself, or choose to raise the price. Allow the same range of responses for the remaining questions.

goal Examining how supply and demand and seasonal availability affect price

page 272 A variety of responses are indicated on the answer key, but the variety you will get in a good discussion will be even greater. You can acquire insight about the youngsters' values in the discussion group. You will perhaps have to play the role of peace-keeper. Children have a right to their own opinions, but they also must learn to be tolerant of others' views.

USE THE PRICE LISTS GIVEN FOR THE CANDY AND FRUIT SHOP TO HELP YOU DO THE FOLLOWING EXERCISES.

1. The shopkeeper has just raised the price of fudge. Why do you suppose he did that? High demand; his rent went up; reduced supply; higher production costs
2. He just lowered the price of mints. Why? Low demand and large supply; lower production costs
3. Fifty watermelons just came into the store. What will happen to the price of watermelons? It may go down if 50 is too many for the store.
4. Few people want to buy licorice, but the store still sells it. Might the shopkeeper change the price? Yes—it might go down.
5. Toffee makers have quit making toffee. How much toffee did your class buy? What might happen to the price of the remaining toffee? It might stay the same, go up, or down. (Be sure to get reasons for answers given.)
6. You own the last apple left in the world. What will you do with it? Whatever you'd like If you sell it, how much will you charge for it? A high price Let's hope someone decides to save it and plant the seeds.

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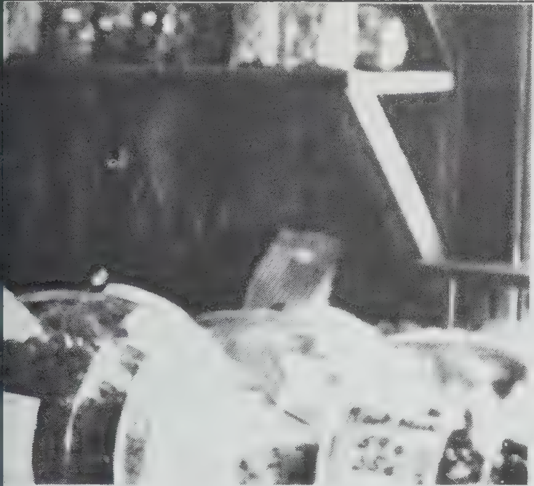
Fruits are picked at different times of the year. Most apples are picked in the fall. Oranges are picked in the winter. Watermelons are picked in the summer.

7. In the winter, stores have lots more oranges because the growers have to sell them right away. They will spoil if they are not sold. In what season are oranges the cheapest? Why? Winter Large supply
   - a When are apples the cheapest? Why? Fall Large
   - b In what season would watermelons cost the most? Winter—season in which they are least plentiful
8. Can you think of any other prices that change as the seasons change? Air conditioners, winter coats, flowers, cars just before new models come out, corn on the cob, etc.



1. So far we have found two factors that might raise or lower the prices in our candy and fruit shop.
   - a If many people want to buy something and there is not much available, the price might go up. If no one buys an item, the price might go down.
   - b If it is winter, oranges will probably be cheap and watermelon will be expensive.

   There is another factor that makes some goods in the shop cheap and others expensive. Figure out what it is. Shipping costs and taxes

(You may not want the pupils to do any more than THINK, but discussion is O.K. too.)

Think about these questions.

2. Which costs more?
   - a Toffee made by the shop owner or by a factory? Why? A factory can save on larger amounts
   - b Fish caught locally or fish caught farther away? Why? Transportation is costly.
   - c Meat packaged locally or meat packaged 2000 km away? Why? Transportation is costly.
3. How much would a block of ice cost at the equator? at the North Pole? Free
   A lot
4. Would a carton of eggs be more expensive in a large city or in a small town? Why?
   They should be cheaper near the source.
5. Do hamburgers cost more at the foot of a mountain or at the top of the mountain? Maybe the same
   Why? If higher at the top of the mountain, it would be because of higher costs or lack of competition.
6. Why are homemade chocolate chip cookies often cheaper than those bought in a store?*
   Why are they better? Fresher

*Labor time at home is not counted; no handling or packaging cost either.

goal Examining how shipping distance and import taxes affect price

page 273 Many of these questions will challenge the logical thinking of those in the discussion group. Answers for questions 3 through 6 will vary, but every answer should be backed up with good reasons.

goal Examining how shipping costs can affect price

page 274 It may surprise you, and the pupils, to know that prices of apples, oranges, eggs, bread, hamburger, and milk are very much the same in widely separated parts of the country.

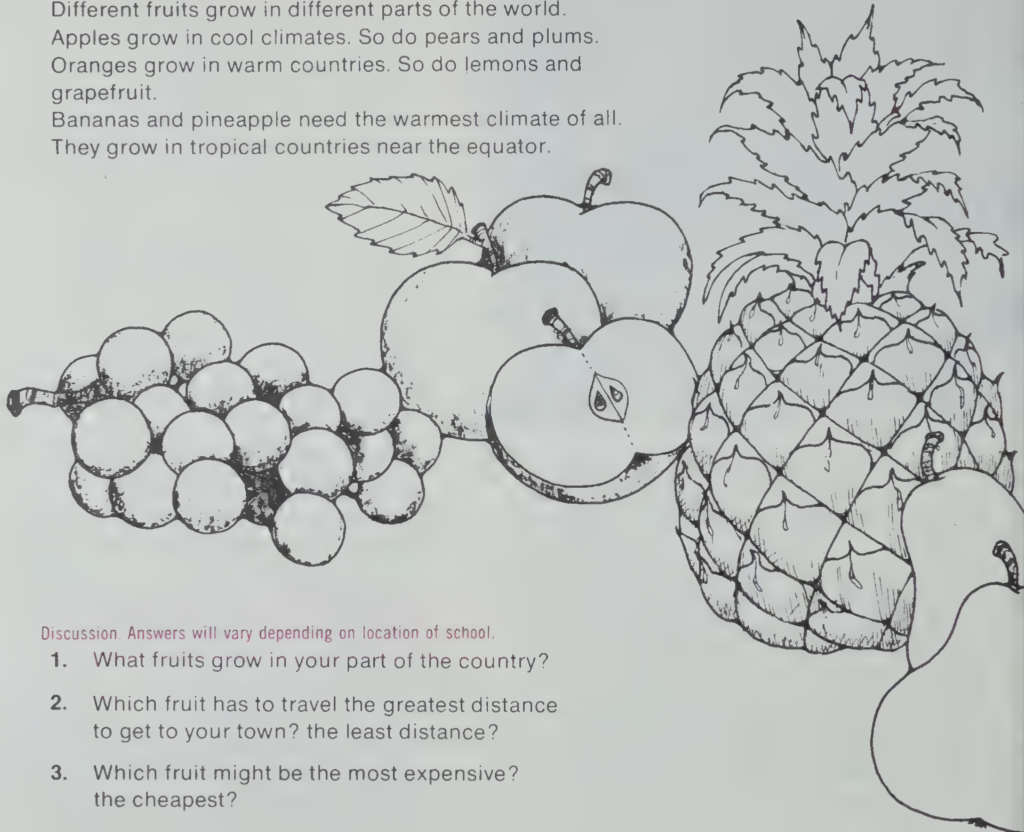
Perhaps transportation costs within a geographic area do not affect prices as much as is supposed. You have probably all seen the highest quality of produce being shipped out of the locale where it is grown. Prices may be the same but quality may vary.

It is still common, however, for manufactured items such as cars and electrical appliances to cost more west of the Rockies. Let eager beavers look in magazines for advertisements that make this statement and then find out why costs are greater.

This is also a good time to look at the price of an item, such as a kilogram of hamburger, in just one neighborhood. One store might have a special for 69¢, the store down the street might charge 79¢, and still another might charge 98¢. Find out whether the pupils know why this happens and how it can affect their family.

Notice how frequently words such as **may be, will probably**, and **often** are used. There are very few hard-and-fast rules that can be passed on to our young consumers.

Different fruits grow in different parts of the world.
Apples grow in cool climates. So do pears and plums.
Oranges grow in warm countries. So do lemons and grapefruit.
Bananas and pineapple need the warmest climate of all.
They grow in tropical countries near the equator.



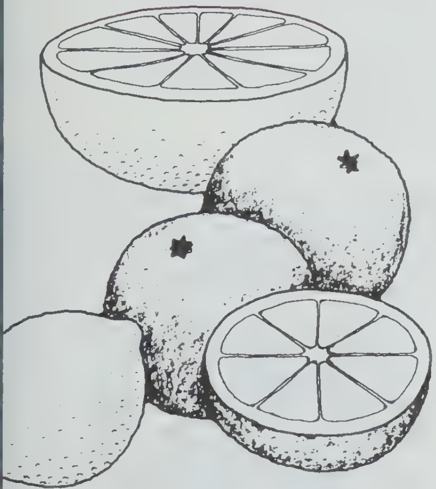
Discussion. Answers will vary depending on location of school.

1. What fruits grow in your part of the country?
2. Which fruit has to travel the greatest distance to get to your town? the least distance?
3. Which fruit might be the most expensive? the cheapest?
4. Make a price list for a shop in your town which sells these fruits.

274

You have learned so much about prices that the owner of the candy and fruit shop has decided to let you run his shop. The first thing you want to do is make sure that everything is priced right. What have you learned so far that will help you to set prices wisely?

ipply and demand, spoilage factors, source of thing to be sold.



(More discussion. There are many good answers. Only some are given.)

There are a couple of other things you will need to know before you are an expert.

1. Do you know where chocolate grows? Do chocolate candies cost more than candies that aren't chocolate? Why? In tropical countries Usually Shipping costs and import duty
2. Which candy on the list do you think is the hardest to make? How much does it cost?
3. Which candy is the easiest to make? How much does it cost?

Discuss: Source and cost of ingredients is main factor on cost. We don't know about the process.

4. Do you think a candy that is harder to make would cost more? Why? Yes Labor costs go up because more time is needed.
5. The peanut-cluster maker has to pay 10¢ less per kilogram for peanuts than he used to. Do you think this will affect the price of peanut clusters? It may. Cost savings are usually passed on to the consumer.
6. The banana crop this year was much larger than usual. What could happen to the price of bananas? It could go down.
7. Lack of rain ruined the crop of peaches from a large fruit-growing area. What might happen to the price of peaches? The price will probably rise

SUPER SLEUTH

Many ads in the newspapers will say that the price is lowered or reduced. Bring in as many ads as you can that show prices.

275

goal Investigating how costs of parts and availability affect the price of the end product

page 275 Again, actual numbers are not offered, yet the cost of ingredients is a major factor in determining the price of the end product. Surplus of perishable items is as important a factor as low supply in determining price levels. There are no single right answers to these questions.

Challenge pupils to do some research work. For example, find out where chocolate grows. Really get going on the Supersleuth suggestions. Use these as independent or small-group activities outside of classtime.

goal Examining factors that might lower prices: oversupply, lack of storage space, perishability, seasonal item

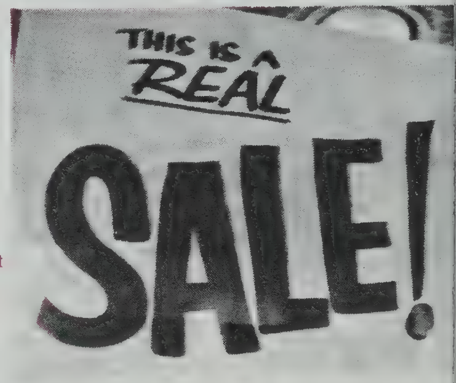
page 276 Number is related to each discussion question, but computation is not required. Good critical thinking is necessary to solve some of these very real problems that affect all consumers. Does lack of storage area affect price? Can an oversupply be returned? At whose cost?

Why might you want to have a sale?

Why might you need to have a sale?
To sell a surplus; to pay bills; to get new customers

1. You have thirty crates of oranges sitting in your shop. They are not selling. What can you do?
Advertise; lower the price
2. It is a week before Halloween. You have hundreds of bags of candy corn in your shop (40¢ a bag), but so do many other shops in town. How can you get people to buy Halloween candy in your shop? Advertise low prices
Why can you lower the price of the candy and still make money? It costs you to keep it. Plus you might get people to buy other things not on sale.
3. It is a week after Christmas. You have fifty chocolate Santa Clauses left over. Each one costs $1.00. You know they will spoil by next Christmas. What can you do to get rid of the Santa Clauses?
Answers will vary. You might even give one away free with another purchase.

In each case the product can be put on sale. That means the price will be lower than usual. People will come to your shop to buy because they are getting more for their money. And when they are buying something on sale, they just might buy some lemon drops or a box of chocolates as well. Right? Right



Some of the advertisements you brought in probably say that a store is having a sale. Does the ad say why the store is having a sale? From what you have learned, can you guess why it might be having a sale? Probably not
Surplus; no one buying; need for cash to pay bills; seasonal items that will spoil

276

Apply all the things you have learned about pricing to another store, a dime store.

1. If one pencil costs 5¢, how much might be charged for 10 pencils? Probably less than 50¢
2. Scissors used to cost 60¢, but no one bought them. What happened to the price? Probably went down
3. The store also sells books, about $4.00 for a hard-cover book and 95¢ for a paperback. Why are hard-cover books more expensive? They cost more to make and ship; they last longer.
4. How will you get people to buy more than one item when they buy a product? Make unit cost on one less when buying two
5. How will you attract people to your store? Advertise, have sales and contests, etc.

CHECKOUT

Answer these. Use any ideas you have. Answers will vary. Accept reasonable answers.

1. What things can affect the price of an item? Supply and demand, shipping costs, season, taxes, etc.
2. What are some of the reasons why items go on sale at a lower price? No demand; large supply; in season; to attract people to store
3. Can advertising help sell items? How? Yes—informs people of where your store is and your low prices.
4. What is your idea of the best commercial on TV or radio? Why do you think that it is best? Answers will vary.
5. You bought one thing for $2.98 and something else for $.79. (The tax was included in each price.) You gave the clerk four one-dollar bills. How much money should you get back? 23¢

277

STOP See activity 1, page 277a.

See activity 2, page 277a.

lesson Page 277

goal Reviewing and applying the concepts developed in the chapter; **Checkout**—understanding of consumer concepts

page 277 Once again, you are the best judge of how this page should be assigned to your specific individuals or groups.

The focus is not on a mastery of skills or concepts. It is on helping pupils become aware of number as the basis for making some decisions in the real world.

Many of the ideas explored in this chapter should spill over into other areas of study as well as form bases for more independent study.

The Checkout questions involve judgments based on broad concepts gained by experience. It is not possible to direct the learner to specific assignments to gain the kinds of experiences necessary. Continue examining ads in your local area. Watch for opportunities in social studies and reading.

RESOURCES

another form of evaluation

for Checkout — page 277

Pretend you own a store.
Answers will vary. Accept reasonable answers.

1. What kind of store do you own?
2. What things will affect the prices you charge? Supply and demand, shipping costs, season, expenses, etc.
3. When might you have a sale? Why? When you are overstocked; to attract customers
4. What kind of advertising will you use? Radio and newspaper ads Why? To attract customers and inform them of stock carried and sales
5. You go to the hobby shop and buy an airplane model for $1.49 and glue for $.27; the tax is $.09. You give the clerk $2.00. How much change should you get? 15¢

activities

1. Pupils work in pairs for this exercise in comparative shopping. Have each pair select an item they would like to buy. For example: a pair of tennis shoes, a candy bar, a box of cereal — whatever they like. Have the youngsters check out three different brands for the same item..What is the price of each brand? What is the measure of the contents of each brand? How does the packaging differ between brands? Check each brand in three different stores. Record the three prices for each brand.

2. Extend activity 1 by having your more capable pupils compile their results in a report and analyze the results. Could the type of packaging affect the cost? Find the cost per unit of weight. Which brand is the better buy? How much money can be saved?

additional learning aids

concept — chapter objectives 4, 6, 7, 8

other learning aids (described on page 288g)
Learning about Measurement

operation — chapter objectives 3, 5

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit master: M 4
Diagnosis: an instructional aid — mathematics level A, SRA (1973)
Probe: L-19
Skill through Patterns, level 4, SRA (1974)
Spirit master: 8

other learning aids — Easy Money, Money Matters, Pay the Cashier Game, Spin-A-Coin


B.C.
BY JOHNNY HART
?
4
4
ZANG
ALL RIGHT, B.C., WRITE THE NUMBERS UP TO ONE HUNDRED, IN TENS.
OK!
MATH CLASS
© Field Enterprises, Inc., 1972
10·22
HOLD IT! HOLD IT!
10, TWENTY, 30, 40, FIFTY SIXTY, SEVENTY, 80, NINE
YOU CANT MIX UP WORDS AND NUMBERS LIKE THAT YOU HAVE TO BE CONSISTANT!
ERASE THE WHOLE THING AND DO IT OVER
IEN, 2WENTY, TH3RTY, 4OURTY, 5IFTY, 6IXTY, 7EVENTY, 8IGHTY 9INETY, 1HUNDRED.

13 PROBABILITY

before this chapter the learner has—

Extracted information from a bar graph

in chapter 13 the learner is—

1. Predicting the outcome of a probability experiment
2. Deciding whether outcomes for an experiment are equally likely
3. Performing a probability experiment
4. Mastering tallying the results of a probability experiment on a chart
5. Constructing a bar graph from the frequency chart of a probability experiment

in later chapters the learner will—

Master preparing a bar graph from a frequency table showing the results of a probability experiment

Notes & Things

The two words that can probably strike fear in the hearts of elementary school teachers more quickly than any other are *probability* and *statistics*. And for good reason. The college classes that dealt with those two subjects were many times the most difficult of any to get a decent grade in. A specialized text immediately launched into new symbols, an unbelievable number of formulas, and tables that went on forever. If any of you shared those experiences, the title of this chapter alone might be enough to make you want to postpone it as long as possible.

But you really don't have to worry. Start thinking about the weather report that you listen to every day. Think about that last consumer report that you read. Remember those valuable tables you use in the almanac, the last tax referendum, your insurance, the discussion about the population explosion, planning the next field trip, or the newspaper reports about the next election—and put your mind at ease. Probability and statistics function in our everyday world to make our lives a little easier. Children need to know some of the very basic ideas so that they can control some of the information that comes to them every day.

The exploratory chapter will develop probability concepts from a game approach. You won't find any functional terminology to be memorized. But you will find a lot of experimentation to be done. An activities approach is employed to introduce the learner to making predictions of the outcome of an experiment that are based on information, not merely on a wild guess. The likelihood of an outcome is examined. Learners perform an experiment, tally results, and then prepare a bar graph to illustrate the results—sophisticated ideas—with no emphasis on technical language.

Independent mastery of learning objectives is not intended with this chapter. The purpose is to build readiness for work at the next level and to develop critical thinking.

things

for each group: 15 beads of one color, 15 of another; container; rubber ball; baskets, pails or boxes of two sizes; container with soft plastic top; 3 circles to match spinners on pupil page 280; heavy straight pin; arrow indicator
for each pupil: coin, graph paper, crayons
for each pair of pupils: 2 pencils or wood cubes, felt marker

For the extra activities you will want to have these things available:
- bottle caps
- plastic spoons and cups
- game board of pictures

goal Think about and explore ideas through a picture clue

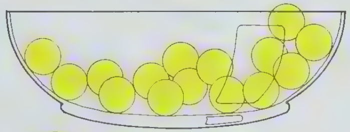
page 278 Probability is still a part of weather forecasting even though the satellite that is circling earth is a tattletale on mother nature. The premium we pay for different kinds of insurance, for example, is directly related to the probability of a car accident, the probability of illness, or the probability that each of us lives as long as he is expected to live. Las Vegas is built on the laws of probability and we make many decisions based on our intuitive sense of probability. All of this is too complicated for children, but keeping the weatherman honest is certainly something that would be fun for everyone.

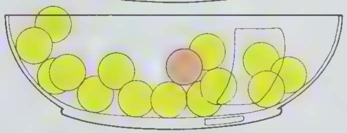
If some youngsters don't have TV in their homes, they will have the job of listening to weather reports on radio. If there is more than one TV station, get someone to monitor each weatherman. What does each weatherman say about the five-day forecast? about tomorrow's forecast? What actually happens to the weather over a five-day period? Share the information and find out which forecaster was right most days. Be sure to challenge the children to wonder if the same results would be true the next week. That, after all, is the essence of probability that you want your learners to know.

Suppose you look outside on a hot day.
Dark clouds cover the sky.
Thunder is booming. Would you expect rain? Yes

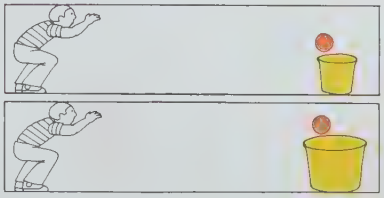
Suppose instead that the sun is shining. There isn't a cloud in the sky. Would you expect rain? No

Do you ever watch the weatherman on television? He tries to predict tomorrow's weather. (A prediction is a guess about what's going to happen in the future.) Often he's right. Is he ever wrong? Yes





1. Let's predict some things that are simpler than the weather. The picture shows a bowl of candy. There are 15 pieces. All are lemon-flavored. If you reach in and pull one out, what flavor will you get? Lemon
   - a Suppose instead that the bowl has 14 lemon candies and 1 root beer candy. If you pull one out without looking, what flavor would you predict you'd get? Probably lemon
   - b There are many more lemon candies than root beer candies. So you're *more likely* to pull out a lemon candy. Does this mean you couldn't pull out the root beer piece? No



2. Here's a simple game. You stand near a pail. Try to bounce a ball into the pail.

   Suppose you use a bigger pail, like the one in the picture. Will this make the game easier or harder?

What Are the Chances? Your goal is to learn.

279

lesson Page 279

goal Thinking about PREDICTIONS and examining the LIKELIHOOD of an EVENT

memo Experience is worth a thousand words. The activities presented on the page are fun and certainly a change of pace for math time.

things for each group:
15 beads each of 2 colors
container, rubber ball
baskets, pails, or boxes of 2 sizes

page 279 *What do you predict the weather will be today? Will it rain? snow? . . . On what do you base your prediction? . . . Does this mean it will not* ***rain*** (or whatever is appropriate)? *Does the weatherman simply guess?* Sometimes it seems as if he does.

Emphasize that a **prediction** does not mean that the event will actually happen. Also, we usually base our predictions on some information.

The candy-bowl situation may be simulated. Have small groups carry out this experiment. Continue with small groups for the second experiment. This is an excellent recess activity. Compare the results of the various groups.

Extend the bead experiment by varying the number in each color group. How does this affect the likelihood of selecting a given color? The results should be tallied on the chalkboard.

lesson Pages 280, 281

goal Making and TALLYING predictions for a probability experiment

memo Sure the page can be completed as a discussion, **but** it's more fun to actually perform the experiment. Note that pages 280 and 281 work together.

things for each group:
- container with soft plastic top
- 3 circles to match spinners in text
- heavy straight pin
- arrow indicator

page 280 Spinners can be easily constructed by using any container with a soft plastic top such as one from margarine. Cut sets of three circles of colored paper the same size as the top. Now shade these circles to match the shaded part of the three spinners illustrated in the text. Fit a color section like that in exercise **A** inside each lid. Snap the cover back onto each container. Attach an arrow indicator with a heavy straight pin. This is an excellent construction project for the youngsters.

Divide into small groups—each group with a spinner. Have pupils record their prediction for the spinner. Save these predictions for page 281. Each group member then has one chance to spin the spinner. Have them record the results.

Change the color sections to match exercise **B** and repeat making a prediction and doing the experiment. Change again to match exercise **C**. Then compare with our results.

Lots of games use spinners. You spin the spinner. When it stops, you know how far you can move.

Here's a picture of a spinner. Part of it is plain. The other part is shaded. Do you think the spinner is more likely to stop on the plain part or the shaded part?

Try your prediction power on the following problems. Each spinner has two parts. If you could spin the spinner, where do you think it will stop? Predict whether it will stop on the plain part or the shaded part. Then look on page 288. You will see where the spinner stopped when we spun the spinner. Answers will vary.

280

Make a chart like the one below. Record how well you predicted where the spinner would stop.

Charts will vary.

	right	wrong
a.		
B.		
C.		
D		

1. Which group did you predict the best? Why? Compare your record with the records of some of your neighbors. If someone did much better than the others, try to figure out why. Results will vary.

2. Look at the spinners in group A. The shaded part is bigger than the plain part. So the spinner is more likely to stop on the shaded part. In group B, is it more likely to stop on the plain part or the shaded part?

3. In group C the parts are exactly the same size. In this case the spinner is as likely to stop on the shaded part as on the plain part. "Shaded" and "plain" are equally likely outcomes.

goal Comparing predictions; examining the likelihood of an OUTCOME

page 281 How good a predictor is each person? Use either the results of the experimentation or the results on page 288. Do listen to some of the reasoning on why one youngster did much better than the others.

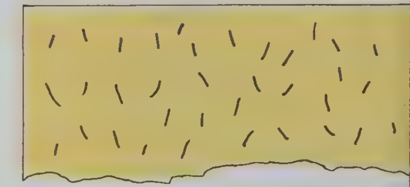
lesson Pages 282, 283

goal Developing a system for tallying

memo Let's talk about this page together.

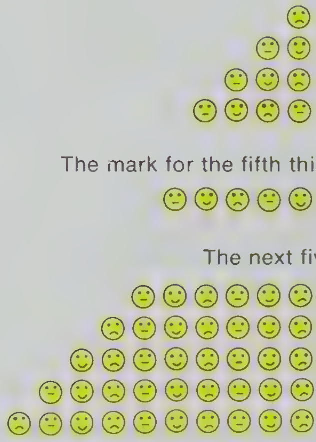
page 282 The focus is on learning how to tally and organize information. Point out STANDARD NUMERAL and make sure that the pupils understand its meaning. Sure there are other systems. The one developed here is widely used and generally understood.

Ruth had to count the people in the class play. She put a mark on a piece of paper for each person in the room. Soon her paper looked like this:



It wasn't easy to figure out how many there were!

A better way to count is to *tally* the results.
You make a short vertical mark for each thing being counted.



Things	Tally	Standard numeral
	|	1
	||	2
	|||	3
	||||	4

The mark for the fifth thing is put across the marks for the first four.

Things	Tally	Standard numeral
	~~||||~~	5

The next five are tallied in the same way.

Things	Tally	Standard numeral	
	~~||||~~ |	6	(5 + 1)
	~~||||~~ ||	7	(5 + 2)
	~~||||~~ |||	8	(5 + 3)
	~~||||~~ ||||	9	(5 + 4)
	~~||||~~ ~~||||~~	10	(5 + 5)

1. The tally for 11 looks like this: ~~||||~~ ~~||||~~ |

   a Show 12 with tally marks. ~~||||~~ ~~||||~~ ||
   b Show 13 with tally marks. ~~||||~~ ~~||||~~ |||
   c Show 15 with tally marks. ~~||||~~ ~~||||~~ ~~||||~~

2. Write standard numerals for each of the following.

   a ~~||||~~ |||| 9
   b ~~||||~~ ~~||||~~ ||| 13
   c ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ | 21

282

Tally the number of girls in your class. Then tally the number of boys. Results will vary.

Tally the number of times the letter *e* appears on this page. 67

Flip a coin ten times. Tally the number of heads you get. Results will vary.

Suppose you flip a coin six times. How many times do you think it will be heads? How many times do you think it will be tails? About 3 About 3

Actually flip a coin six times.

a Tally the number of heads. Results will vary.

b Also tally the number of tails. Results will vary

c Was your prediction correct? Answer will vary.

Repeat the experiment. That is, flip a coin six more times. Tally the number of heads and the number of tails. Were your results the same? Probably not, but they could be.

Flip a coin 30 times. Tally the number of heads. Is your result what you expected? Answers will vary

Look at your friends' results for problem 7. Make a tally like this Show how many got 12 heads, how many got 13 heads, and so on. Which result came up most often? Answers will vary

12 heads ///
13 heads ~~////~~ //
and so on

283

goal Practice in tallying results from an activity

things for each pupil:
a coin

memo This is an activity page.

page 283 Most coins in circulation are not perfectly balanced. The force with which the coin is tossed also affects the outcome. At least a hundred tosses are necessary to indicate whether each outcome is **equally likely** or whether one outcome is **more likely.** The greater the number of trials, the better the prediction.

You may wish to repeat problems 4 and 5, increasing the number of tosses substantially. Thirty tosses are required for problem 7. As the number of tosses increases, pupils may want to change their predictions. Have pupils work in pairs for this activity.

things bottle caps; plastic spoons and cups

Vary the coin-toss experiment by using any one of the above. *Which way will the cap* (or whatever) *land most often—up or down?* Make a prediction. Perform the experiment. Tally the results. *Was your prediction correct?*

See activity 1, page 288a.

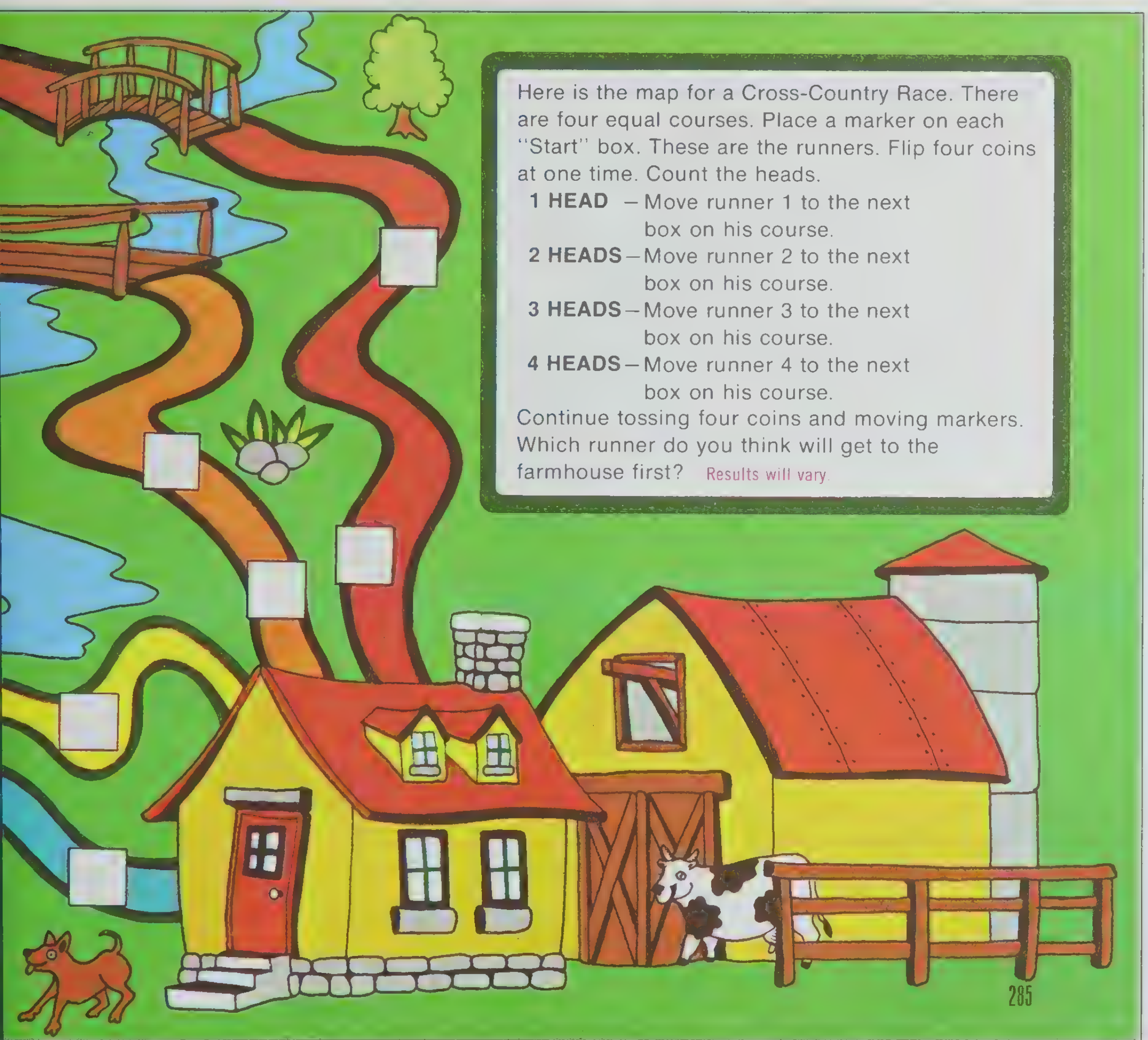
goal Performing a probability experiment that has four possible outcomes

memo The activity lends itself to groups of four, pairs of students, or individual organization. Note that pages 284 and 285 work together.

page 284 Before beginning the coin toss for the race consider the possible outcomes of tossing four coins at the same time. What are these possible outcomes?

Heads	0	1	2	3	4
Tails	4	3	2	1	0

Perhaps your pupils will want to try a few tosses before making a prediction. If they don't think of this themselves, prod them. One objective of these activities is to develop critical thinking. Predictions should be based on information.



goal Comparing results of a probability experiment

memo Not everyone will know the meaning of **course** as it is used on this page. Ask if anyone has heard of a golf course. If someone has, ask what they think a golf course is. If you can't get the answer that a course is a path that has points marked on it and if the word continues to bring blank looks, say to heck with it and substitute the word path or road.

page 285 Concentrate only on the course that brought in a winner most often. A tally chart will help organize this information.

Course	1	2	3	4
Winner				

Each time the course brings in a winner make a tally in the box below. Each group or person who performed the experiment will need to report their winner. How do the results compare with the predictions?

lesson Pages 286, 287

goal Performing a probability experiment and making a tally chart of the results

memo The results of the experiment on page 286 are necessary to complete page 287. Have pupils work in pairs for the experiment.

things for each pair of pupils:
2 pencils or wood cubes
felt marker

page 286 Pencils or cubes can be taped with adhesive tape or numbered directly with a felt marker. Pair pupils so that tasks may be shared. The objective is to perform the experiment described and make a tally chart of the results. Have the "tallier" prepare his chart headings before the pencil roll begins. List all possible sums. The tallies will have to be totaled before the graph on page 287 is made.

Sum	Tally	Total
2		
3		

1. Get two six-sided pencils.
   Number the sides of each pencil from 1 through 6.
   (Put a different numeral on each side.
   Instead of pencils, you can use cubes.)

   a Roll one pencil. When it stops, what number is on the top side?
   Results will vary.

   b Roll the other pencil. When it stops, what number is on the top side?
   Results will vary

   c What is the sum of the two numbers you just rolled? Results will vary.

2. Roll the two pencils again. What is the sum of the numbers on top this time? Results will vary.

3. If you roll two pencils, what's the smallest sum you could get? What's the largest? 12
   2

4. Make a chart like the following. Put in all the sums you could get by rolling two pencils.

SUM	TALLY
2	|||
3	
4	

   a Now roll both pencils together 36 times.
   Record the number of times each sum occurs.
   (Use your chart. Make tally marks.)

   b What sum did you get most often? least often? Results will vary.
   Results will vary.

286

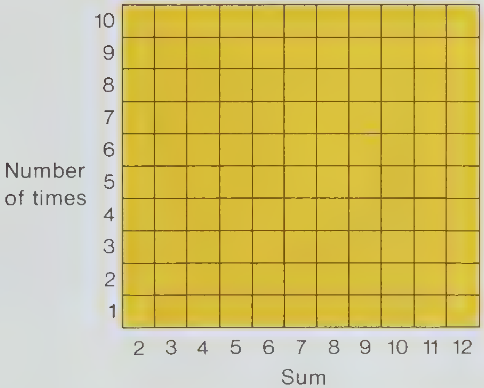
See activity 2, page 288a.

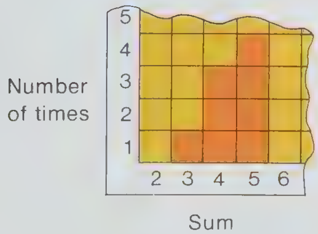
Copy this chart. Use graph paper. Or draw the lines yourself.

a How many times did you get the sum 2? Find 2 along the bottom of the chart. Shade in one box above 2 for each time you got the sum 2. Then shade in boxes above 3 to show how many times you got the sum 3. Do the same for 4, 5, and so on. Results will vary

Example:

SUM	TALLY		
2		=	0
3	|	=	1
4	|||	=	3
5	||||	=	4





b The graph you made is called a bar graph. Is the sum that was rolled most often the easiest to find on your graph? the sum that was rolled the least often? Explain. Yes Yes The sum rolled most often has the highest bar The sum rolled least often has the shortest bar

If you roll the pencils only once, what sum would you predict? Why? 7 There are more combinations whose sum is 7 than any other number

287

goal Showing the results of a probability experiment on a bar graph

things for each pupil:
graph paper
crayons

page 287 Making a graph may be a new experience for some youngsters. Examine the labels necessary for the graph. Each group member should have the experience of preparing his own graph. Use crayons to shade the number of boxes.

Compare completed graphs. Were the results from all groups identical? Which sum was rolled most often? least often?

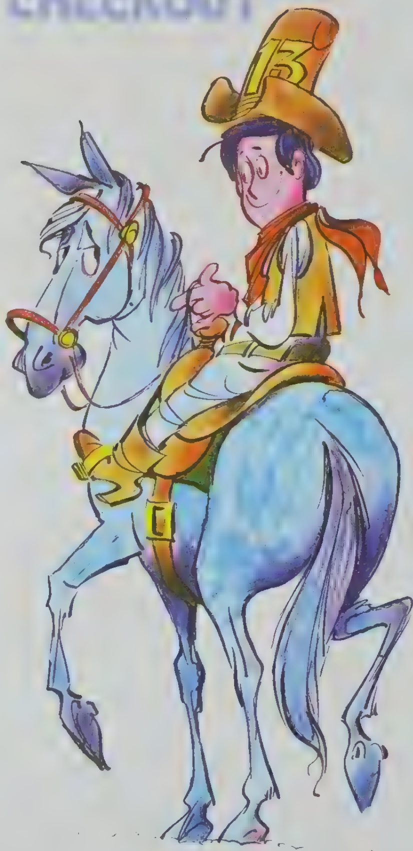
Will this information help with predicting the outcome from rolling the two pencils only once?

lesson Page 288

goal **Checkout** – determining equally likely outcomes; performing a probability experiment, tallying results, and making a graph to show these results.

page 288 The focus of this exploratory chapter is on concept, on attaining a feeling of confidence. Independent mastery is not an objective at this time. You might consider continuing the small-group work for the Checkout. Circulate. Observe those pupils who require additional experiences.

CHECKOUT



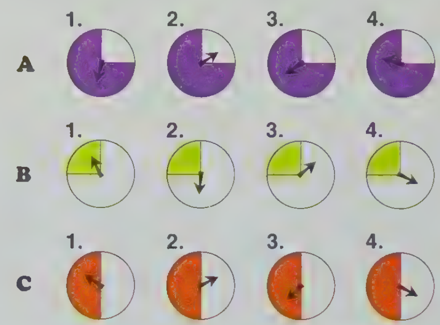
Skill: Determining likely outcome

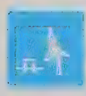
1. Bob slid into second base. He said he was safe. The second baseman said he was out. Since there was no umpire, they decided to flip a coin:
   Heads ⟶ Bob is out.
   Tails ⟶ Bob is safe.
   Are "out" and "safe" equally likely outcomes? Yes

Skills: Performing experiment, making tally chart, making bar graph

2. If you roll three pencils, what's the smallest sum you could get? 3
   - **a** What's the largest? 18
   - **b** Roll them 36 times. Make a chart of the sums. Results will vary.
   - **c** Make a bar graph to show the sums. Bar graphs will vary.

This is where the spinners on page 280 stopped.





See activity 3, page 288a.

things for each group: 1 die (or cube with faces numbered from 1 through 6)

Rules: Roll the die and tally which side lands faceup. Make a bar graph to show the results.

Variation: Use a pair of dice (or numbered cubes) and multiply the 2 numbers.

RESOURCES

another form of evaluation

for Checkout—page 288

1. A bag of candy has 3 pieces of peppermint and 3 pieces of licorice. If you reach in and get a piece, are you equally likely to get a peppermint as a licorice? Yes
2. Another bag has 3 red-hots and 4 jellies. If you reach in this bag, are you equally likely to get a red-hot as a jelly? No
3. Get 2 dice.
   a) If you roll them, what sums could you get? 2 through 12
   b) Roll them 24 times. Make a tally chart of the sums. Results will vary.
   c) Make a bar graph of the sums. Graphs will vary.
   d) What sum did you get most often? Answer will vary.

activities

1. things equal quantities of beads (or marbles) of two colors; container

Rules: Select 2 beads at a time. Are they the same color? Are they different colors? Predict which outcome will happen most often. Perform the experiment. Replace the beads after each draw. Tally the results. *Was your prediction correct?*

Variation: Use equal quantities of straws cut in 2 lengths. Hold so that they all appear even. Draw 2 at a time. Replace straws after each draw.

2. things beads (or marbles) of two colors (20 of each color) in a container; 3 smaller containers

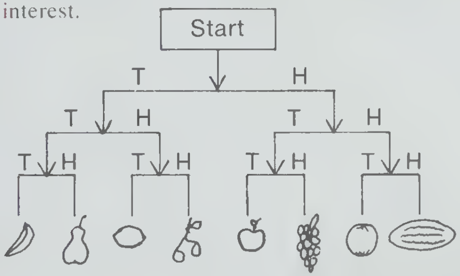
Mix equal quantities of two colors of beads (or marbles) in a container. Label the small containers: yellow, mixed, green (or whatever colors are used).

Rules: Select 2 beads at a time. If both are of the same color, place them in the appropriate container. If one of each color is drawn, they go in the container marked "mixed." Predict which container will have the most beads when all the beads have been drawn.

Perform the experiment. Tally the results. Was the prediction correct?

3. things penny; marker; game board

Have each pupil prepare a game board as shown. The pictures can be of anything of interest.



Rules:
- Begin at start. Follow the path to an arrowhead.
- Toss your penny. If heads land up, turn right. If tails land up, turn left.
- Follow the path, placing your marker on the next arrowhead. Toss your penny again to decide which way to turn.
- Continue until you reach a picture.
- Make a tally chart. Record where you landed.
- Play the game again. Record the results.
- Continue to play until you reach each picture.

How many times did you have to play the game in order to reach each picture? Compare your results with those of others who played the game too.

additional learning aids

concept—chapter objectives 1, 2, 3

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
 Spirit masters: S 1, 2
Mathematics Involvement Program, SRA (1971)
 Cards: 364, 367

notation—chapter objectives 4, 5

other learning aids (described on page 228g)
Block Graph, Good Time Mathematics, Histogram Board, Probability Maze

multiplication

A game for two, three or four people.

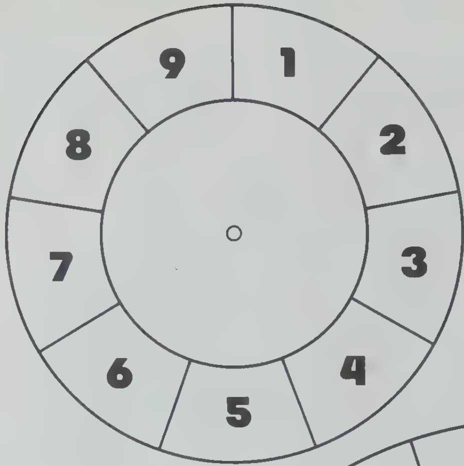
Here is what you play with:
(1) A playing board
(2) One token for each player
(3) A spinner marked off into nine equal parts numbered 1 through 9

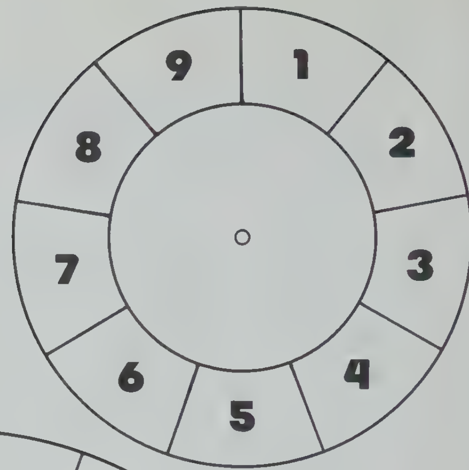
Each player places his token on START. The first player spins the spinner and says aloud the number it stops on. He spins again and tells the number it stops on. Then he multiplies these two numbers, calling out their product. The last digit in the product is the number of squares his token is to move. When he finishes, it's the next player's turn.

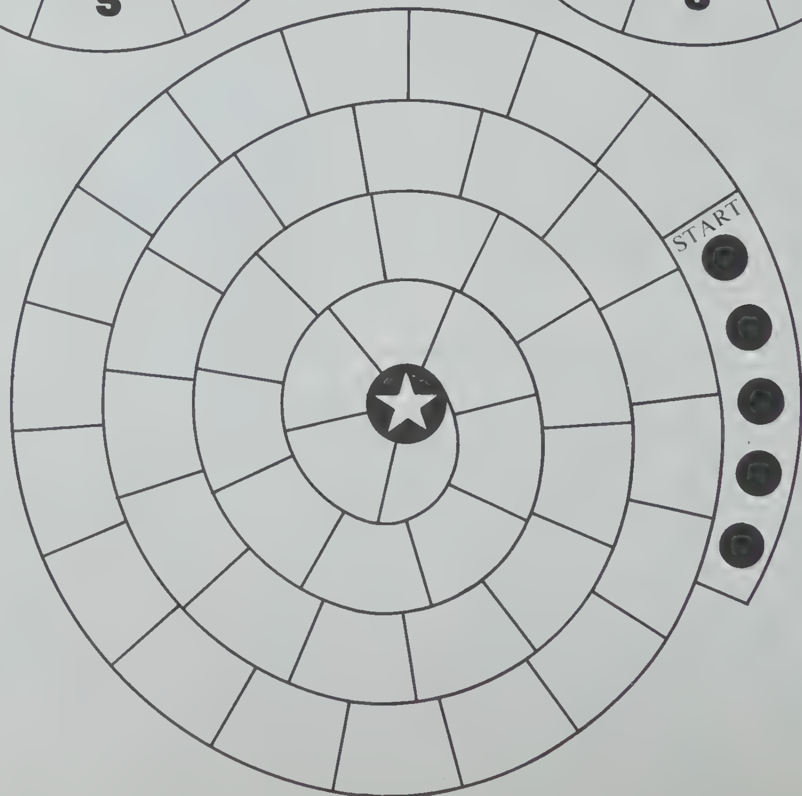
If a person lands his token on a square occupied by another token, he gets an extra turn. If he makes a mistake in multiplying so that he lands on the wrong square, his opponents can correct him *after he finishes moving.* The player who makes the mistake must retract his move and lose his turn.

The game continues until some lands in the center of the spiral by exact count. That person is the winner.

A player can be too close to the center to move the full number of squares indicated by spinning and multiplying. For example, if a player is two squares away from winning, he can't move eight squares if he spins 7 and 4. He uses up his turn without moving. If he's on a square with another token, he doesn't get an extra turn.







puzzle pasttime

+	1	2	3	0	5	6	7	4	8	9
0										
9										
2										
7										
4										
5										
6										
3										
8										
1										

1	2	3	0	5	6	7	4	8	9
10	11	12	9	14	15	16	13	17	18
3	4	5	2	7	8	9	6	10	11
8	9	10	7	12	13	14	11	15	16
5	6	7	4	9	10	11	8	12	13
6	7	8	5	10	11	12	9	13	14
7	8	9	6	11	12	13	10	14	15
4	5	6	3	8	9	10	7	11	12
9	10	11	8	13	14	15	12	16	17
2	3	4	1	6	7	8	5	9	10

To save time and work, remove this sheet from the guide. Paste it on light-weight cardboard. Cut out the board and the pieces. The puzzle can be stored in an envelope. No directions are needed. The children will know what to do. (The puzzle is more of a challenge than it appears. Why don't you try it after the pieces are cut out and scrambled?)

puzzle pasttime

×	1	3	5	7	9	0	2	4	6	8
1										
3										
5										
7										
9										
0										
2										
4										
6										
8										

1	3	5	7	9	0	2	4	6	8
3	9	15	21	27	0	6	12	18	24
5	15	25	35	45	0	10	20	30	40
7	21	35	49	63	0	14	28	42	56
9	27	45	63	81	0	18	36	54	72
0	0	0	0	0	0	0	0	0	0
2	6	10	14	18	0	4	8	12	16
4	12	20	28	36	0	8	16	24	32
6	18	30	42	54	0	12	24	36	48
8	24	40	56	72	0	16	32	48	64

To save time and work, remove this sheet from the guide. Paste it on light-weight cardboard. Cut out the board and the pieces. The puzzle can be stored in an envelope. No directions are needed. The children will know what to do. (The puzzle is more of a challenge than it appears. Why don't you try it after the pieces are cut out and scrambled?)

14 GEOMETRY CONGRUENCE

before this chapter the learner has—

1. Sorted figures and objects that are alike in one or more ways
2. Decided how two or more objects are alike or different
3. Had tracing experiences
4. Measured with a ruler and used a straightedge
5. Performed paper-folding activities

in chapter 14 the learner is—

1. Making a congruent model of a plane figure by tracing or by paper folding
2. Testing a plane figure for congruence by tracing or measuring with a ruler
3. Determining whether a plane figure has two congruent halves by folding
4. Making patterns, using one or more congruent shapes for the pattern

in later chapters the learner will—

1. Master finding a line of symmetry by folding
2. Master identifying congruent figures by tracing and matching

Notes & Things

Congruence is a big word, and congruence is a big idea in geometry. This chapter will explore the idea and provide a wide variety of activities simply to get acquainted with the ways that congruence is used in our everyday world.

In the study of geometry that comes at a later level, the learners will find that congruence is a property that belongs to two-dimensional figures. They will find that one figure is congruent to another if one is the image of the other. There is plenty of time later for that level of abstraction. For now the emphasis will be on things that are the same size and shape. The exploration will focus on two-dimensional shapes, but congruence certainly applies to three-dimensional objects as well. There will also be time later to investigate that idea.

This is an activity chapter. There are lots of things for individuals as well as small groups to do. There are lots of ideas to talk over. And there are some pretty tricky problem-solving situations scattered throughout. It is hoped that the idea of congruence will take on real meaning and, most importantly, that everyone will enjoy this chapter.

things

pair of gloves
boxes
½″ squared paper
crayons
centimetre ruler
for each pupil: paper for tracing, rulers, scissors, paste, paper of two different colors, 2 circular regions (coffee filters), graph paper

goal Think about and explore ideas through a picture clue

page 289 Not many people have had the chance to visit a factory where items are mass produced. If any of your pupils have, they probably already have an understanding of same size, same shape. The idea of congruence should be reserved for two-dimensional shapes, but who can pass up such a good opportunity to relate a math idea to the real world?

It's doubtful if the youngsters will immediately identify these objects as part of a seat-belt buckle. That's O.K. We would all have a hard time telling what the individual parts of a manufactured item are. The photograph is intended to get the pupils thinking about the manufacturing process. What things are manufactured? Are they made in one piece? Can you guess how the machines might work?

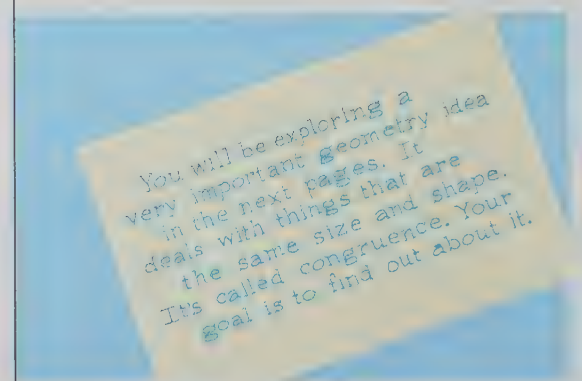
And the resulting research can be an interesting task for everyone. Find a worn-out or discarded manufactured object that has more than one part. Take it apart. Tell the function of each part. Tell how you think it might have been made, step-by-step.

A visit to a manufacturing plant sometime during the study of this chapter would be a real bonus if it could be arranged.

goal Survey – identifying likenesses and differences; establishing the learning goal of the chapter

page 290 You may want to examine pairs of students. Ask the same or similar questions as in the text. Watch the word **could** in the questions about shoes and shirts. Does this mean that they **do** have the same size shoes?

Look for objects in the room that are the same size or the same shape. Discuss their likenesses and differences. For example, compare a sheet of notebook paper a pupil has with a sheet of paper you have. Then compare two like sheets. One pair of sheets may be CONGRUENT. Decide after more has been learned about CONGRUENCE. Let the pupils hear both words even though they will not be used for several days. For now the emphasis is on things that are the same size and shape.



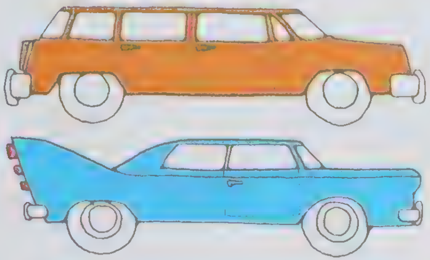
Bill and Tom want to see whether they are the same height. They measure each other. They find they are both 4 feet 6 inches tall. Then they weigh themselves. Tom weighs 8 pounds more than Bill. They have found that they are the same in one way but different in another.

How are they the same? Same height
How are they different? Different weight, different features

Could the two boys have shoes of the same size? Ye
Could they wear the same size in a shirt? Maybe

In what other ways could they be alike or different
Answers will vary. Examples: color of hair, eyes, intelligence, strength, personality, etc.

1 Bill and Tom decide to find other things that are very much alike. Their fathers each have a car. One car is newer than the other. One car is a station wagon. The other is a sedan.



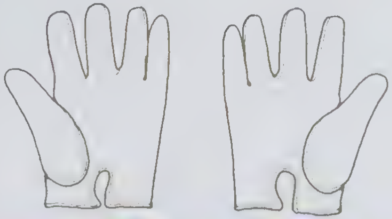
a Are the cars the same size? Same length

b Are they the same shape? No

c Is one bigger inside than the other? Yes

d In what other ways can the two cars be compared?
Speed, color, horsepower, gas consumption, etc.

2 Fred has a pair of gloves. He says they are the same. Are they? The gloves are the same size, but one is for the right hand and the other is for the left hand.

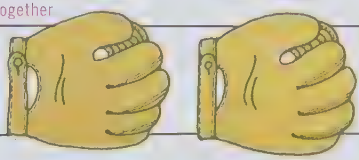


a Are they the same shape? No

b Are they the same shape if you flip one of them over? No

c Are they exactly alike? No
Consider valid discussion on the pieces being alike before they are sewn together

3 Here are pictures of two baseball gloves. They are both fielders' gloves. Are they both for the same hand? How are they different?
Yes Could have different room inside, etc.



4 Bill is setting the table for his mother. He wants the table to look nice. He uses glasses that are alike. Tell which three he used.

5 Name some pairs of things in your kitchen at home that are the same size and shape.
Answers will vary. Examples: cups, plates, forks

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goal Examining pairs of things to determine whether they are the same size and shape

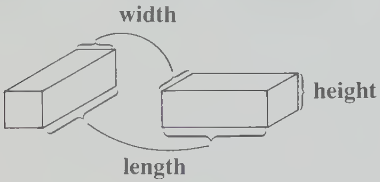
things pair of gloves

page 291 Careful of part **a** in problem 1. **Size** in cars may mean price range to some, compact versus full size, overall length, weight, or wheelbase to others.

It may be necessary to examine a real pair of gloves to satisfactorily answer the questions in problem 2. Rubber gloves can easily be turned inside out. A change in color will show as well, and this will help.

goal Verifying size, shape, and height of a 3-dimensional shape by measuring

memo Be careful of words. We use **length, width,** and **height,** without remembering that these words may be new to the youngsters. Length is usually the longest dimension on the surface of a rectangular solid. Width is the shorter, second dimension on the surface. Height is used most often in geometry to name the third dimension, although thickness or depth are equally as appropriate.



For a box, the word **height** would sound best. If it were a picture of a book, perhaps **thickness** would be a better descriptive word. If it were a hole in the ground, most people would call it **depth.**

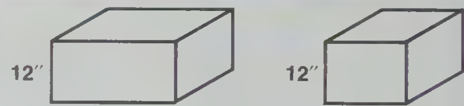
things rulers and boxes

page 292 Before anyone can determine whether two objects actually are the same size and shape, some measurements will have to be compared. In the case of 3-dimensional objects, the measurements of length, width, and height of both objects will have to be made and compared.

Hands-on experience with boxes is important. Youngsters have difficulty "seeing" a 3-dimensional object shown on a 2-dimensional page.

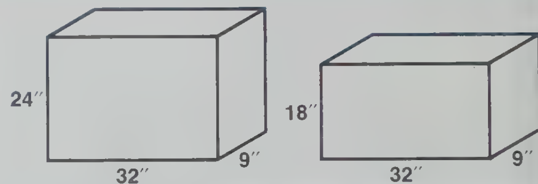
1 Bud has a box. He wants to find another the same size and shape. He will look for a second box at the grocery store. But he does not want to take his first one along. How can he be sure the box he finds is the same size and shape? Measure height, width, and length.

a Bud measures his box and finds it is 12 inches high. Can he be sure another box 12 inches high will be the same as his? Are these boxes the same size? No
No



b He measures his box and finds it is 26 inches wide. This will help him find what he wants. What else must he know?
Length

2 Same length and width
How are these boxes the same?
How are they different? One is higher.



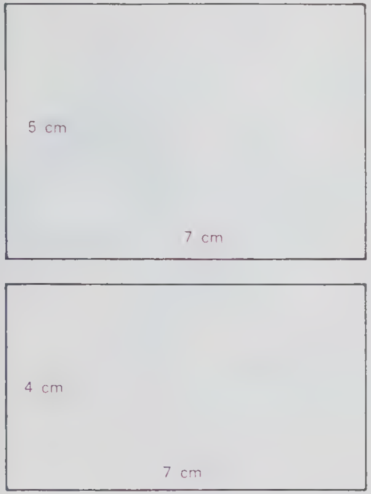
3 Measure one of your books. How wide is the front cover? How long is the front cover? How thick is the book? Answers will vary.

a Now measure a book of a friend. Are the measurements the same? Are the books the same size and shape? Answers will vary.

b Is your teacher's book different from yours? What is the difference? Answers will var
Greater thickness (

1

Measure the two rectangles below. Write the measurements on a sheet of paper.



How are the two rectangles the same? Are they exactly the same? Are the opposite sides of each rectangle the same length? Yes

Same length
No

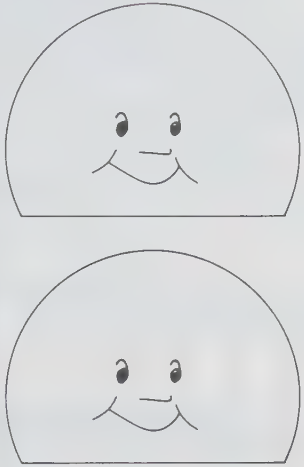
2

Take a piece of notebook paper and draw this shape on it.

The two shapes will not be the same unless you trace. How can you check whether they are the same size and shape? By placing one over the other.

3

Sally cut out place cards for her party. She wanted them the same size and shape. Here are two of them. Are they the same size and shape? Yes



Two things that are exactly the same size and shape are *congruent.*

Are the place cards pictured above congruent? Yes

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lesson Pages 293, 294

goal Making a congruent figure by tracing

memo Congruence is a property of 2-dimensional figures, but the idea of congruence applies to many everyday objects that are 3-dimensional. When the book refers to the mathematical concept, the models will be 2-dimensional. The emphasis starts on this page.

things paper for tracing

page 293 Careful of the questions at the end of problem 1! The rectangles are the same length, but they are not exactly the same because they have different widths.

A simple test for congruence in problems 2 and 3 is to make a tracing (a movable model) and lay it over the figure to be tested. Some learners may have difficulty making an exact tracing because of poor muscular control. This is not unusual.

When these youngsters answer that their tracing and the figure are not congruent, they could be correct. If they understand the concept of congruency, do not belabor this point. Simply accept their answer as true for that particular model. More tracing experiences will help correct the situation.

goal Testing for congruency by tracing or by measuring

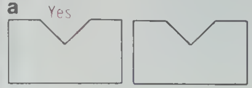
page 294 The discussion-activity pages continue. Tracing is theoretically the best test for congruency because of possible error in measurement. But since this is exploration and since the pupils aren't too good at tracing or measuring, why not provide both methods? Let them use whichever method is more appropriate for a given problem. But don't let them get by with a quick answer that is not verified in some way.

If you don't have good answers to problem 8, we are all in

TROUBLE

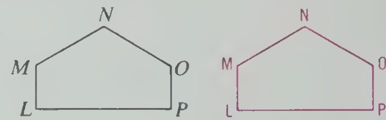
1. Is your right shoe congruent to your left shoe? No (The soles might be if you flipped one.)
2. Is the cover of your math book congruent to your neighbor's math book? Yes, if it's the same text.
3. Are these figures congruent?

a Yes



b No

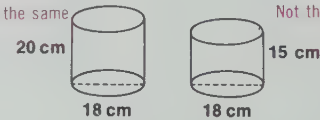
4. Name five pairs of things in your classroom that are congruent. Answers will vary. Examples: desks, chairs, rulers, shades
5. Draw a figure congruent to this shape. Label it the same.



6. How are these cans the same? How are they different?

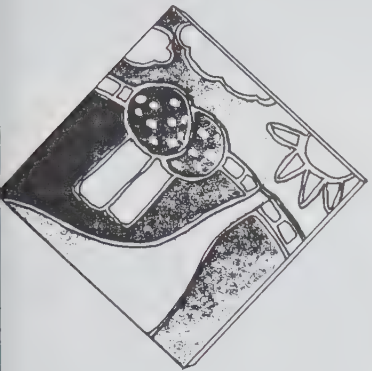
Top and bottom are the same

Not the same height



7. Name at least two things that are alike in some way but that are not congruent. Answers will vary. Examples: bike wheels, books, coins
8. Name at least five pairs of things in your classroom that are alike in some way but are not congruent. Tell why each pair of things is not congruent. Answers will vary.

1. Alice Brandon loves old things, especially old furniture and things from old houses. One day she bought an old door with a beautiful window pane of stained glass. The pane of glass measured 60 cm on a side. Here is a drawing of it.



Alice wants to put the pane of stained glass in a window frame in her house. Can she do it?
Here is a diagram of her window frame.

Alice thinks the stained glass pane will not fit in the frame. Will it? Prove it. No. Trace and fit to see.

2. Do the following shapes look different? Are they congruent? Yes One is turned.

Trace the first shape.
Now turn the paper a little and see whether it fits over the second shape.

lesson Pages 295, 296

goal Examining congruent figures that have been turned

things paper for tracing

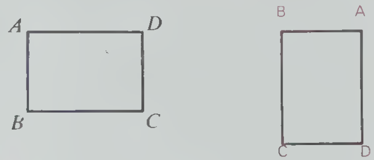
page 295 Another discussion-activity page. *Any ideas how we can prove whether Alice is right or wrong?* Suggest to the pupils that they can make a tracing of the window pane and then try to fit it into the drawing of the window frame. Can they do it? Encourage them to turn the figure any way they want. *Does turning help?*

goal Examining similar and congruent figures; identifying congruent figures that have been turned

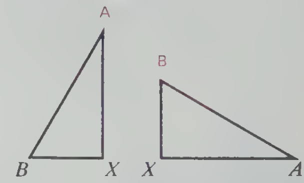
things paper for tracing
rulers

page 296 Careful of problems 1, 2, and 3. Before the figures are labeled, learners should check whether they are congruent. Only congruent figures can be labeled in exactly the same way. Tracings should be labeled exactly as the figure that was traced. Check problem 2 carefully. One side of each triangle is congruent, but the other two are not.

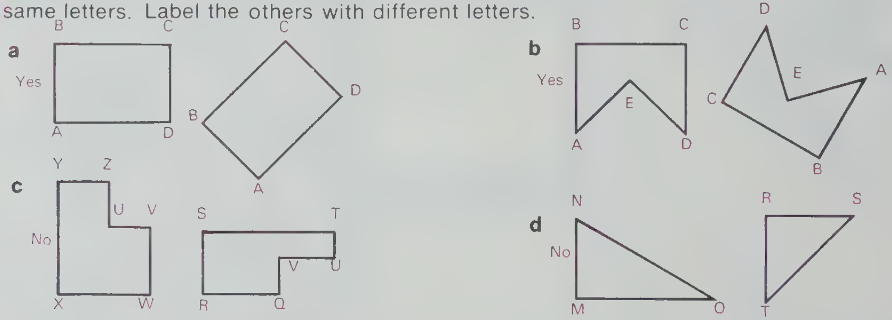
1 The first rectangle below is labelled *ABCD*. Trace the second rectangle and label it the same way. Are these two rectangles congruent? Yes
How can you tell? They match



2 Trace these two triangles and finish labelling them. They should be labelled the same way.
Are they congruent? No.



3 Which pairs of figures below are congruent?
Trace the figures. Label the congruent figures with the same letters. Label the others with different letters.

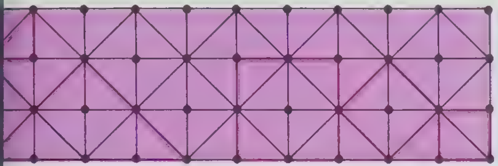


(Other letters might be used.)

4 Take two rules of the same length. How many centimetres long is each one? Measure the width of your mathematics book with the first rule. How many centimetres is it? Measure the width with the second rule. How many centimetres this time? Are your two answers the same? Why? Yes Centimetres are congruent units of measure.
Answers will vary.

296

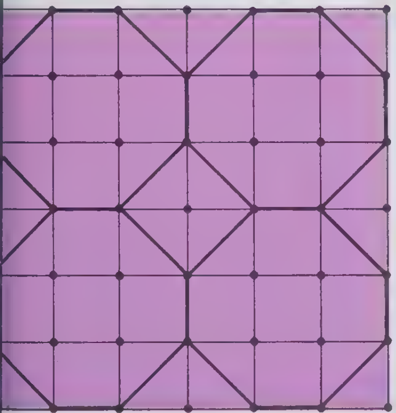
teresting patterns can be made on
quared paper using congruent shapes.
ere's an idea to get you started.



1 a Can you see congruent squares in the outline? If you made a pattern like this and colored the square regions, you would get one design. Yes There are 4 different sizes. Large one not marked.

b Look again. Can you see congruent triangles? If you colored the triangular regions, you would get another design. Yes. There are 4 sizes again.

c Now look for congruent rectangles. See them? Coloring the rectangular regions would make one more design. Yes – many different sizes



2 This is another interesting shape that could be used to make a design.

a Can you see congruent squares in the outline? Yes

b Can you see congruent triangles, too? Yes

3 Get some squared paper.
Make some of your own designs.

lesson Page 297

goal Recognizing congruent squares and triangles

things $\frac{1}{2}''$ squared paper
crayons

page 297 You'll quickly spot youngsters who have a geometric eye and those who may have a perceptual problem. Perhaps you will need to reproduce the triangular pattern at the top of the page on a spirit master. This will enable youngsters who are having a hard time to use crayons and experiment until they can make some sense of the whole thing.

The page should stimulate the youngsters to want to try for themselves. Half-inch graph paper, if you have it available, is easier to work with than the smaller size.

lesson Page 298

goal Exploring some properties of a square

things paper
centimetre ruler
scissors

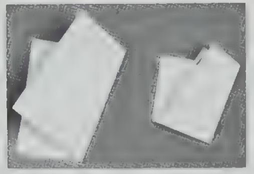
page 298 To make a square-corner tester, fold a piece of paper any way (as shown). Now make a second fold so that the first fold lies on top of itself. (Do not unfold the paper.) The corner that results from this folding is a square corner. The children may need to watch you make one before they make their own.

Directions for the remaining activities are self-explanatory. These activities lend themselves to small-group work. Problem 7 is a real challenge. Why don't you try it without peeking at the answer key? Finding all the shapes could take

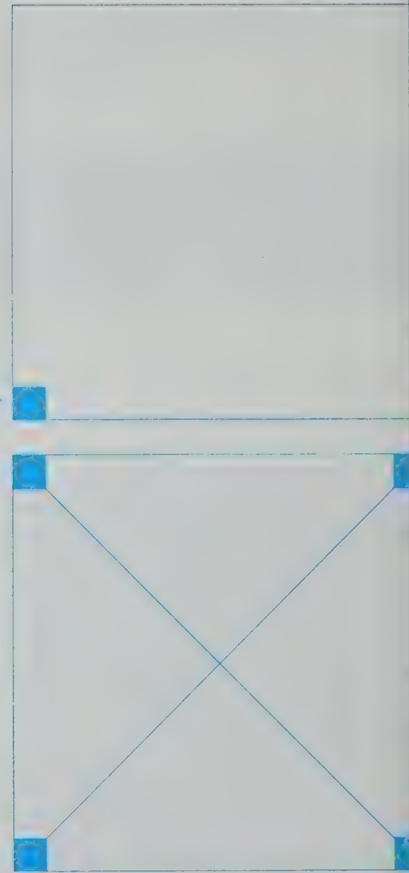
forever

1. Make a square with sides that are at least 6 cm long. The sides can be longer if you wish.

2. Make a square-corner tester by folding paper.



Mark each of the corners that is a square like this



3. Now draw straight line segments on your square that connect opposite corners. These line segments are called diagonals.

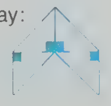
4. Measure each diagonal. Is one the same length as the other? Is one diagonal longer or shorter than one side of the square? Yes

5. Use your square-corner tester again. Look at the point where the diagonals cross. Can you find any square corners? Mark each one that you find in the same way you did the other square corners. Yes

All corners are square.

6. Cut on the diagonal lines. Make four triangles. Are the triangles congruent? Yes

7. Fit them together so that one edge matches another edge. You can fit them together in six different ways. Here is one way:

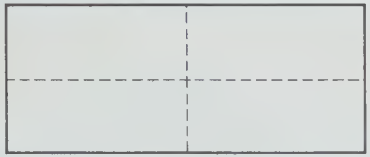


Find the other five.

298

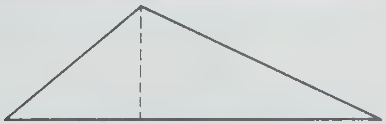
Some capital letters of our alphabet can be folded to have two parts the same. Which ones? Draw them on your paper. Then draw a line through each one to show that the two parts are congruent. A B C D E H I M O T U V W X Y (consider K)

Some shapes can be divided in more than one way. Each way makes two congruent parts. Trace this figure on paper. Fold the figure along one of the dotted lines. The two parts are congruent. Unfold and fold the figure along the other dotted line. Again the two parts are congruent.

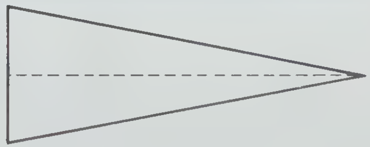


This is a special kind of triangle. Trace the triangle. Fold it in half along the dotted line. Do the two parts match? Yes

Are the two parts of this triangle the same? No
Trace to prove your answer.



Are the two parts of this triangle the same? Yes
Trace to prove your answer.



lesson Pages 299, 300, 301

goal Identifying figures with congruent parts

things paper for tracing

page 299 As the pupils make their folds, they will actually be finding **lines of symmetry.** This term will not be introduced in the written materials at this level. It is a word some learners may be familiar with from other sources. It's up to you whether you introduce it here.

Show which capital letters have congruent halves by having the pupils draw their discoveries on the chalkboard.

goal Finding some properties of an EQUILATERAL TRIANGLE

things for each pupil:
- paper for tracing
- 2 different-colored pieces of paper
- rulers
- scissors
- paste

page 300 The directions are self-explanatory. You might want to try the activity yourself before using it with the children. This will alert you to possible difficulties they could have. You'll need patience. Rulers will help keep the sides of the triangle straight for problem 2.

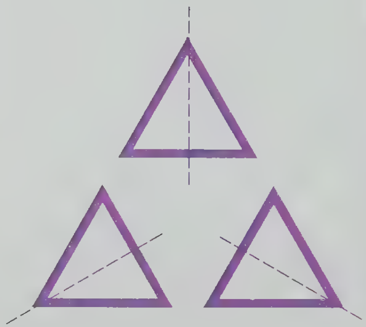
After completing the page, you may want the group to examine the triangles on pages 299 and 300. *Does anyone notice something about the triangles?* (3 sides congruent, 3 folds; 2 sides congruent, 1 fold; 0 sides congruent, 0 folds)

1

One kind of triangle can be folded in half in three ways. Each time the two parts are congruent.

Why can this triangle be folded in three ways? (Hint: Measure the length of each side.)
Each side is the same length.

A triangle with each side the same length is called an equilateral triangle.



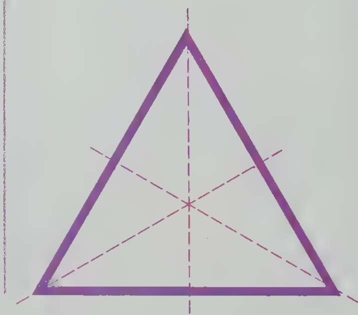
2

Is the triangle below an equilateral triangle? Yes

a Trace it. Then cut out your triangle. Fold it in the three ways shown in problem 1. Then cut along each fold mark.

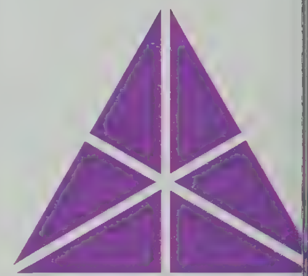
b How many triangles do you have now? 6

c Are the smaller triangles congruent? Yes



3

Get two different-colored sheets of paper. Make the same-size equilateral triangle on both sheets. Fold and cut each as you did before. You will have six triangles of one color and six of another color. Fit them together to make a pattern. When you find one you like, paste it on another sheet of paper.



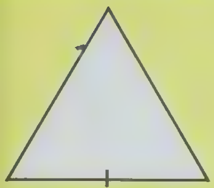
300

things tracing paper

Have the pupil trace the large triangle on the page and then make an **X** in one corner. *Remember for which corner you made the* **X**. *Find how many ways you can get the tracing to fit back on the triangle in the book.*

Turning, sliding, flipping the paper—all these motions are allowed.

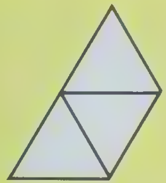
Repeat for the figures formed on pages 302 and 303.



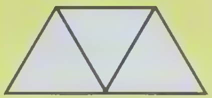
Crease here only.

Make another equilateral triangle. Cut it out.

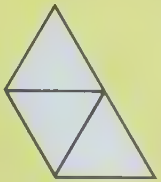
FOLD just one side so that you can find the halfway point on the edge.



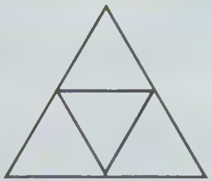
NOW FOLD the right corner up. Make a crease mark.



NOW FOLD the top corner of the triangle down to meet the halfway point. Make a crease. Then open the triangle again.



OPEN AGAIN. Then fold the left corner up in the same way. Open again. You should have three fold marks.



How many small triangles in the large triangle? 4
What fractional part is one small triangle of the large one? $\frac{1}{4}$ Cut on the fold marks so that you have four smaller triangles. Are the smaller triangles congruent? Yes

Designs like this can be made with triangles of these two sizes.

Can you find others?
There are many others.

goal Finding more properties of an equilateral triangle

things paper for tracing
rulers
scissors

page 301 The directions are self-explanatory. You may want to dictate the directions. This technique frees the learner to **do.**

Creative designs make excellent room decorations.

lesson Pages 302, 303

goal Making a square and an OCTAGON from a circular region

things for each pupil:
2 circular regions (coffee filters)

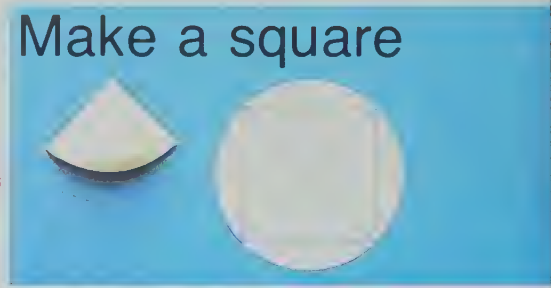
page 302 The directions are self-explanatory; but you are likely to avoid trouble if you dictate the directions, rather than have the children read them. Take advantage of this paper-folding activity to reinforce some fraction concepts:

- Halves folded in half (doubled) form fourths.
- 2 halves make a whole.
- 4 fourths make a whole.
- 2 fourths are the same as a half.
- Fourths folded in half (doubled) form eighths.
- 8 eighths make a whole.
- 2 eighths make a fourth.
- 4 eighths make 2 fourths.
- 4 eighths make a half.
- The more pieces folded, the greater the denominator.
- The greater the denominator, the smaller the size of the parts.

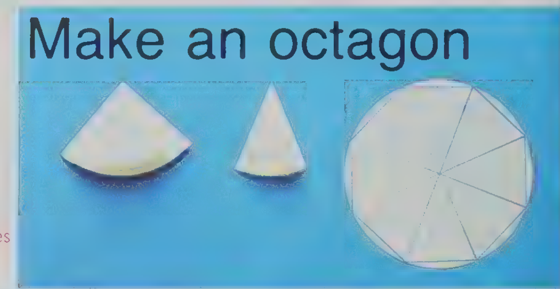
MAKING MORE SHAPES

Coffee filters are great to use for the next activity. But any circular piece of paper will be fine.

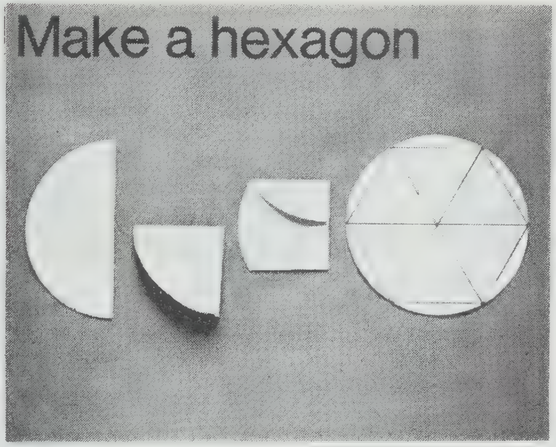
1. Use a circular shape.
Fold into fourths. Open it out flat.
Mark the fold marks with pencil.
Then connect the points on the edge.
Also mark diagonals of the square.
How many triangles are formed? 4
Are they congruent? Yes



2. An octagon has eight sides.
Use another circular shape.
Fold into fourths. Then fold in half again.
You will have folded the shape into eighths.
Open it out flat.
Mark the fold marks as before.
Then connect the points on the edge.
How many triangles are formed? 8
Are they congruent? Yes



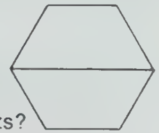
Make a hexagon



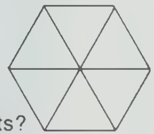
1. **A hexagon has six sides.**
Make one. Use another circular shape.

Fold it into fourths.
Open it that so one-half shows.
Fold edges to meet at the center.
Open the circle.
Connect the points on the edge.
You can divide this shape into triangles, too.
Try it.

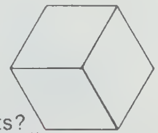
2. **The hexagon can be cut in many ways.**



Congruent parts? Yes



Congruent parts? Yes



Congruent parts? Yes

3. Can you find other ways to divide a hexagon into congruent parts? Try it.

4. Get some paper of different colors.
Choose a part of one of the hexagons.
Use it to make lots of congruent shapes.
Make designs.

goal Making a HEXAGON from a circular region

things for each pupil:
1 circular region (coffee filter)
colored paper
scissors
paste

page 303 Activity page. Proceed with whatever successful technique you have developed with your pupils for this kind of activity.

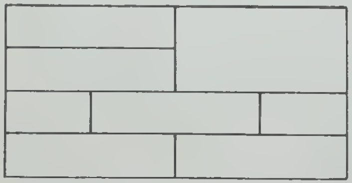
Display some of those creative designs. Congruent parts of hexagons present many good design possibilities.

lesson Pages 304, 305

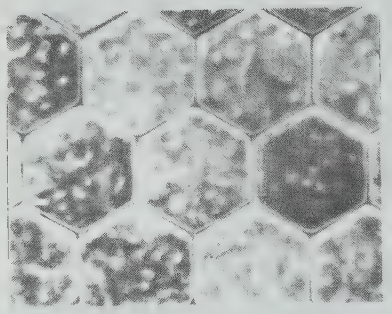
goal Applications of congruence in real-world situations

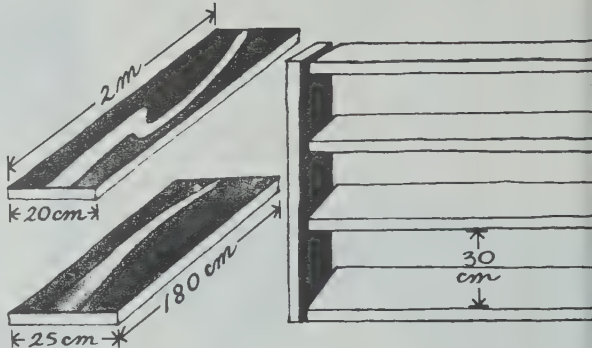
page 304 *What shape are the tiles on Joan's bathroom floor?* The tile patterns could be made of construction paper and pasted on cardboard. Add these to the display in the room.

Watch out for problem 2. It is hard! You may not want all pupils to try it. In fact, it probably should be done by pairs of pupils so that they can share their problem-solving ideas. Examine a bookcase in the room or in the school library. *Is the space between shelves the same? What would happen if the space between shelves were not kept the same at each end of two shelves?* (Slanted shelf) *Are the shelves the same length?* Find an example of a bookcase where all the shelves are not the same length. *What does it look like?*



or some variation.





1

The tiles on the bathroom floor in Joan's house are all the same size and shape. Are the tiles congruent? Yes

Make a pattern for a floor, using only a rectangular shape.

Answers will vary. Example:

People use the idea of congruence in building things. Boards and bricks are congruent. Walls should be the same size and shape.

2

Jack wants to build a bookcase. He wants it to have four shelves. He had two boards 2 m long and 20 cm wide. He got two more boards 180 cm long and 25 cm wide.

a How can the shelves be made the same length? Cut 20 cm off the longer boards.

b How can they be made the same width? Saw off 5 cm width from the 180 cm boards

c Will the shelves be congruent if they are the same length and width? Only if same thickness too

d The shelves in the bookcase should be 30 cm apart. How high will the sides of the bookcase be? Look out! More than 90 cm (include thickness of 4 boards)

304

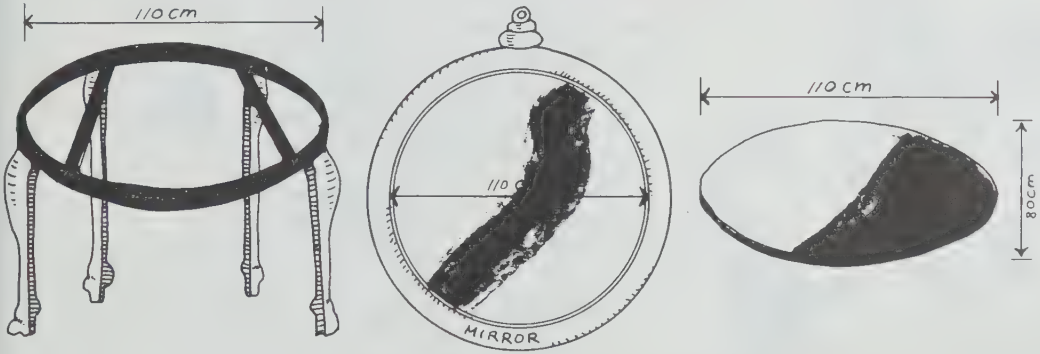
1 Here's another way that the idea of congruence is used in our world.

Mrs. Clear has an oval table with a glass top.
The top has been broken, so she needs a new glass.
Mrs. Dark offers to let her use her mirror, but
Mrs. Clear says it won't fit. Why not? Not the same shape

a The new glass must fit the ledge around the top of the frame. The distance across from one side to the other is 110 cm . How big must the glass be? 110 cm across

b Mrs. Clear decides to have the glass cut especially for her table. What shape must the glass be cut? Oval 110 cm by 80 cm

c The glass comes in two thicknesses — 10 mm and 8 mm . Does it matter which she chooses? Probably not



2 Name at least five other things that use the idea of congruence. Answers will vary

goal Examining more real-world applications of congruence

page 305 Here is a **research** page. You may not want all pupils to tackle problem 1. It provides a challenge for your best independent learners. Be sure that they understand the meaning of oval, and the difference between oval and round shapes before they tackle the problem.

Consider providing time to research the answer to problem 2 also. The information could be summarized in a chart.

lesson Page 306

goal **Checkout** – identifying congruent shapes by applying the concept of congruence to form tile patterns

things for each pupil:
graph paper or paper for tiles
scissors
paste

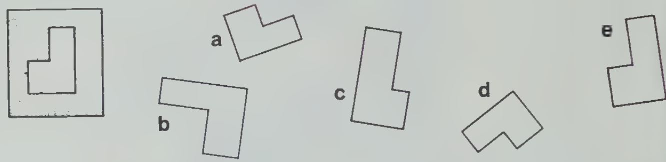
page 306 You decide how pupils should complete this page. Graph paper would be helpful and satisfactory for problem 2. You may choose to have them make patterns from shapes of two different colors and then paste the patterns on construction paper.

Lots of hands-on experience with real tiles or paper shapes will help sharpen the children's perception of shapes that match exactly (congruence). Attribute blocks (if you have them) will help too. But keep this sort of thing fun. Mastering the concept of congruence is not expected at this level.

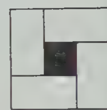
CHECKOUT

Skill: Identifying congruent figures

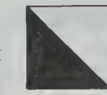
1. Name the shapes below that are congruent to the shape in the box. a, d



2. This tile pattern is made up of more than one shape. How many shapes are there in the pattern? 2
Use two shapes to make up a pattern of your own.
Patterns will vary.

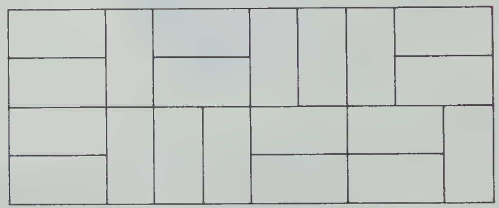


3. Beth made four clay tiles in art class. They were all alike. One looked like this:



You make four copies on paper. Show at least five different designs that Beth could have made with her tiles. Designs will vary.

4. Are the shapes in this design congruent? Yes



306

See activity, page 306a.

things wire coat hangers; string; stiff paper

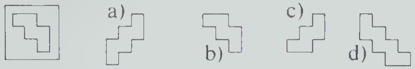
An opportunity for everyone to be creative. Have your youngsters paste the various figures made throughout the chapter on stiff paper, then use the figures to make a mobile.

RESOURCES

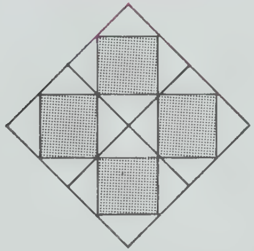
another form of evaluation

for Checkout—page 306

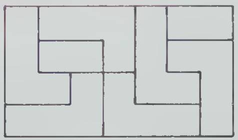
1. Name the shapes below congruent to the shape in the box. b, c



2. What two shapes are used in this pattern? Make up your own pattern with these shapes. Squares and triangles
Patterns will vary.



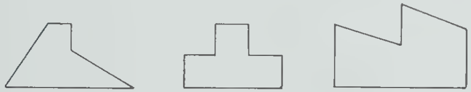
3. Are the shapes in this design congruent? Yes, (but there are 2 shapes).



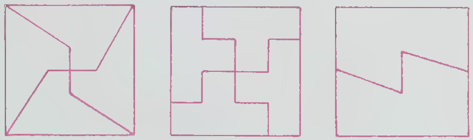
activities

things cardboard

Cut a set of 4 of each of the shapes given from cardboard. Challenge. Arrange each set of 4 congruent shapes to form a square.



solutions:



additional learning aids

concept—chapter objectives 1, 2, 3, 4

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit master: G 2
Skill through Patterns, level 4, SRA (1974)
Spirit masters: 7, 11, 19, 48, 69

other learning aids (described on page 336i)
Geoboards and Motion Geometry Resource Book and Activities, Good Time Mathematics, Learn to Fold—Fold to Learn, Mira, Mira Math for Elementary School

15 A REVIEW

before this chapter the learner has—

1. Mastered the skills to be reviewed
2. Developed the concepts and practiced the skills to be mastered in this chapter

in chapter 15 the learner is—

1. Reviewing the addition and subtraction mastery skills developed
2. Extending addition and subtraction to advanced skills
3. Reviewing the multiplication mastery skills developed
4. Mastering showing that the order in which any two 2-digit factors are multiplied does not affect their product
5. Mastering showing that the way any three 1- or 2-digit factors are grouped for multiplication does not affect their product
6. Mastering finding the quotient for any 3-digit number and any 1-digit number
7. Extending division skills to division of a 4-digit number by a 1-digit number
8. Solving a one-step word problem involving any one of the four operations of arithmetic
9. Mastering comparison of two fractions when given an appropriate model
10. Practicing renaming fractions
11. Reviewing addition and subtraction of two fractions with like denominators
12. Practicing renaming appropriate fractions as mixed numbers

in later chapters the learner will—

1. Maintain the skills mastered
2. Apply and extend the concepts and skills mastered

Notes & Things

A complete computational review of this level's work is put into the context of a mystery story. Computational skill will help the learner find the Phantom, the awful thief who lives in a town of 801 friendly and 7 unfriendly persons. The clues to solving the mystery contain the skill diagnosis. The learner who, according to the diagnostic check, needs instructional review and practice will be directed to self-study material. The learner will have to perform the skill in order to find the clue. Youngsters who have mastered their skills will get one clue quickly and go immediately to the next.

This chapter is mostly a self-study review. Directions for the pupil are found in the book. The simple self-direction chart that was used in chapter 8 is used again so that pupils can work independently.

There is a lot of reading in this chapter, but the story is so exciting and motivation to solve the mystery is so great that the usual reading problems will at least be put into a different context. But you may be surprised how many poor readers will get involved and be able to decode messages that you would not expect they could do. Nonreaders will of course need help, especially with the directions.

Emphasize each learner's responsibility to himself. Answers to the diagnostic checks are provided at the end of the chapter. This will help the learner to become more self-reliant. The learner who is confused, who does not understand, or who after additional review and practice is still not succeeding, has the responsibility to ask for help. The design of the chapter frees you to tutor individually or in small groups. Very capable learners are free to "run with the ball." Learners who solve the mystery early have an obligation to keep it secret. Let everyone discover the solution for himself.

It is hard to determine how much time will be spent on this chapter, since the pupils are on their own. Encourage the pupils who finish early to be your helpers. They might help read sections for your poor readers. They might help give computation guidance to others. Helping others is a learning experience in itself.

For the extra activities you will want to have available a set of 4 congruent shapes.

goal Think about and explore ideas through a picture clue

page 307 There is no reason to explore the living habits of our friendly neighborhood spider but this spiderweb will afford you the chance for a language lesson. Since you will find some youngsters finishing this self-study chapter ahead of others, please let this photograph simply be the introduction to the mystery story and then come back and use it for an activity after your speedy ones finish the chapter itself.

Your precocious pupils should enjoy investigating pictorial stereotypes. What does a picture of a spiderweb make you think of? What about a dark cave? Or what about shaky handwriting? Now that should be all that's necessary for some private investigation. Use advertisements in magazines and newspapers. Find pictorial stereotypes and tell how they are (or might) be used.

lesson Pages 308, 309

goal Introduction to the goal and theme of the chapter

memo The introduction to this chapter continues on page 309.

page 308 Read this page to the children with all the expression you have; or get one of your good readers to prepare in advance to read the introduction to the group. This is not the way math chapters usually begin. But this is not the usual review chapter. By this time, everyone needs a little more incentive to practice computation.

There are two reasons for you to finish this chapter.

1. **To review what you have learned this year**
2. **To find the Phantom**

The Phantom must be found. He is making trouble for the 801 friendly people and the 7 unfriendly people who live in a tiny town tucked away in the mountains.

One of the 801 friendly people is a little old lady. She just loves to help the sheriff solve crimes. The sheriff doesn't know for sure which of the 401 little old ladies in the town is always helping him. He knows it is one of them, however, because she is always leaving notes for him on paper that little old ladies use. Her notes always contain some sort of clue. (She is a nosy little old lady. She always knows what's going on.) She wants the sheriff to work for his salary. He always has to do some work to follow up on her messages.

The sheriff got mighty sick yesterday right in the middle of a whole series of robberies. You're just going to have to help him. The little old lady's clues will be coming to you. Follow her directions carefully and you won't have to do all the work the sheriff sometimes has to do. Don't worry. If you miss a clue, there will be other directions to help you.

308

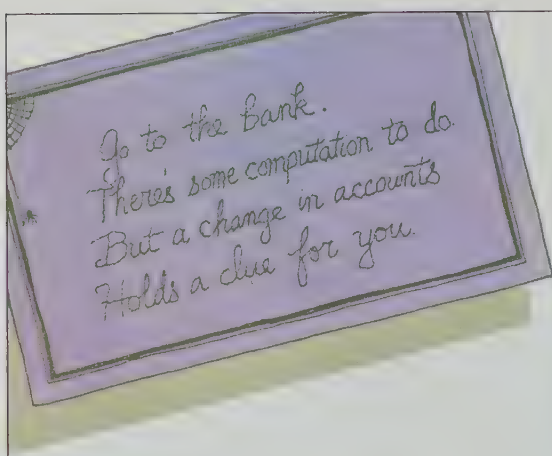
goal Introduction to the theme and the organization of the chapter

page 309 Continue the dramatic reading. Have fun with the suspects' names. There is no one correct pronunciation. Let the youngsters decide the easiest way to pronounce and therefore remember each name. (One editor remembered Franksanwine by thinking he was on a picnic in France where he ate hotdogs and drank wine—Franks an(d) wine—but that may not be an appropriate way to help the youngsters remember.)

goal Diagnosis of ability in advanced subtraction skills

page 310 Checking out each suspect's bank account is a diagnostic check of advanced subtraction skills. Be sure learners follow the self-direction chart at the bottom of the page. (This same type of chart was used in chapter 8. You should have very little trouble with it.)

The answers also are available again. By this time each person should be confident that the following work will be of specific help and that only the practice he needs will be assigned. (Please don't let him down by making him do more work than he needs in any particular skill.)



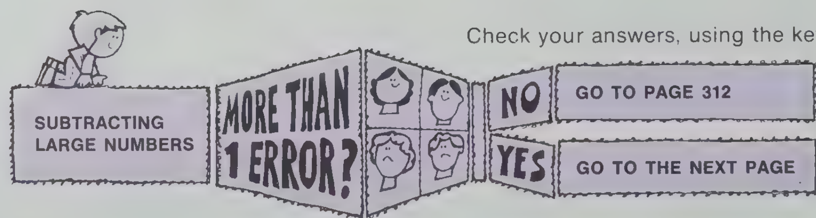
Here is a list of the amounts of money in the suspects' bank accounts on March 31 and on April 30. How much more or less is in each account on April 30 than was in it on March 31?

Diagnostic check on subtraction skills

	Balance		
	March 31	April 30	
a Baron Von Drup	$ 3,279.58	$ 3,741.82	$462
b Birdman	$ 1,226.00	$ 1,273.55	$47.
c Blockhead	$ 182.95	$ 167.95	$15.
d Count Dragulot	$ 15,666.66	$ 17,129.79	$14
e Franksanwine	$100,000.00	$115,000.00	$15,
f John Goldstone	$ 569.20	Account close Owner died	
g Sergeant Sargent	$ 2,497.00	$ 1,948.27	$54
h Weird Wolf	$ 6,931.50	$ 7,123.41	$19

Did you eliminate one suspect from your list?
Yes — John Goldstone

Check your answers, using the key on page 336.



310

gnose specific subtraction skills to be practiced

ıbtract

	tens	ones
	2	1
−		4
	1	7

b

	tens	ones
	8	2
−	4	7
	3	5

c

	hundreds	tens	ones
	4	5	7
−		6	8
	3	8	9

d

	hundreds	tens	ones
	6	1	8
−	2	7	4
	3	4	4

e

	hundreds	tens	ones
	3	7	3
−	1	9	5
	1	7	8

ieck your answers on page 336.

ır each problem you missed, do the row of problems
low that has the same letter. For example, if you
ssed **c**, do only the **c** row.

I: Renaming tens

1. 22 − 6 = 16 **2.** 35 − 7 = 28 **3.** 51 − 3 = 48 **4.** 48 − 9 = 39 **5.** 92 − 8 = 84

I: Renaming tens

6. 61 − 13 = 48 **7.** 70 − 26 = 44 **8.** 82 − 55 = 27 **9.** 44 − 28 = 16 **10.** 76 − 37 = 39

I: Renaming tens and hundreds

11. 164 − 38 = 126 **12.** 468 − 59 = 409 **13.** 813 − 67 = 746 **14.** 545 − 77 = 468 **15.** 200 − 46 = 154

I: Renaming hundreds

16. 215 − 121 = 94 **17.** 332 − 192 = 140 **18.** 704 − 651 = 53 **19.** 828 − 372 = 456 **20.** 629 − 199 = 430

I: Renaming tens and hundreds

21. 522 − 256 = 266 **22.** 473 − 195 = 278 **23.** 666 − 388 = 278 **24.** 654 − 267 = 387 **25.** 227 − 149 = 78

Now turn to the next page.

goal Diagnosis of ability and practice in basic subtraction skills

page 311 Learners who are not quite ready for advanced subtraction are checked out here for very basic skills. The top row of problems will diagnose the range of minimal subtraction skills. Note that there is a row of practice problems below that corresponds with each problem on the diagnostic check. The learner can find out for himself which practice rows he should do, because he will correct his own diagnostic check. An error means practice.

After the learner has completed the first two examples in any row of practice problems, check his success before having him complete the row. Practice without understanding accomplishes nothing. Provide teacher or peer tutoring for those learners who continue to have difficulty. You may want to work with them in small groups.

For learners who continue to make errors in this practice set, check the following:

- Mastery of subtraction facts
- Understanding of renaming

goal Diagnosis of ability and practice in advanced subtraction skills

page 312 Here are the most difficult of all subtraction problems. These skills are not included in the mastery objectives for the year. You will **not** want your pupils who are having trouble to do these problems, but surely you will want your top students to check themselves out.

Make sure that all pupils who do this page follow the self-direction chart.

Diagnostic check on advanced subtraction skills

Subtract

a 567,124 − 73,843 = 493,281

b 781,546 − 267,538 = 514,008

c 856,523 − 396,736 = 459,787

Check your answers using the key on page 336.

SUBTRACTION WITH REALLY BIG NUMBERS

ANY ERRORS?

NO → GO TO THE NEXT PAGE

YES → DO **a**, **b**, AND **c** BELOW

Diagnose specific subtraction skills to be practiced.

a

	hundred thousands	ten thousands	one thousands	hundreds	tens	ones
		5	9	5	3	4
−		2	6	8	1	9
		3	2	7	1	5

b

	hundred thousands	ten thousands	one thousands	hundreds	tens	ones
	3	6	8	2	4	5
−		3	4	1	5	7
	3	3	4	0	8	8

c

	hundred thousands	ten thousands	one thousands	hundreds	tens	ones
	7	7	4	0	0	2
−		9	2	6	8	8
	6	8	1	3	1	4

Check your answers with the key on page 336.

For each problem you missed,
do the row of problems below that has the same letter.

Skill: Renaming (alternating positions)

a
1. 43,671 − 1,847 = 41,824
2. 182,449 − 28,153 = 154,296
3. 763,951 − 5,270 = 758,681
4. 935,627 − 509,436 = 426,191

Skill: Renaming (two positions side by side)

b
5. 758,431 − 25,670 = 732,761
6. 583,547 − 124,934 = 458,613
7. 356,812 − 167,489 = 189,323
8. 425,907 − 14,957 = 410,950

Skill: Renaming (emphasis on zero)

c
9. 247,730 − 38,940 = 208,790
10. 423,868 − 115,974 = 307,894
11. 564,004 − 125,163 = 438,841
12. 752,005 − 231,946 = 520,059

Go to the next page if you have time.

312

You really should know something about your suspects. These problems will tell you a bit.

1

Weird Wolf really is weird. He collects things. He has 100,000 gum wrappers, for example. Birdman loves gum wrappers too. But he only has 83,498 of them. Weird Wolf always teases that he has more. Lots more. How many more? 16,502

2

Believe it or not, Franksanwine is 1420 years old. Count Dragulot is only 952 years old. Franksanwine always teases the count that he is only a little kid. He can't do all the things that Franksanwine can until he is as old as Franksanwine. How many years younger is the count? 468

3

Birdman had 249 pigeons. Then Weird Wolf "collected" 157 of them. How many pigeons did Birdman have left? 92

4

Blockhead had a water bed—a big one! He filled it with 232,647 gallons of water. Weird Wolf "collected" 43,752 gallons of it. How many gallons were left in the bed? 188,895

5

In the past 20 years Count Dragulot has invited 72,679 people to his castle. 14,795 people have left. How many people did not leave the castle? 57,884

6

Baron Von Drup is the little old lady's favorite character. She is always snooping in his affairs. She has counted the number of letters he has received in the last ten years. She has counted 3650 pink envelopes out of the 36,500 letters he has had in his post office box. How many letters were not in pink envelopes? 32,850

goal Application of advanced subtraction skills to solving word problems

page 313 Solving word problems is an important skill too. Less capable readers don't have a chance without help. These problems are intended to be pleasant nonsense. Another dramatic reading would be great! Any pupil who did not do page 312 should not be expected to do these either.

lesson Pages 314, 315, 316

goal Diagnosis of ability in advanced addition skills

page 314 Another note from the little old lady. You will need to review the directions together—they are involved. Youngsters who understand how to add and how to rename for addition should have no problems with the large number of digits.

Suggest that the youngsters check by counting the digits of the numerals on the page and the digits they copied. It's easy to lose one along the way. Why do all that addition until they make sure they are adding the correct numbers?

Add the house number
and the telephone too.
It is the only work you have to do
To cross one more suspect off your list.
If you find the name,
you haven't missed.

Add the number of the house and the number of the telephone to decode the message. (Only one sum forms a message. The others are nonsense. That doesn'[t] mean the other sums are wrong.)

Diagnostic check on addition skills

	Code
1	Baron Von Drup
2	Birdman
3	Blockhead
4	Not
5	Is
6	Phantom
7	The
8	Suspect
9	Stole
10	Jewels

Name	Address	Phone number
Birdman	654496 Tree St.	4291971
Blockhead	1993 Quarry Ave.	6657455
Dragulot, Count	34892 Coffin Rd.	2176345
Franksanwine	9289 A Street	7826187
Goldstone, John	572 Shady Rest	7843866
Sargent, Sergeant	17823 Main Ave.	9848504
Von Drup, Baron	653982 Cedar St.	9018349
Wolf, Weird	2647 Woods Court	9379783

If you missed and didn't find the name, go on to the next page. If you have one less suspect, turn to page 316. Start in the middle with problem 1.

7 8 The Suspect   3 5 Blockhead Is   4 7 6 Not The Phanto[m]

314

Compute.

Diagnose specific addition skills to be practiced.

a

	hundreds	tens	ones
		6	4
+		2	8
		9	2

b

	hundreds	tens	ones
	6	3	6
+		9	4
	7	3	0

c

	hundreds	tens	ones
	3	5	5
+	3	4	5
	7	0	0

d

	thousands			ones		
	hundred	ten	one	hundreds	tens	ones
			4	2	3	9
+			8	5	7	6
		1	2	8	1	5

e

	thousands			ones		
	hundred	ten	one	hundreds	tens	ones
		5	4	7	8	9
+		6	2	5	4	3
	1	1	7	3	3	2

f

	thousands			ones		
	hundred	ten	one	hundreds	tens	ones
	6	7	2	0	8	2
+		9	1	9	6	9
	7	6	4	0	5	1

Use the key

on page 336 to check your answers.

For each problem you missed, do the row of problems below or on the next page that has the same letter as the problem you missed.

Skill: Renaming ones

a 1. $61 + 83$ = 144 2. $58 + 35$ = 93 3. $45 + 27$ = 72 4. $93 + 56$ = 149

Skill: Renaming ones and tens

b 5. $789 + 55$ = 844 6. $538 + 63$ = 601 7. $842 + 78$ = 920 8. $666 + 99$ = 765

Skill: Renaming twice

c 9. $543 + 618$ = 1161 10. $739 + 287$ = 1026 11. $398 + 575$ = 973 12. $598 + 752$ = 1350

315

goal Diagnosis of ability and practice in adding two addends, with renaming required

page 315 Only the top row of problems is needed to diagnose the minimal additional skills, but maybe everyone would like to show off a bit and do the second row too. No unnecessary practice please. Make sure that those directions are followed.

Check learners who continue to have difficulty.

- Have they mastered the addition facts? Check particularly facts with a 2-digit sum.
- Do they understand the steps necessary in renaming?
- Do they understand place value?

Pupils having trouble should not be expected to do problems as difficult as **e** and **f**. These are far more advanced than the mastery objectives for this level. Help them enough to recall the clues on page 314. Everyone must eliminate Blockhead as a suspect.

goal Practice in advanced addition skills and application of these skills to solving word problems

page 316 Pupils need only to complete through row **d** satisfactorily to reach the mastery objectives for this level. You will want to challenge independent learners with rows **e** and **f.**

Youngsters have already been directed to the word problems on page 314. It's up to you whether or not you want to use them with other pupils too.

Skill: Renaming twice (4-digit sums)

d	**13.** $\begin{array}{r} 1268 \\ +1376 \\ \hline \end{array}$ 2644	**14.** $\begin{array}{r} 6124 \\ +\ 958 \\ \hline \end{array}$ 7082	**15.** $\begin{array}{r} 2368 \\ +6499 \\ \hline \end{array}$ 8867	**16.** $\begin{array}{r} 3573 \\ +3599 \\ \hline \end{array}$ 7172

Advanced skill: Renaming two or three times (6-digit sums)

e	**17.** $\begin{array}{r} 83451 \\ +\ 7808 \\ \hline \end{array}$ 91,259	**18.** $\begin{array}{r} 77799 \\ +62166 \\ \hline \end{array}$ 139,965	**19.** $\begin{array}{r} 59816 \\ +87452 \\ \hline \end{array}$ 147,268	**20.** $\begin{array}{r} 63543 \\ +50507 \\ \hline \end{array}$ 114,050

Advanced skill: Renaming alternate positions (6- and 7-digit sums) (Not appropriate for all pupils)

f	**21.** $\begin{array}{r} 582084 \\ +893295 \\ \hline \end{array}$ 1,475,379	**22.** $\begin{array}{r} 739387 \\ +227071 \\ \hline \end{array}$ 966,458	**23.** $\begin{array}{r} 236806 \\ +\ 80759 \\ \hline \end{array}$ 317,565	**24.** $\begin{array}{r} 789028 \\ +154674 \\ \hline \end{array}$ 943,702

Solve these problems and you'll know still more about your suspects.

1

Count Dragulot has an Egyptian mummy. He bought it for $29,350. He thought it must be lonesome all by itself, so he bought another for $789,122. How much money in all does he have wrapped up in his mummies? $818,472

2

Weird Wolf has lots of money too. He counts it each month when the moon is full. Last month he counted $482,769. This month he added $180,962 to his money. How much money will he count when the moon is full this month? (Where could that money have come from?) $663,731

3

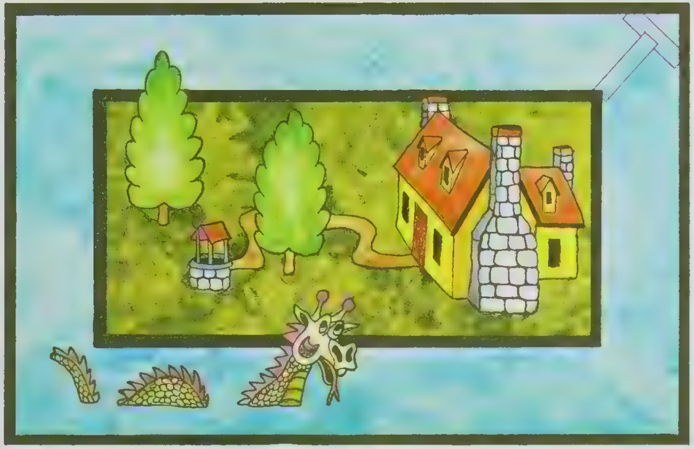
The old lady is the only one who knows who the Phantom is. She has watched him for a long time. She knows he used many different names. He used 1109 names last year. And he used 901 different names this year. How many different names has he used all together? 2010

4

You've had enough practice now to be able to correct your mistakes on page 314. Then decode the message on that page and eliminate one suspect.
After that, go to page 317.

Now go along to Franksenwine's place,
And try to solve this confusing case.
If you really try to think and think,
You'll cross the moat quick as a wink.

The picture shows the house taken from the air. The moat goes all around the property. There is no bridge across. The water is 20 feet wide and very cold. You don't know what might be in the water, either. You can't swim across. You find 2 boards. One board is $19\frac{1}{2}$ feet long. The other board is 19 feet long. You don't have a hammer and nails. Can you use the boards to cross the moat? Tell how.



lesson Page 317

goal Challenging the ability to solve problems

page 317 This situation represents real problem solving. Some pupils will "luck out" and get the answer immediately. Others will need lots of time. Urge those who got the answer to keep it a secret. But don't let anyone get discouraged either. Rather than give away the answer, suggest that they hire a helicopter, take the boards with them, and get across the moat. They can figure out how to get out of there later.

lesson Pages 318, 319

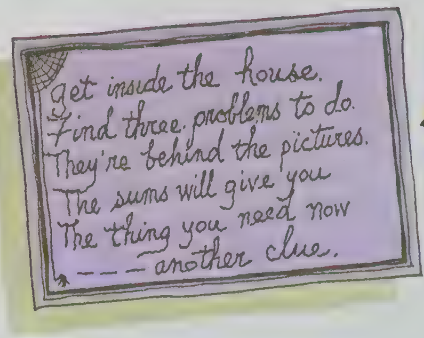
goal Diagnosis of ability to add three and four addends requiring renaming

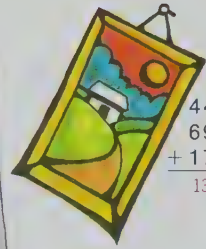
page 318 About now, you will have some youngsters who will ask, "Will this story really work out with the clues that are given?" Assure them that following all the directions is the **only** way they will find the criminal.

There are a lot of alternatives in the directions on this page. But if pupils go step by step and do the job that applies to each step, they will be able to cross the next suspect off the list on the very next page.

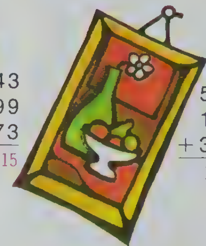
Diagnostic check on addition of more than two addend

You're given a message to go to Franksanwine's house.

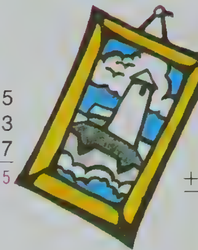




443
699
+ 173
1315



5265
1283
+ 3767
10,315



18
34
18
+ 60
131

Check your answers with the key on page 336. If you made an error, go to page 319 for another clue. If you didn't make any mistakes, you have just found the combination to Franksanwine's safe.
Use the sum in problem 3 to open the safe.

DIRECTIONS FOR OPENING THE SAFE

1. Start at zero.
2. Turn forward to the number in the thousands place. 1
3. Turn back to the number in the hundreds place. 3
4. Turn forward to the number in the tens place. 1
5. Turn back to the number in the ones place. 5

The number you stopped on tells the number you'll start on on page 31

318

Skill: 1- and 2-digit addends

1.	2.	3.	4.
6	3	6	17
7	5	4	25
1	9	21	46
+3	+6	+47	+20
17	23	78	108

Skill: 2-, 3-, and 4-digit addends

5.	6.	7.	8.
22	532	674	5293
20	638	140	7250
32	57	685	+7324
+42	+ 72	+781	19,867
116	1299	2280	

Check your answers with the key on page 336.
Make sure your answers are correct. Take only the digits in the ones place in the sums for problems 5, 6, 7, and 8. Add them. Use that sum. Match it with the list of suspects. The number that matches is the next guy you can cross off the list.

$$\begin{array}{r} 6 \\ 9 \\ 0 \\ +\,7 \\ \hline 22 \end{array}$$

21—Count Dragulot	23—Weird Wolf
22—Sergeant Sargent	24—Baron Von Drup

NOW EVERYBODY FIND THE ANSWER TO THIS ONE.

Count Dragulot is adding bats to his batroom.
He bought 439 bats from Louie's Bat and Ball Store,
291 from the Downtown Bat and Owl Shop,
5503 bats from Creature Supplies Company,
and 1306 used bats from Honest John's Used Bat Lot.
How many bats did he buy in all? 7539

319

goal Practice in adding three and four addends requiring renaming

page 319 Check youngsters directed to this practice closely. Those operating with minimal skills may be able to compute only problems 1, 2, 3, and 4. All other pupils should be able to compute the entire set of problems. Mastery objectives for the level include all of these skills.

Encourage those who are having trouble to change the order and the grouping of the addends to simplify the computation.

lesson Pages 320, 321, 322, 323

goal Diagnosis of ability to multiply by a 1- or 2-digit factor

page 320 Mastery objectives for this level include problems 1 through 3 only.

Beware of one man who goes to the store.
He buys not one package but many more.
He uses the stuff in his top-secret work.
Help find the totals.
Now don't you shirk!

Diagnostic check on multiplication skills

1. He bought 7 jars that contained 34 each. How many in all? 238
2. He bought 9 bags that had 376 in each bag. How many in all? 3384
3. He bought 64 boxes that had 82 in each. How many in all? 5248
4. And 76 packs that had 387 in each. How many in all? 29,412
5. Then he bought 61 cartons with 583 in each carton. How many in all? 35,563

Take the digit in the tens place in each product. List them in order. Use each digit in order to break the code listed below. You'll know one more suspect to cross off your list. 38416 BIRDMAN

CODE

0	1	2	3	4	5	6	7	8	9
T	A	WE	BI	M	DR	N	VO	RD	LF

If you got the message, go on to page 322. If you don't have the name to cross off your list, go to the next page for another chance.

320

You can use either of these methods to complete your multiplication.

METHOD 1

$$\begin{array}{rl} 78 & \\ \times 13 & \\ \hline 24 & 3 \times 8 \\ 210 & 3 \times 70 \\ 80 & 10 \times 8 \\ 700 & 10 \times 70 \\ \hline ? & 1014 \end{array}$$

METHOD 2

$$\begin{array}{rl} 78 & \\ \times 13 & \\ \hline 234 & 3 \times 78 \\ 780 & 10 \times 78 \\ \hline ? & 1014 \end{array}$$

Diagnose specific multiplication skills to be practiced.

a $\begin{array}{r} 453 \\ \times \quad 6 \\ \hline \end{array}$ 2718 **b** $\begin{array}{r} 45 \\ \times 78 \\ \hline \end{array}$ 3510 **c** $\begin{array}{r} 895 \\ \times \ 39 \\ \hline \end{array}$ 34,905

Use the key on page 336 to check your answers.
For each problem you missed, do the lettered row of problems that is the same as the letter of the problem you missed.

Skill: 1-digit multiplier

a **1.** $\begin{array}{r} 28 \\ \times \ 4 \\ \hline \end{array}$ 112 **2.** $\begin{array}{r} 43 \\ \times \ 7 \\ \hline \end{array}$ 301 **3.** $\begin{array}{r} 287 \\ \times \quad 5 \\ \hline \end{array}$ 1435 **4.** $\begin{array}{r} 751 \\ \times \quad 2 \\ \hline \end{array}$ 1502

Skill: 2-digit × 2-digit

b **5.** $\begin{array}{r} 39 \\ \times 50 \\ \hline \end{array}$ 1950 **6.** $\begin{array}{r} 65 \\ \times 27 \\ \hline \end{array}$ 1755 **7.** $\begin{array}{r} 26 \\ \times 35 \\ \hline \end{array}$ 910 **8.** $\begin{array}{r} 67 \\ \times 19 \\ \hline \end{array}$ 1273

Skill: 2-digit × 3-digit

c **9.** $\begin{array}{r} 484 \\ \times \ 12 \\ \hline \end{array}$ 5808 **10.** $\begin{array}{r} 349 \\ \times \ 23 \\ \hline \end{array}$ 8027 **11.** $\begin{array}{r} 758 \\ \times \ 75 \\ \hline \end{array}$ 56,850 **12.** $\begin{array}{r} 686 \\ \times \ 56 \\ \hline \end{array}$ 38,416

Use the digits in problem 12 to break the code on page 320.

321

goal Diagnosis of ability and practice in multiplying by a 1- or 2-digit factor

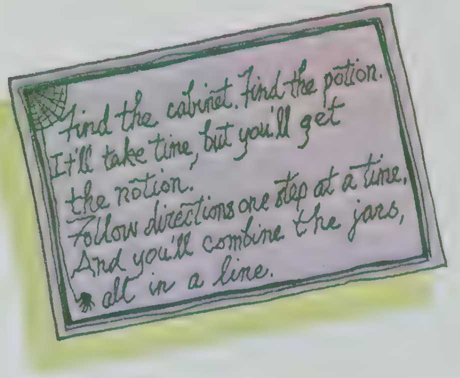
page 321 The instructional review will help most of those in trouble, but you will want to check learners having trouble multiplying by a 1-digit factor. Most likely causes for error:

- Has not mastered multiplication facts
- Confused about how to rename
- Lacks understanding of place value

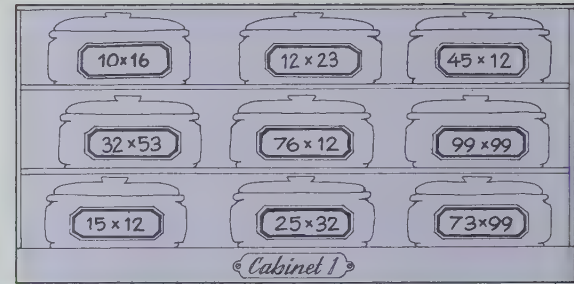
Row c is not included in the mastery objectives for this level. Omit practice now with pupils operating with minimal skills.

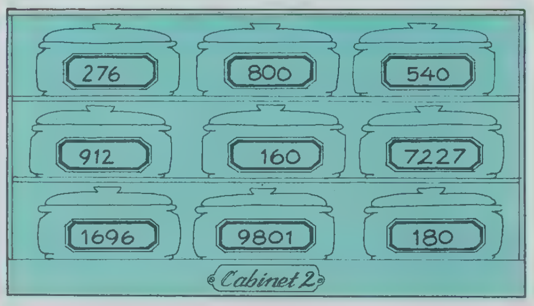
goal Practice with the order (commutative) property of multiplication

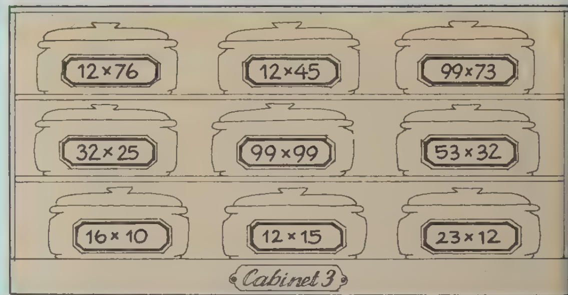
page 322 The focus here is not on the name of the property but on the concept that changing the order of the two factors does not change the product.



Cabinets 1, 2, and 3 each have 9 jars of potion. 27 jars in all. Each jar is marked differently. But there is one jar in cabinet 1 that is like a jar in cabinet 2. And you'll find still another jar just like your first and second ones in cabinet 3.



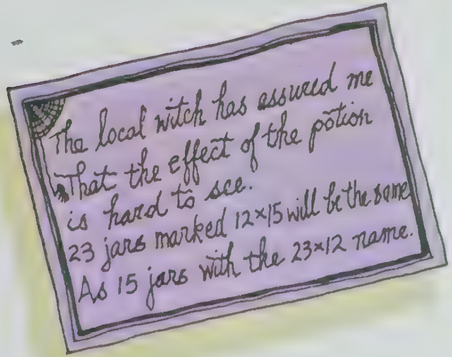


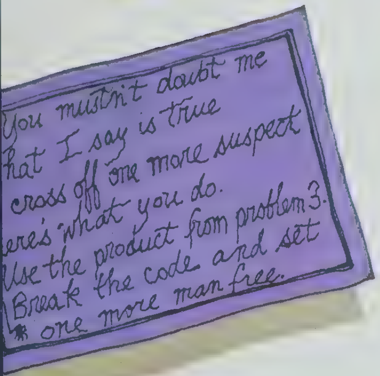


Find the jars with the same potion. Write your answers like this:

Cabinet 1	Cabinet 2	Cabinet 3
10 × 16	160	16 × 10
12 × 23	276	23 × 12
45 × 12	540	12 × 45
32 × 53	1696	53 × 32
76 × 12	912	12 × 76
99 × 99	9801	99 × 99
15 × 12	180	12 × 15
25 × 32	800	32 × 25
73 × 99	7227	99 × 73

322





Can the little old lady be right? Multiply to prove whether she is right or wrong.

1. 25 jars of 32×53 should be the same as 53 jars of ? $\times$? . Multiply to show that it's true. *25×32 or 32×25 42,400

2. 45 jars of 12×76 should be the same as 76 jars of ? $\times$? . Multiply to show that it's true. **$12 \cdot 45$ or $45 \cdot 12$ 41,040

3. 73 jars of 99×99 should be the same as 99 jars of ? $\times$? . Multiply to show that it's true. ***99×73 or $73 \cdot 99$ 715,473

CODE

0	1	2	3	4	5	6	7	8	9	715473
DRA	E	VON	OLF	RD	I	CO	W	BA	FRA	WEIRD WOLF

*$25 \times (32 \times 53) = 25 \times 1696 = 42{,}400$
$53 \times (32 \times 25) = 53 \times 800 = 42{,}400$

**$45 \times (12 \times 76) = 45 \times 912 = 41{,}040$
$76 \times (12 \times 45) = 76 \times 540 = 41{,}040$

***$73 \times (99 \cdot 99) = 73 \cdot 9801 = 715{,}473$
$99 \times (73 \cdot 99) = 99 \cdot 7227 = 715{,}473$

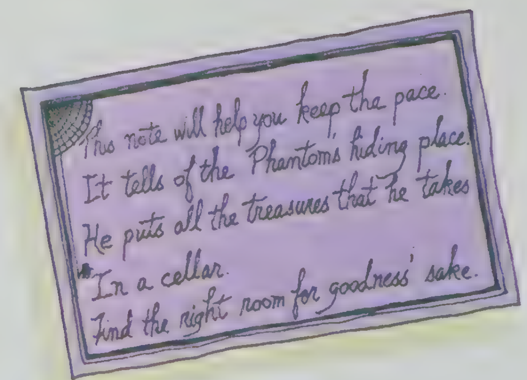
goal Practice with the grouping (associative) property of multiplication

page 323 The purpose of this work is to emphasize that when three factors are multiplied, the way the factors are grouped will not change the product.

lesson Pages 324, 325

goal Diagnosis of ability to divide with a 1-digit divisor

page 324 Mastery objectives for this level include problems 1 and 2 only. But everyone should be confident enough to try the remaining problems. They have to find that next clue, don't they?



Diagnostic check on division skills

Before you can check out the cellar, you must do these problems. Divide.

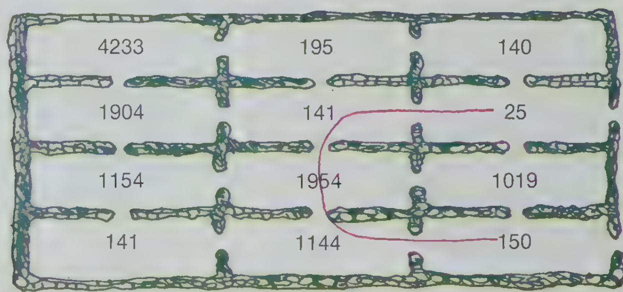
1. $3\overline{)75}$ 25 **2.** $6\overline{)846}$ 141 **3.** $4\overline{)7816}$ 1954

4. $8\overline{)9152}$ 1144 **5.** $9\overline{)1350}$ 150

Below is a picture of the cellar. Each room has doorways. Each room has a number. Some of them just happen to have the same number as the answers to the problems you just worked. Find the room marked with the same number as your first answer.

Use tracing paper.

Trace the path from that room to the next room that has the same number as your second answer. Move in order of your answers. If you can't find a room that is the same number as one of your answers, complete the next page.



You have found the right clue if your path forms the shape of a letter of the alphabet. Take your clue to page 326.

324

Diagnose specific division skill to be practiced

O.K., so you had some trouble. Show that it wasn't big trouble. *Complete* each problem below. Be sure to write the remainder beside the quotient (if there is a remainder).

a $4\overline{)213}$? 53 R1
200 (50 × 4)
13
12 (3 × 4)
?
1

b $7\overline{)854}$? 122
700 (100 × 7)
154
140 (20 × 7)
?
14
14 (2 × 7)
0

c $6\overline{)4121}$? 686 R5
3600 (600 × 6)
?
521
480 (80 × 6)
41
36 (6 × 6)
5

Check your answers with the key on page 336. For each problem you missed, do the row of problems below with the same letter as the problem you missed.

Skill: 2-digit quotients

a 1. $3\overline{)125}$ 41 R2 2. $6\overline{)159}$ 26 R3 3. $5\overline{)476}$ 95 R1 4. $4\overline{)307}$ 76 R3

Skill: 3-digit quotients

b 5. $4\overline{)840}$ 210 6. $7\overline{)815}$ 116 R3 7. $5\overline{)705}$ 141 8. $8\overline{)914}$ 114 R2

Skill: 3- and 4-digit quotients (not appropriate for all pupils)

c 9. $5\overline{)5648}$ 1129 R3 10. $3\overline{)9176}$ 3058 R2 11. $8\overline{)2364}$ 295 R4 12. $7\overline{)6983}$ 997 R4

Go back to page 324 and get back on the right path.

NOW ANSWER THIS ONE.

Everyone wondered how many Venus's-flytrap plants Count Dragulot owned. The little old lady kept track of the number of flies the count caught. She knew each plant needs 8 flies each day. The count caught 152 flies today. How many Venus's-flytrap plants must the count own? 19

325

goal Instructional review of and practice in dividing by a 1-digit divisor

page 325 Prerequisite skills to check out with learners in real trouble:

- Multiplication facts
- Ability to multiply
  $40 \times 3 = \underline{?}$ $400 \times 3 = \underline{?}$
  $50 \times 6 = \underline{?}$ $500 \times 6 = \underline{?}$
- Ability to estimate the quotient

You may not want your pupils-in-trouble to do problems as difficult as those in **c.** This type of problem is not a mastery expectation for this level.

lesson Pages 326, 327, 328

goal Diagnosis of ability to compare two fractions without models

things for each pupil: tracing paper

page 326 Make sure that everyone doing this page has tracing paper so that no marks will be made in the books.

Pick the directions that have the same letter you found.

a

You must have been working very hard. You must be tired. Because you are tired, you made a wrong turn. In fact, you're reading the wrong directions. Go back to page 324 and get back on the right path.

b

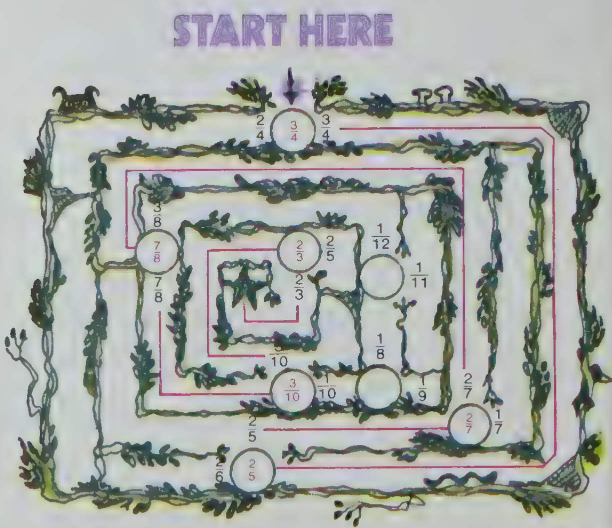
The lights in the cellar are very dim. Your eyes played a trick on you. You have lost your way. You are reading the wrong directions. Go back to page 324 and get back on the right path.

c

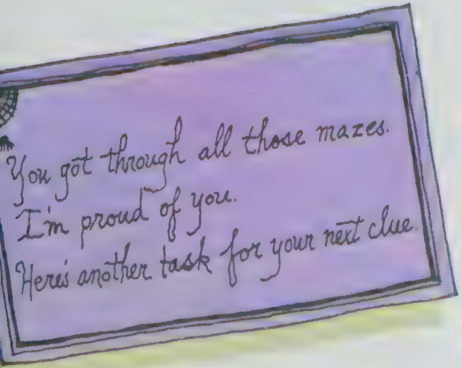
You must find your way to the center of the maze below. At each circle you must decide which way to go. Go in the direction of the larger fraction. Keep track of all your turns. Get a sheet of paper. Write the largest fraction at each turn.

Diagnostic check (combined with page 327)

Skill: Comparing fractions

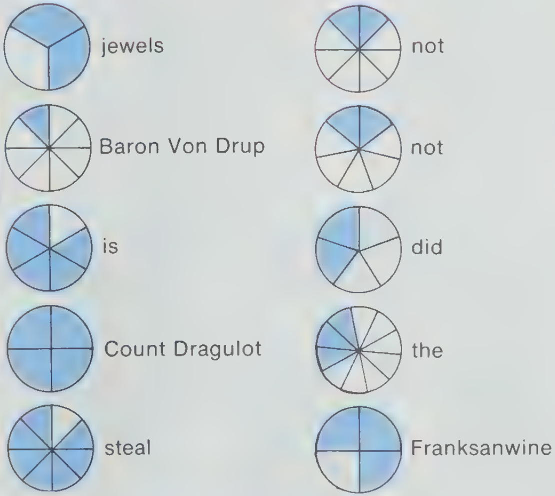


326

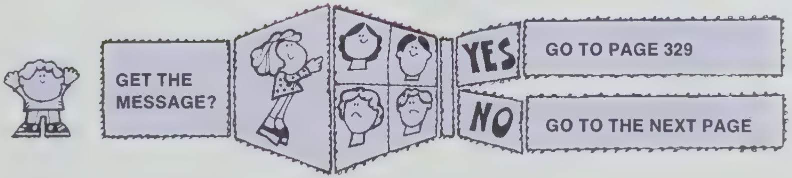


Your list of fractions will help decode the next message. The fractional part is shown as a region. Find the region that shows the first fraction on your list. If you continue, you'll find the name of one more suspect to cross off your list.

Skill: Recognizing a fractional part of a region



Franksanwine did not steal the jewels.



327

goal Diagnosis of ability to recognize a fractional part of a region

page 327 The focus here is on identifying a fractional part of a region.

Did anyone have trouble getting the correct list of fractions from the maze on page 326? Those who did will find it helpful to use the region models on this page or to go back to the maze and start over.

It's easy to get the wrong guy in this exercise. And if it's wrong here, the pupil will never know who the thief is.

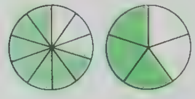
goal Instructional review of and practice in comparing two fractions

page 328 If you have pupils who have trouble completing rows **a** and **b**, it is necessary to get out some manipulatives. The models are as much help as can be given in a book. Felt pieces may help. Or perhaps the pupil would benefit from making his own rectangular regions, using graph paper.

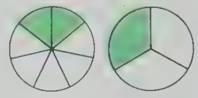
Diagnose specific skill to be practiced

Complete each of these statements with > or <.

a $\frac{7}{10} \overset{>}{?} \frac{3}{5}$



b $\frac{2}{7} \overset{<}{?} \frac{1}{3}$



c $\frac{3}{5} \overset{>}{?} \frac{4}{7}$

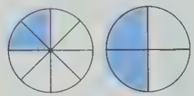
Need a model this time? Draw your own.

Use the key on page 336 to check your answers. For each problem you missed, do the row of problems below with the same letter as the problem you missed.

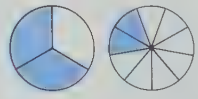
Skill: Comparing fractions using models

a

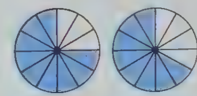
1. $\frac{2}{8} \overset{<}{?} \frac{2}{4}$



2. $\frac{2}{3} \overset{>}{?} \frac{2}{9}$

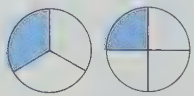


3. $\frac{9}{12} \overset{>}{?} \frac{8}{12}$

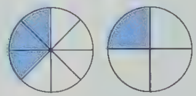


b

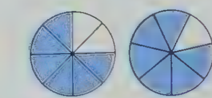
4. $\frac{1}{3} \overset{>}{?} \frac{1}{4}$



5. $\frac{3}{8} \overset{>}{?} \frac{1}{4}$



6. $\frac{6}{8} \overset{<}{?} \frac{6}{7}$



Skill: Comparing fractions—no models

c If you need a model for any of these problems, draw your own.

like denominators

7. $\frac{5}{6} \overset{>}{?} \frac{1}{6}$ **8.** $\frac{5}{8} \overset{>}{?} \frac{3}{8}$ **9.** $\frac{2}{2} \overset{>}{?} \frac{1}{2}$

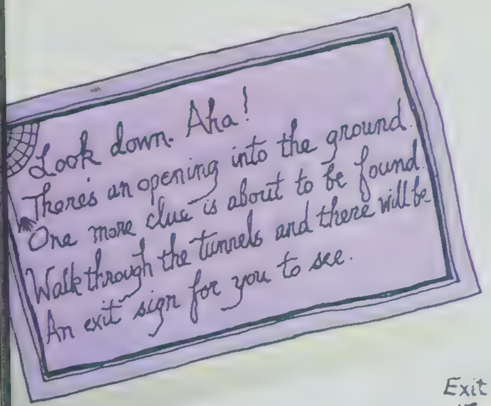
like numerators

10. $\frac{3}{4} \overset{>}{?} \frac{3}{5}$ **11.** $\frac{2}{7} \overset{<}{?} \frac{2}{5}$ **12.** $\frac{5}{7} \overset{<}{?} \frac{5}{4}$

one denominator a factor of the other

13. $\frac{3}{2} \overset{>}{?} \frac{2}{2}$ **14.** $\frac{1}{8} \overset{<}{?} \frac{1}{2}$ **15.** $\frac{1}{2} \overset{<}{?} \frac{3}{4}$

You've practiced enough now to be able to correct your mistakes on page 327. Then you'll eliminate one more suspect. After that, go to page 329.

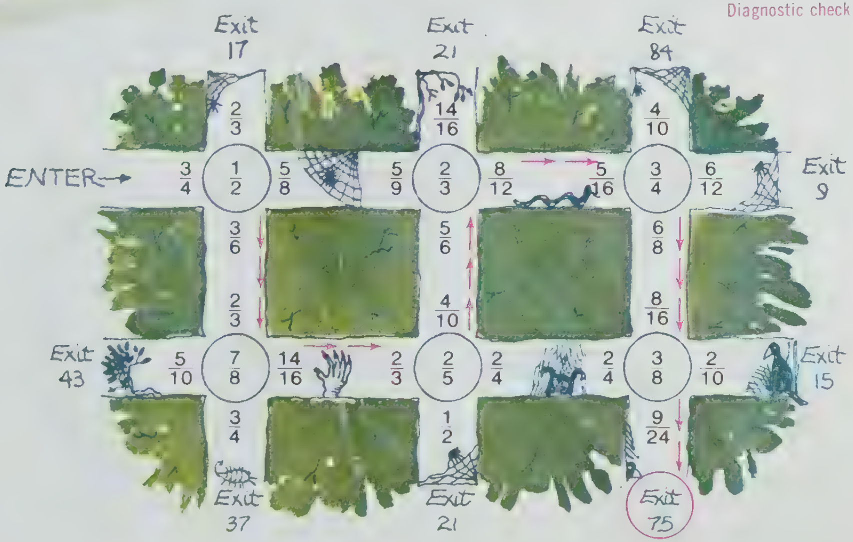


Once again you have to decide which paths to follow.
At each crossing you will see a fraction in a ring.
Find another name for the fraction.

Go in that direction.

If you take the wrong path, scorpions may bite you.
Take your exit number and go to the next page.

Diagnostic check



329

lesson Pages 329, 330, 331

goal Diagnosis of ability to rename fractions without a model

page 329 Look out for pencil marks in the books again. This should be a mental-computation page, so urging youngsters to trace the path with a finger should do the trick.

goal Practice in renaming fractions with the aid of a model

page 330 If there is still a problem after using the models on the page, try another approach:

$$\frac{1}{2} = \frac{\square}{4}$$

2 times what number is equal to 4?
If the denominator is multiplied by 2, you must multiply the numerator by that same number. So—

$$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

Use at least 3 more examples. Also consider this presentation:

$$\frac{3}{5} = \frac{\square}{15}$$

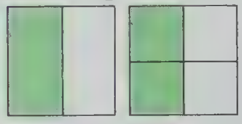
$$\frac{3 \times ?}{5 \times ?} = \frac{\square}{15} \qquad 5 \times ? = 15$$

$$\frac{3 \times 3}{5 \times 3} = \frac{9}{15}$$

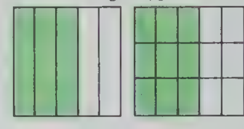
If your exit number was 75, go to the next page now.
If your exit number was *not* 75, you have to have time to recover from those scorpion bites. Do this page.

Complete each of these sentences.

a $\frac{1}{2} = \frac{■}{4}$ 2



b $\frac{3}{5} = \frac{■}{15}$ 9

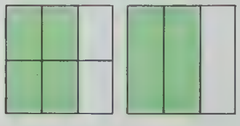


c $\frac{3}{4} = \frac{■}{16}$ 12
Need a model again?
Draw your own.

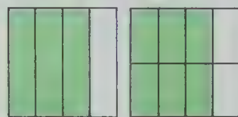
Use the key on page 336 to check your answers. For each problem you missed, do the row of problems below with the same letter as the problem you missed.

Skill: Renaming fractions

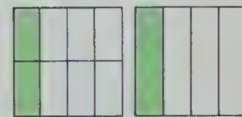
a **1.** $\frac{4}{6} = \frac{■}{3}$ 2



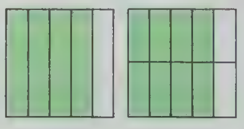
2. $\frac{3}{4} = \frac{■}{8}$ 6



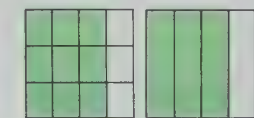
3. $\frac{2}{8} = \frac{■}{4}$ 1



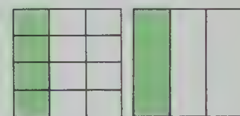
b **4.** $\frac{4}{5} = \frac{■}{10}$ 8



5. $\frac{9}{12} = \frac{■}{4}$ 3



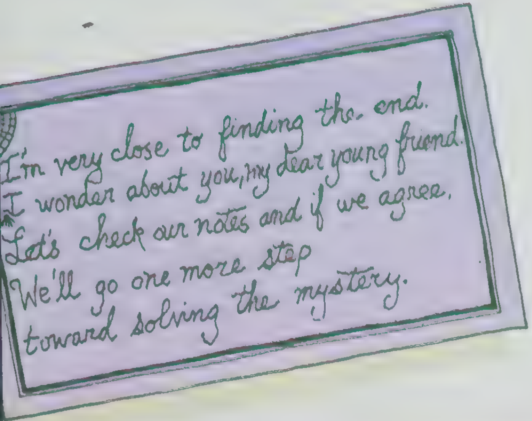
6. $\frac{4}{12} = \frac{■}{3}$ 1



c Draw your own models if you need them.

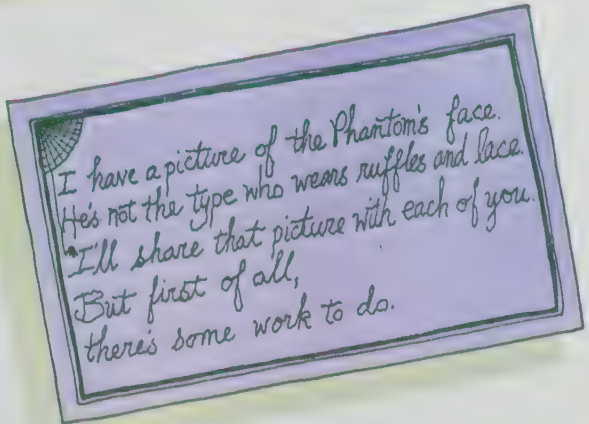
7. $\frac{2}{5} = \frac{■}{10}$ 4 **8.** $\frac{6}{9} = \frac{■}{3}$ 2 **9.** $\frac{5}{10} = \frac{■}{2}$ 1 **10.** $\frac{4}{6} = \frac{■}{9}$ 6 **11.** $\frac{10}{14} = \frac{■}{7}$ 5 **12.** $\frac{2}{12} = \frac{■}{18}$ 3

Exit 75 has led to another note from the little old lady.



1. One suspect is no longer alive. John Goldstone (page 310)
2. One suspect was eliminated on page 314. Blockhead
3. One man less on the list after page 319. Sergeant Sargent
4. Another was crossed off the list on page 320 or page 321. Birdman
5. Still another was freed on page 323. Weird Wolf
6. And one more suspect was eliminated on page 327. Franksanwine

There are only two suspects left. Who are they?
Baron Von Drup and Count Dragulot



goal Summarizing the clues to the phantom

page 331 We're sure even you don't know which suspects are left—unless you used a check list. Praise any of the youngsters who had this foresight. No fair telling anyone else. Let each person learn by experience.

lesson Pages 332, 333

goal Practice in renaming a fraction with its simplest name or as a mixed number

memo Page 333 is dependent on the work from page 332.

page 332 Check progress after the first row in each set. You may find people who need to review some of the techniques for renaming found in chapter 11. Group these youngsters for review while the others proceed independently.

To get the picture of the Phantom in focus, complete the problems in set 1. Write the simplest name for each fraction.

SET 1

1. $\frac{2}{6}$ $\frac{1}{3}$	2. $\frac{6}{8}$ $\frac{3}{4}$	3. $\frac{5}{10}$ $\frac{1}{2}$	4. $\frac{4}{10}$ $\frac{2}{5}$	5.
6. $\frac{8}{18}$ $\frac{4}{9}$	7. $\frac{10}{12}$ $\frac{5}{6}$	8. $\frac{6}{20}$ $\frac{3}{10}$	9. $\frac{0}{3}$ 0	10.
11. $\frac{6}{16}$ $\frac{3}{8}$	12. $\frac{2}{10}$ $\frac{1}{5}$	13. $\frac{7}{7}$ 1	14. $\frac{8}{12}$ $\frac{2}{3}$	15.
16. $\frac{20}{25}$ $\frac{4}{5}$	17. $\frac{2}{20}$ $\frac{1}{10}$	18. $\frac{6}{15}$ $\frac{2}{5}$	19. $\frac{3}{18}$ $\frac{1}{6}$	20.

SET 2

You'll need answers from set 2 also. Write a mixed number for each of these.

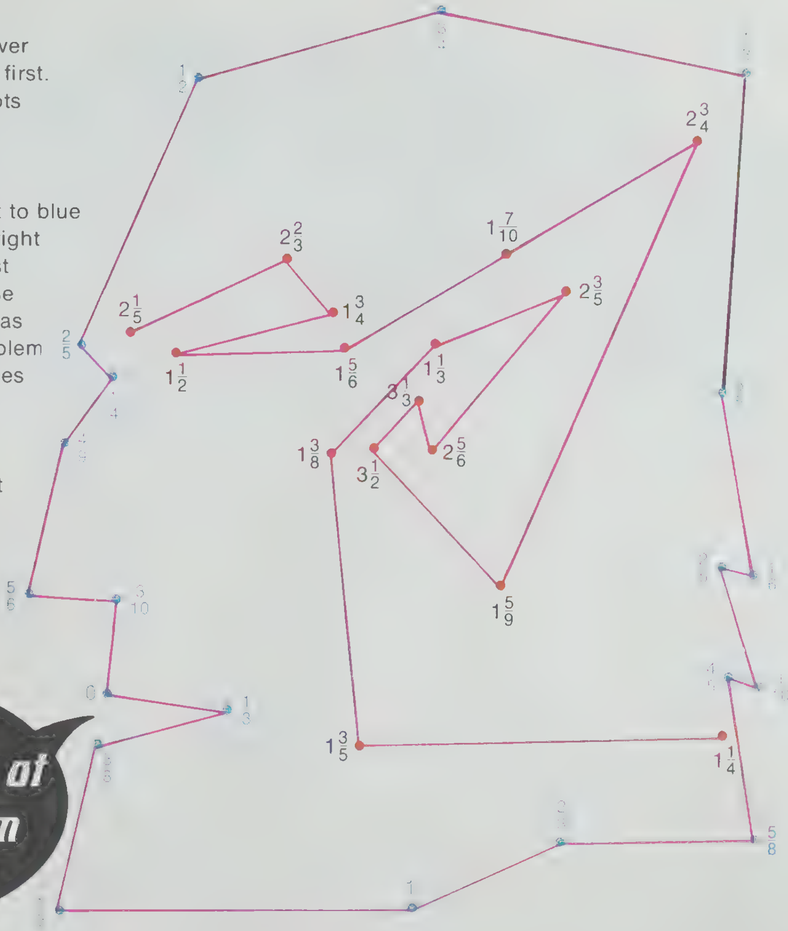
1. $\frac{5}{4}$	2. $\frac{8}{5}$	3. $\frac{11}{8}$	4. $\frac{4}{3}$
5. $\frac{13}{5}$	6. $\frac{17}{6}$	7. $\frac{10}{3}$	8. $\frac{7}{2}$
9. $\frac{14}{9}$	10. $\frac{11}{4}$	11. $\frac{17}{10}$	12. $\frac{11}{6}$
13. $\frac{6}{4}$	14. $\frac{14}{8}$	15. $\frac{24}{9}$	16. $\frac{22}{10}$

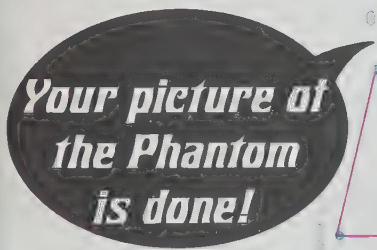
332

ace a plain sheet of paper over
is page. Trace the blue dots first.
your paper slips, the blue dots
ll help you get your paper
the right place again.

he answers for set 1 are next to blue
ots. Find the number to the right
a dot that matches your first
swer. Start there. Draw a line
om that dot to the dot that has
e same number as your problem
Continue in order. These lines
ll show you part of the
hantom's face.

he answers for set 2 are next
the red dots. Find the
umber to the right of a red
ot that matches your first
swer. Connect the dots
order.





333

goal Discovering how the phantom looks

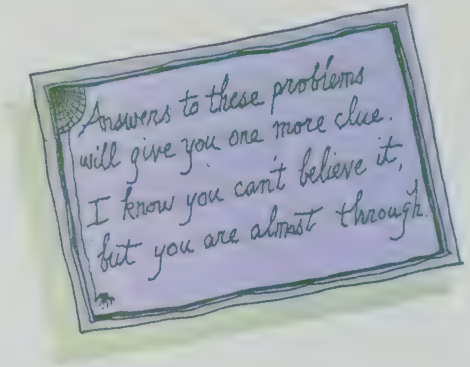
page 333 Make sure the paper used to cover page 333 is thin enough so pupils can easily see the numbers. There are a lot of dots to connect. The Phantom is such a lovely character! Every child deserves to get a good picture. He will unless the tracing paper gives him a hard time.

Isn't he a handsome phantom? The youngsters have worked hard to complete this picture. Let them know what a **great job** they've done!

lesson Pages 334, 335, 336

goal Diagnosis of ability and practice in adding and subtracting fractions with a like denominator

page 334 Recopying the problems is a lot of writing and is unnecessary. Direct the youngsters to write only the answers to these problems and leave sufficient space to decode the message.



Diagnostic check on addition and subtraction of fractions

Add. Write the answers in the order in which they occur. Then decode.

1. $\frac{4}{5}+\frac{1}{5}$ $\frac{5}{5}$
2. $\frac{3}{8}+\frac{2}{8}$ $\frac{5}{8}$
3. $\frac{1}{4}+\frac{1}{4}$ $\frac{2}{4}$
4. $\frac{2}{10}+\frac{1}{10}$ $\frac{3}{10}$
5. $\frac{10}{15}+\frac{3}{15}$ $\frac{13}{15}$
6. $\frac{2}{8}+\frac{3}{8}$ $\frac{5}{8}$
7. $\frac{1}{15}+\frac{6}{15}$ $\frac{7}{15}$
8. $\frac{20}{100}+\frac{1}{100}$ $\frac{21}{100}$
9. $\frac{3}{5}+\frac{2}{5}$ $\frac{5}{5}$
10. $\frac{1}{9}+\frac{3}{9}$ $\frac{4}{9}$
11. $\frac{4}{8}+\frac{2}{8}$ $\frac{6}{8}$
12. $\frac{2}{10}+\frac{1}{10}$ $\frac{3}{10}$
13. $\frac{7}{12}+\frac{8}{12}$ $\frac{15}{12}$
14. $\frac{4}{12}+\frac{1}{12}$ $\frac{5}{12}$
15. $\frac{1}{10}+\frac{2}{10}$ $\frac{3}{10}$
16. $\frac{5}{100}+\frac{16}{100}$ $\frac{21}{100}$
17. $\frac{3}{9}+\frac{1}{9}$ $\frac{4}{9}$
18. $\frac{2}{5}+\frac{3}{5}$ $\frac{5}{5}$
19. $\frac{1}{10}+\frac{2}{10}$ $\frac{3}{10}$

Subtract.

1. $\frac{15}{15}-\frac{2}{15}$ $\frac{13}{15}$
2. $\frac{7}{12}-\frac{5}{12}$ $\frac{2}{12}$
3. $\frac{5}{9}-\frac{2}{9}$ $\frac{3}{9}$
4. $\frac{6}{9}-\frac{1}{9}$ $\frac{5}{9}$
5. $\frac{4}{10}-\frac{1}{10}$ $\frac{3}{10}$
6. $\frac{90}{100}-\frac{69}{100}$ $\frac{21}{100}$
7. $\frac{6}{9}-\frac{2}{9}$ $\frac{4}{9}$
8. $\frac{3}{8}-\frac{2}{8}$ $\frac{1}{8}$
9. $\frac{8}{10}-\frac{5}{10}$ $\frac{3}{10}$
10. $\frac{80}{100}-\frac{59}{100}$ $\frac{21}{100}$
11. $\frac{7}{9}-\frac{3}{9}$ $\frac{4}{9}$
12. $\frac{6}{9}-\frac{3}{9}$ $\frac{3}{9}$
13. $\frac{12}{15}-\frac{5}{15}$ $\frac{7}{15}$
14. $\frac{4}{4}-\frac{1}{4}$ $\frac{3}{4}$

CODE (Careful! There's a trick in the message.)

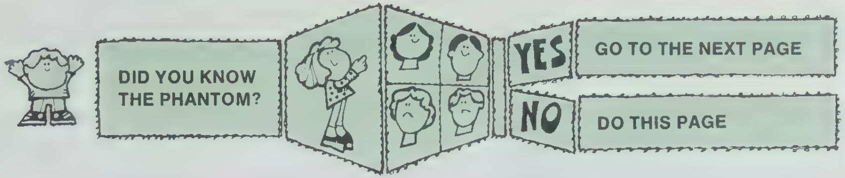
A	B	D	E	H	I	M	N	O	P	R	S	T	U	V	Space
$\frac{7}{15}$	$\frac{3}{4}$	$\frac{5}{9}$	$\frac{2}{4}$	$\frac{5}{8}$	$\frac{15}{12}$	$\frac{6}{8}$	$\frac{21}{100}$	$\frac{4}{9}$	$\frac{13}{15}$	$\frac{3}{9}$	$\frac{5}{12}$	$\frac{5}{5}$	$\frac{2}{12}$	$\frac{1}{8}$	$\frac{3}{10}$

Who is the Phantom? Go to the next page.

THE PHANTOM IS NOT PURD NOV NORAB.

The Phantom is Count Dragulot.

334



Diagnose specific skill to be practiced.

Find the answer to each of these problems.

a $\frac{2}{9} + \frac{5}{9}$ $\frac{7}{9}$ **b** $\frac{6}{12} - \frac{2}{12}$ $\frac{4}{12}$ or $\frac{1}{3}$

Use the key on page 336 to check your answers. For each problem you did incorrectly, do the set of problems below with the same letter as the problem you missed.

Skill: Adding fractions with like denominators

a Add.

1. $\frac{2}{4} + \frac{1}{4}$ $\frac{3}{4}$	**2.** $\frac{1}{6} + \frac{3}{6}$ $\frac{4}{6}$	**3.** $\frac{4}{6} + \frac{1}{6}$ $\frac{5}{6}$	**4.** $\frac{4}{8} + \frac{3}{8}$ $\frac{7}{8}$	**5.** $\frac{3}{10} + \frac{6}{10}$ $\frac{9}{10}$
6. $\frac{3}{9} + \frac{2}{9}$ $\frac{5}{9}$	**7.** $\frac{5}{12} + \frac{2}{12}$ $\frac{7}{12}$	**8.** $\frac{6}{12} + \frac{5}{12}$ $\frac{11}{12}$	**9.** $\frac{5}{14} + \frac{7}{14}$ $\frac{12}{14}$	**10.** $\frac{7}{16} + \frac{5}{16}$ $\frac{12}{16}$

Skill: Subtracting fractions with like denominators

b Subtract.

11. $\frac{5}{6} - \frac{2}{6}$ $\frac{3}{6}$	**12.** $\frac{5}{9} - \frac{3}{9}$ $\frac{2}{9}$	**13.** $\frac{3}{4} - \frac{2}{4}$ $\frac{1}{4}$	**14.** $\frac{7}{12} - \frac{4}{12}$ $\frac{3}{12}$	**15.** $\frac{7}{12} - \frac{3}{12}$ $\frac{4}{12}$
16. $\frac{7}{8} - \frac{6}{8}$ $\frac{1}{8}$	**17.** $\frac{7}{10} - \frac{2}{10}$ $\frac{5}{10}$	**18.** $\frac{11}{12} - \frac{6}{12}$ $\frac{5}{12}$	**19.** $\frac{11}{12} - \frac{9}{12}$ $\frac{2}{12}$	**20.** $\frac{14}{15} - \frac{9}{15}$ $\frac{5}{15}$

Now that you've had more practice, you should be able to correct your errors on page 334. Then you'll know who the Phantom is. After that, go to page 336.

335

goal Practice in adding and subtracting fractions with a like denominator

page 335 Good grief! More practice after page 334? Yep—**but** only if it is absolutely necessary in order to master this skill.

goal Providing mixed practice with word problems

page 336 This page is really just for fun. We do hope the kids will enjoy these problems and maybe giggle once or twice.

This chapter is certainly not a typical textbook chapter. Here are some things that are different about it:

- It's self-study.
- It's a lot of reading.
- It's corny.

Before judging us too harshly, please ask yourself these questions:

- Did the chapter work?
- Was the diagnosis effective?
- Did it motivate?
- Did high motivation minimize reading problems?

EVERYONE—
pupils and teachers alike—
DESERVES PRAISE
FOR THE HARD WORK DONE!

Answer Key

page
310 **a** $462.24 **b** $47.55 **c** $15.00
d $1463.13 **e** $15,000.00 **f** account closed
g $548.73 **h** $191.91

311 **a** 17 **b** 35 **c** 389 **d** 344 **e** 178

312 top: **a** 493,281 **b** 514,008
c 459,787
bottom: **a** 32,715 **b** 334,088
c 681,314

315 **a** 92 **b** 730 **c** 700
d 12,815 **e** 117,332 **f** 764,051

318 **1.** 1315 **2.** 10,315 **3.** 1315

319 **1.** 17 **2.** 23 **3.** 78 **4.** 108
5. 116 **6.** 1299 **7.** 2280 **8.** 19,867

321 **a** 2718 **b** 3510 **c** 34,905

325 **a** 53 R1 **b** 122 **c** 686 R5

328 **a** $\frac{7}{10} > \frac{3}{5}$ **b** $\frac{2}{7} < \frac{1}{3}$ **c** $\frac{3}{5} > \frac{4}{7}$

330 **a** $\frac{1}{2} = \frac{2}{4}$ **b** $\frac{3}{5} = \frac{9}{15}$ **c** $\frac{3}{4} = \frac{12}{16}$

335 **a** $\frac{7}{9}$ **b** $\frac{4}{12}$ or $\frac{1}{3}$

Some more interesting information about the 7 unfriendly people who lived in the tiny mountain town:

1. During an especially difficult operation, Franksanwine took 42 jolts of electricity each hour for 13 hours straight. How many jolts in all did he take? 546
2. Count Dragulot's mummy kept getting lost in the count's huge house. The count had to go look for his mummy 9999 times the first year, 6752 times the next year, and 728 times the last year. How many times did he have to go look for his mummy before he bought his bloodhound to do the job for him? 17,479
3. Birdman ate a very special diet. In the past month (31 days) he ate 26 bushels of birdseed each day, 17 pecks of berries each day, and 12 boxes of Brady's Birdy Biscuits each day. How many bushels of birdseed, how many pecks of berries, and how many boxes of Brady's Birdy Biscuits did he eat in all? 806 bushels of birdseed, 527 pecks of berries, and 372 boxes of biscuits
4. Blockhead joined the circus once. He was shot out of a cannon 9 times a day. He quit after he had been shot 774 times. Enough is enough! How many days did he work? 86
5. Weird Wolf's favorite store had a sale on wolfsbane. If you buy 125 packages you get a FREE silver bullet. Each package contained 23 grams of wolfsbane. How many grams of wolfsbane did Weird Wolf buy to his get free silver bullet? 2875

336

RESOURCES

additional learning aids

concept—chapter objectives 8, 9, 13

SRA products

Mathematics Involvement Program, SRA (1971)
  Cards: 154, 85

operation—chapter objectives 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 15

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
  Spirit masters: P 5, 6, 7
  W 11, 14
Computapes, SRA (1972)
  Module 4, Lessons: MD 26, 35, 36
  Module 5, Lessons: Fr 9, 13, 14
Computational Skills Development Kit, SRA (1965)
  Division cards: 1, 2, 3, 4, 5, 6, 7, 8
Cross-Number Puzzles (Whole Numbers), SRA (1966)
  Division cards: 1, 2, 3, 4, 5, 6, 7, 8, 9
Skill Modes in Mathematics, SRA (1974)
  Level I, Molecules: A, B, C, E, F, I, J
  Level II, Molecules: A, B, E, F, K
Skill through Patterns, level 4, SRA (1974)
  Spirit masters: 31, 45, 46, 53, 61, 64, 72

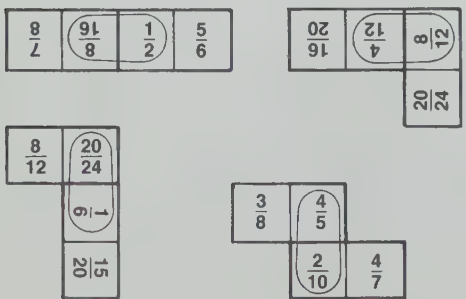
fractionoes

This game is for two, three or four players.

Here is what you play with:
40 rectangular pieces called fractionoes. Each piece is divided in half. There is a fraction printed on each half.

One person is the dealer. He deals five fractionoes facedown to each player, seven if only two are playing. He places one fractiono faceup in the center of the playing area. The remaining pieces belong facedown in a pile to the side. If possible, each player holds his fractionoes so no one else can see what numbers are printed on them.

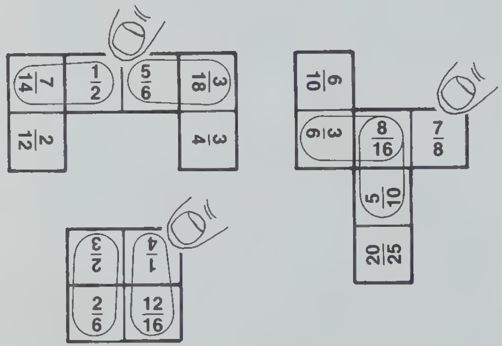
The playing now begins. The person to the left of the dealer goes first. He takes one of his fractionoes and places it faceup in the playing area so:
(1) One square of one fractiono and one square of the other match sides.
(2) The fractions printed on the sides that match have a sum of 1.



Notice that the fractions don't have to appear right-side up.

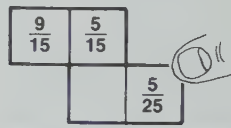
The players play in turn. Whenever it is a person's turn to play, he places one of his fractionoes in the playing area so that one of its squares touches the side of one other fractiono, and so that the fractions printed on the two adjacent squares have a sum of 1. Then it is the next player's turn.

Sometimes a player places a fractiono so that there is more than one pair of adjacent squares. In this case it is necessary for each pair of fractions to add up to 1.



Sometimes a player whose turn it is can't play any of his fractionoes. When someone gets stuck like this, he must take a fractiono from the pile at the side of the playing area and then play. If he's still stuck, he takes another fractiono and again tries to play. He does this a third time if necessary. If a player still can't play after taking three fractionoes from the pile, he passes without playing. (If it happens, when a player is stuck, that there are no fractionoes left in the pile, he passes immediately.)

There are two of the forty fractionoes that have only one fraction printed on them. The place where the other fraction belongs is blank. When someone plays one of these two pieces, he assigns a fraction to the blank square—any fraction he wishes. For example, in the diagram below, the player can make the indicated play if he assigns $\frac{2}{3}$, or an equivalent of $\frac{2}{3}$, to the blank square.



Once a blank has been assigned a fraction, that fraction value is unchanged until the end of the game.

Sometimes a player makes a mistake. The fractions don't add up to one. His opponents should point out his error. The player who made the mistake must take back his fractiono and he must draw two fractionoes from the pile at the side of the playing area. That is his penalty. He passes up his turn without playing again.

The game continues until one person plays all his fractionoes. He's the winner. If everyone gets stuck, unable to make a play, and the pile at the side of the playing area is all gone, then the winner is the one with the fewest fractionoes remaining.

playing pieces for fractionoes

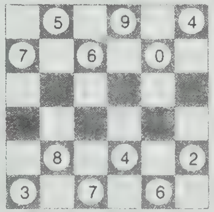
Remove this sheet from the guide. Paste it on light-weight cardboard. Then cut the playing pieces apart.

$\frac{1}{10}$	$\frac{3}{6}$	$\frac{2}{12}$	$\frac{4}{9}$	$\frac{4}{6}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{2}{5}$
$\frac{12}{18}$	$\frac{6}{10}$	$\frac{7}{14}$	$\frac{3}{12}$	$\frac{5}{9}$	$\frac{1}{4}$	$\frac{3}{18}$	$\frac{2}{9}$
$\frac{16}{20}$	$\frac{5}{10}$	$\frac{5}{8}$	$\frac{12}{20}$	$\frac{6}{18}$	$\frac{7}{8}$	$\frac{5}{25}$	$\frac{4}{7}$
$\frac{4}{12}$	$\frac{20}{25}$	$\frac{10}{15}$	$\frac{4}{8}$	$\frac{9}{12}$	$\frac{8}{16}$		$\frac{2}{10}$
$\frac{3}{9}$	$\frac{5}{7}$	$\frac{9}{10}$	$\frac{2}{4}$	$\frac{2}{7}$	$\frac{8}{10}$	$\frac{15}{18}$	$\frac{1}{9}$
$\frac{3}{5}$	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{1}{7}$	$\frac{3}{15}$	$\frac{7}{10}$	$\frac{1}{5}$	$\frac{6}{15}$
$\frac{15}{20}$	$\frac{12}{16}$	$\frac{3}{7}$	$\frac{9}{18}$	$\frac{6}{9}$	$\frac{10}{12}$	$\frac{7}{9}$	$\frac{4}{5}$
$\frac{1}{6}$	$\frac{2}{6}$	$\frac{4}{16}$	$\frac{6}{7}$	$\frac{8}{20}$	$\frac{6}{8}$	$\frac{2}{8}$	$\frac{3}{8}$
$\frac{5}{6}$		$\frac{12}{15}$	$\frac{5}{15}$	$\frac{20}{24}$	$\frac{6}{12}$	$\frac{4}{10}$	$\frac{10}{20}$
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{3}{10}$	$\frac{9}{15}$	$\frac{8}{12}$	$\frac{1}{8}$	$\frac{4}{24}$	$\frac{8}{9}$

number checkers

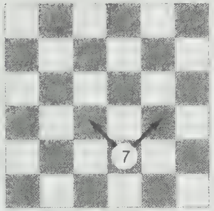
Number Checkers is a game for two people. It is played with 20 checkers numbered from 0 to 9. (Use standard checkers. Put a piece of tape on one side. Mark the numerals on the tape.) The playing board is placed so that each player has a dark square in his lower left-hand corner. One player takes the red pieces and the other player the black.

At the beginning of the game each player turns over his checkers so that the numerals are facedown. Then each player places six of the checkers on the playing board on the dark squares of the two rows nearest him. Each player then turns over his checkers so that the numeral sides are up.

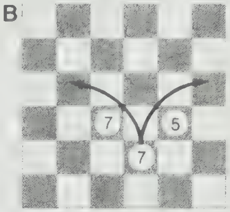


Red and black move alternately, red moving first. Each player in his turn moves one of his own checkers. A checker can be moved *forward* to a vacant, adjacent square (diagram A below). If an adjacent square in front of a checker is occupied by a hostile piece, if the checker's number is greater than or equal to the number on the hostile piece, and if the square behind the hostile piece is vacant, then the checker can and must jump over the hostile piece, landing on the vacant square (diagram B below). The hostile piece is removed from the board. It is said to be captured.

A



B

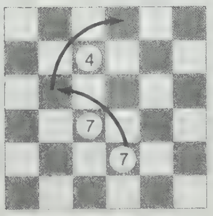


If the checker can continue jumping in this fashion from the square it has just landed on, it must do so (diagram C below). The player removes from the board all hostile pieces he jumps over this way.

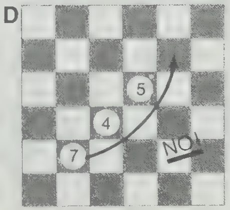
A player is obligated to jump if he can. However, if two jumps are possible, he is free to pick the one he wants.

A player can never jump over one of his own checkers. Neither can he jump over a hostile piece if that piece has a larger number than his own checker. Also, if two pieces are back-to-back, they cannot be jumped (diagram D below).

C



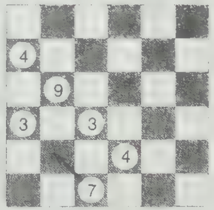
D



A numbered checker can never move backward. It can only move forward, away from the side of the board it starts at. This means, of course, that the red and black checkers move in opposite directions. When a checker reaches the far edge of the board, however, it is turned over. That piece becomes a king.

A king moves and jumps like a numbered checker in all respects except one. A king is not limited to the forward direction. It can move and jump backward as well as forward. In numerical value a king should be considered superior to any numbered checker, although it has no number printed on it. It can jump any numbered checker, but no numbered checker can jump it. Only another king can jump a king.

A player wins the game if he captures all his opponent's pieces. He also wins if he blocks his opponent's pieces so that none of them can move.



In this diagram black wins with the move indicated. For red is now left without a move, since a 3 cannot jump a 7.

my name ______________ my teacher's name ______________

	I can do it everytime.	I will be able to do it soon.
I can read and write any number less than one million.	______	______
I can also tell the value of each digit in a number such as 978 654.	______	______

I can estimate the answer and find the exact answer for any problems such as these:

$$\begin{array}{r} 3246 \\ +2739 \\ \hline \end{array} \quad \begin{array}{r} 4532 \\ +4198 \\ \hline \end{array} \quad \begin{array}{r} 6717 \\ +2988 \\ \hline \end{array} \quad \begin{array}{r} 5098 \\ +4982 \\ \hline \end{array}$$
______ ______

$$\begin{array}{r} 314 \\ 452 \\ +123 \\ \hline \end{array} \quad \begin{array}{r} 256 \\ 109 \\ +532 \\ \hline \end{array} \quad \begin{array}{r} 567 \\ 148 \\ +396 \\ \hline \end{array} \quad \begin{array}{r} 465 \\ 987 \\ +101 \\ \hline \end{array}$$
______ ______

$$\begin{array}{r} 2134 \\ 1503 \\ +4360 \\ \hline \end{array} \quad \begin{array}{r} 5016 \\ 2744 \\ +1230 \\ \hline \end{array} \quad \begin{array}{r} 6437 \\ 1586 \\ +2097 \\ \hline \end{array} \quad \begin{array}{r} 7435 \\ 5797 \\ +8598 \\ \hline \end{array}$$
______ ______

$$\begin{array}{r} 7854 \\ -\ 541 \\ \hline \end{array} \quad \begin{array}{r} 8291 \\ -\ 137 \\ \hline \end{array} \quad \begin{array}{r} 5008 \\ -\ 699 \\ \hline \end{array} \quad \begin{array}{r} 3050 \\ -\ 657 \\ \hline \end{array}$$
______ ______

$$\begin{array}{r} 65 \\ \times 21 \\ \hline \end{array} \quad \begin{array}{r} 72 \\ \times 54 \\ \hline \end{array} \quad \begin{array}{r} 57 \\ \times 48 \\ \hline \end{array} \quad \begin{array}{r} 89 \\ \times 67 \\ \hline \end{array}$$
______ ______

$3\overline{)963} \quad 7\overline{)219} \quad 8\overline{)825} \quad 6\overline{)609}$
______ ______

I can show that I can switch the order of numbers when I multiply and still get the same answer. ______ ______

I can also show that I can group 3 numbers differently when I multiply and still get the same answer. ______ ______

I can find the answer to problems such as these:

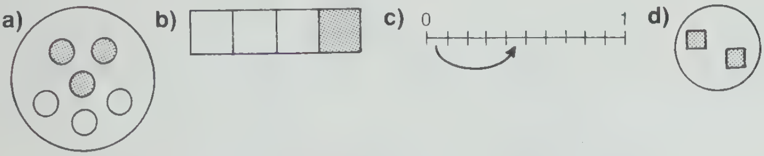
a) $12 + 3 =$ ______ b) $8 +$ ______ $= 15$
c) $27 - 8 =$ ______ d) $20 -$ ______ $= 10$
e) $15 \times 4 =$ ______ f) $4 \times$ ______ $= 36$
g) $62 \div 6 =$ ______ h) $48 \div$ ______ $= 8$
______ ______

I can find answers for word problems such as these:

a) Jack collected 57 kilograms of scrap paper one week and 153 kilograms the next week. How much paper did he collect in those two weeks? ______ ______
b) Jan bought 3 packs of paper. Each pack cost 29 cents including tax. How much did she pay for the paper? ______ ______
c) The groceries cost $6.37. How much change would be returned from a $10 bill? ______ ______

I can name the fraction shown in each of the pictures

a) b) c) d)



And I can tell which of these fractions is more or less than any of the others.

I can rename fractions such as $\frac{2}{2}$, or $\frac{8}{8}$.

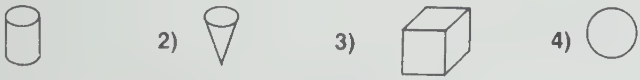
I can find answers to problems like these:

a) $\frac{1}{9} + \frac{3}{9}$ b) $\frac{2}{3} + \frac{1}{3}$ c) $\frac{7}{8} - \frac{5}{8}$ d) $\frac{3}{4} - \frac{0}{4}$

I can match the name of each object with the picture of the object:

a) sphere b) cone c) cylinder d) cube

1) 2) 3) 4)



I know the best unit of measure to use if I have to measure any of the following lengths:
a) my room b) a shoelace c) distance to South America

I know the best unit of measure to use if I have to find the mass of any of the following things:
a) a letter b) myself c) an elephant

I can name objects that could be measured with each of the following units of measure:

a) centimetre b) metre c) kilometre

I can answer these questions about measurement:
a) 1 metre = _______ centimetres
b) 1 kilometre = _______ metres

I can keep track and report the number of times heads turn up when I flip a coin 20 times.

my name ______________________________

date ____________________

individual progress chart for ____________________
(pupil name)

recorded by ____________________ —fourth-level teacher

evaluation of major learning goals	date of 1st try	date of 2d try	comments
Knows the value of each digit of a 6-digit number (p. 7)			
Knows how to appropriately round numbers (p. 13)			
*Knows how to read and write any 6-digit number (p. 20)			
Knows how to add two 2-digit numbers (p. 27)			
Knows how to add two 3-digit numbers (p. 31)			
Knows how to subtract two 3-digit numbers (p. 36)			
*Knows how to add and subtract 3-digit numbers (p. 45)			
Knows the multiplication facts (p. 57)			
Knows the division facts (p. 68)			
*Can use facts to solve a one-step word problem (p. 72)			
Can use appropriate units of measure and name equivalent measures (p. 79)			
Knows how to add or subtract measurements (p. 87)			
Knows how to multiply a 2-digit number by a 1-digit number (p. 106)			
Knows how to multiply two 2-digit numbers to solve a one-step word problem (p. 117)			
*Knows how to multiply any two 2-digit numbers (p. 120)			
Can name a fraction that describes a fraction model and can compare two fractions with common denominators (p. 128)			
Knows how to add and subtract fractions with common denominators (p. 140)			
*Can order, compare, add, and subtract fractions with common denominators (p. 144)			
Knows how to identify solid shapes (p. 152)			
*Can identify rectangular prisms, cylinders, cones, and spheres (p. 159)			
*Knows how to add two, three, or four 4-digit numbers, subtract two 3-digit numbers, and multiply a 3-digit number by a 2-digit number (p. 188)			
Can estimate to find the answer to a division problem (p. 204)			
Knows how to divide a 3-digit number by a 1-digit number (p. 210)			
*Knows how to estimate, divide, and check division (p. 216)			
Can write a math sentence (p. 221)			
Can find the solution to an open math sentence (p. 223)			
Can write true, false, and open sentences (p. 228)			
*Knows how to write a math sentence and solve a one-step word problem (p. 235)			
Knows how to rename fractions (p. 244)			
Knows how to add and subtract fractions with common denominators and rename the answers if appropriate (p. 249)			
Can rename fractions that name mixed numbers (p. 253)			
*Knows how to add and subtract fractions and rename answers if appropriate (p. 258)			
*Knows some elements of consumer economics (p. 277)			
*Can perform an experiment and tally the information (p. 288)			
*Can identify congruent figures (p. 306)			

*Checkout

Other Learning Aids

whole-number concepts

Abacus board (Creative Publications) Counting board useful for teaching place value
Abacus Spinner Game (Math Shop) Game designed to provide practice in recognizing and understanding place value
Chip Trading (Scott Scientific) Game to develop an understanding of place value
Counting Chips (Creative Publications) Plastic chips useful for teaching simple computations
Cuisenaire® Rods (Cuisenaire) Centimetre rods designed to provide practice with arrays
Fundamath (Ideal) Boards with beads that demonstrate addition and subtraction
Japanese Abacus (Creative Publications) An abacus designed to teach place value and basic number operations
Multifax & Quotient (Math Shop) Game in which number sentences based on the multiplication facts are formed
Place Value I and II (Creative Publications) Self-correcting cards to provide practice in reading numbers through hundred millions
Ranko (Math Shop) Game designed to provide practice in ordering numbers
Tally Counter (Creative Publications) Adding device to help children understand place value

whole-number operations

Dial-A-Matic® Adding Machine (Sigma Scientific) A simple calculator for practice in addition and subtraction
I Win (sets 1, 2 and 3) (Scott, Foresman) Card game to provide practice in four basic operations with whole numbers
Mathfacts Games™ (Milton Bradley) Self-instructional and self-checking games that deal with basic multiplication and division facts
Motivator Activity Cards – Multiplication Facts (Singer/SVE) Laminated fact cards for practice with multiplication facts
Multiplying Machine (Math Shop) Self-checking machine to be used for practice with the multiplication facts
Napier's Rods (Sigma Scientific) Rods designed to provide practice in multiplication
Numble™ (Sigma Scientific) Crossword-type number game to reinforce the four basic operations with whole numbers
Numo (Math Shop) A bingo-type game to provide practice in addition and subtraction
Orbiting The Earth (Scott, Foresman) [multiplication and division] Game with vinyl playing field to provide practice in multiplication and division
Rally with Remainders (Math Shop) A self-correcting game providing division practice
Ting (SEE) A jigsaw puzzle for reinforcement of multiplication facts
Veri-Tech Senior (ETA) [addition, subtraction, division, and multiplication books] A self-checking device that provides practice with whole-number operations
Winning Touch (Ideal) Game that provides for reinforcement of multiplication facts

fractional-number concepts

Action Fraction Games (Constructive Playthings) Game designed to develop concepts and increase skills with fractional numbers
Experiments in Fractions (Math Shop) Activities that help the student understand, develop, and use vocabulary, notation, and operations
Fraction Bars Student Activity Book (Creative Publications) Games and activities designed to teach specific objectives for fractions
Fraction Dominoes (SEE) Game involving matching a fractional numeral with its model
Fraction Line Set (Sigma Scientific) A learning activity designed to help students visualize operations by computing with fraction strips
Fraction Wheel (Ideal) Circle showing fractions and their relationships by revolving disks
Student Fraction Sets (ETA) [Circular, Square] Activities that provide experiences in matching a unit with fractional parts and vice versa

geometry

Geoboards and Motion Geometry Resource Book and Activities (Scott, Foresman) Activities dealing with congruence, coordinates, transformations, and area
Geometry Figures and Solids (Creative Publications) Forms to facilitate understanding of basic geometric concepts
Learn to Fold – Fold to Learn (Lyons & Carnahan) Workbook that presents a variety of paper-folding activities
Mira (Creative Publications) An aid for investigating properties of plane geometry
Mira Math for Elementary School (Creative Publications) Activities to be used with the Mira
Polyhedra Model Kit (Creative Publications) Kit to make five basic polyhedra models
Soap-film Shapes (SEE) Wire shapes that are dipped in soapsuds to make six different geometric-shaped soap bubbles
Shape Tracers (Math Shop) Geometric shapes for tracing and practice in shape recognition

measurement

Easy Money (Milton Bradley) Game that provides practice in dealing with money
Good Time Mathematics (Holt, Rinehart & Winston) A multimedia program designed to give activity-based learning experiences
Learning About Measurement (Lyons & Carnahan) A workbook with activities using the metric and customary systems
Money Matters (Math Shop) Book of puzzles providing practice in problem solving
Pay the Cashier Game (Garrard Publishing) Board game centered around counting, adding, and subtracting money
Spin-A-Coin (Math Shop) Game dealing with place value and operations involving money
Spin-A-Yard (Math Shop) Game providing practice with linear measurement

statistics and probability

Block Graph (ESA) Demonstration set for introducing bar graphs, averages, and so on
Histogram Board (ESA) Board for making bar diagrams – to be used with Stern Unit Cubes
Probability Maze (ESA) A board to illustrate probability and statistics

problem solving and applications

Heads Up™ (Creative Publications) Game providing practice with equations
Number Sentence Games (Creative Publications) Booklet of worksheets that provide practice with operations and number combinations
True or False (ESA) A game for deciding true or false statements

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the study of numbers

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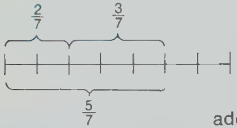
GLOSSARY

addition Putting together



$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$

addend addend sum

congruent Two figures that are the same size and shape are congruent. Trace and match to test.

congruent triangles

decimal Another name for some fractions

$\frac{12}{100} = .12$ ← decimal

decimal point

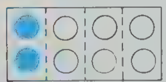
division quotient → 91 R2 ← remainder

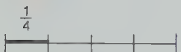
fact: $45 \div 5 = 9$

problem:

```
       1
      90
 5 )457
  − 450   (90 × 5)
      7
    − 5   (1 × 5)
      2
```

fraction A number that tells how much





1 ← numerator
4 ← denominator

$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$. $\frac{1}{2}$ can be renamed as $\frac{3}{6}$.

$\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$. $\frac{1}{2}$ is the simplest name for $\frac{3}{6}$

greatest common factor
factors of 12: 1, 2, 3, 4, 6, 12
factors of 16: 1, 2, 4, 8, 16
4 is the greatest common factor of 12 and 16.

math sentence

true: $3 + 4 = 7$ missing factor: $3 \times \square = 18$
false: $4 \times 2 > 10$ equality: $14 - 6 = 8$
open: $3 + \square = 6$ inequality: $14 + 2 < 20$

mixed number Another name for some fractions

$\frac{7}{6} = 1\frac{1}{6}$ $\frac{13}{2} = 6\frac{1}{2}$

mixed numbers

multiple Another name for product

The multiples of 5 are 0, 5, 10, 15, 20, and so on because $0 \times 5 = 0$, $1 \times 5 = 5$, $2 \times 5 = 10$, and so on.

multiplication

```
   24   factor
 ×  6   factor
   24   (6 × 4)
+ 120   (6 × 20)
  144   product
```

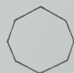
operation Using a pair of numbers to get another number

addition + (plus)	subtraction − (minus)
multiplication × (times)	division ÷ (divided by)

plane figure

all sides congruent

equilateral triangle



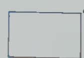
octagon (8 sides)

corner

side

hexagon (6 sides)

square corners



rectangle

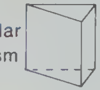
diagonal

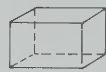
square

prediction A guess of what will happen after doing good thinking and research

prism A special solid shape

triangular prism





rectangular prism

relation symbol $=, \neq, >, <$

$4 = 4$ 4 equals 4	$6 \neq 3$ 6 is not equal to 3
$8 > 2$ 8 is greater than 2	$1 < 5$ 1 is less than 5

rounding

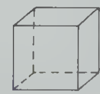
37 rounded to the nearest ten is 40.
37 is rounded up to 40.

428 rounded to the nearest hundred is 400.
428 is rounded down to 400.

solid shape

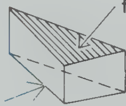
cone

sphere



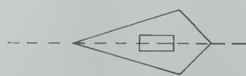
cube

face



edge vertex

symmetry plane figure can be folded on a line to match exactly.



line of symmetry

tally A way of recording a count

23  ← tally marks

INDEX

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